Graded Problem Set 4

Date: 13.03.2014 Due by 18:00 – 20.03.2014

Instructions : In the first page, write your name, Sciper number, list of collaborators, and the name of the other (eventual) person which is in your group for the last problem. Then, write the solution for each problem on a separate page. For the last problem you will be required to submit a small report together with some code. Send everything to our email addresses `{marco.mondelli, ruediger.urbanke}@epfl.ch`

Rules : As concerns the first 4 problems, you are allowed and encouraged to discuss these problems with your colleagues. However, each of you has to write down her solution in her own words. If you collaborated on a homework, write down the name of your collaborators and your sources in the space below. No points will be deducted for collaborations. But if we find similarities in solutions beyond the listed collaborations we will consider it as cheating. Please note that EPFL has a VERY strict policy on cheating and you might be in BIG trouble. It is simply not worth it. As concerns the last problem (Problem 5), you can work in groups of two people, if you wish. A single report has to be handed in for each of the groups.

Grading : Each of the first 4 problems is worth 10 points. The last problem is worth 40 points.

Problem 1. *[10 points]* Show that a tree without a vertex of degree 2 has more leaves than other vertices.

Problem 2. *[10 points]* Consider a connected graph $G = (V, E)$ and its spanning tree $G'(V, E' \subset E)$. Let $d_G(u, v)$, $d_{G'}(u, v)$ denote the distance of two vertices u, v in G and in G' respectively. Show that unless G is a tree itself there is no spanning tree G' for which it holds that $d_G(u, v) = d_{G'}(u, v)$ for every $u, v \in V$.

Problem 3. *[10 points]* Show that a sequence $d_1, d_2, \ldots, d_n$, $n \geq 2$ of positive integers is the degree sequence of a tree with n vertices if and only if $\sum_{i=1}^{n} d_i = 2(n-1)$.

Problem 4. *[10 points]* Let G be a graph s.t. the degree of each vertex is ≥ 3. Then, there is a cycle of even length.

Problem 5. *[40 points]* In this problem, you will need to use your programming skills. You can use your favorite language, e.g., C, C++, Matlab, Mathematica. However, you have to use only low-level functions. For example, if you need to simulate a graph with certain characteristics, you cannot just write one line of Mathematica code or two lines of C just invoking a specific library which does the job for you :-)

Attach to the report an electronic version of your code. Comment your code, so that we can understand what you are doing. Add also a README.txt, so that we know how to run it on our machines. Write in the report the results of the program and, when requested, a comment/explanation of those results.

In this exercise, we will deal with a special class of graphs. In particular, let $G(n, p)$ be a graph with n vertices s.t. for any pair of nodes the edge connecting them is included in the graph with probability p independently from every other edge.

1. *[10 points]* Evaluate the probability that $G(n, p)$ has m edges and compute the expected number of edges.
2. *[10 points]* Write a program which accepts p and n as input and outputs the random graph $G(n, p)$.
3. *[10 points]* Write a program which accepts p and n as input. Then, it outputs the size (number of nodes) of the largest connected component of $G(n, p)$.
4. *[10 points]* Try your program of point 3 for $n = 10000$ and for several values of p in the interval $(0, 2/n)$. For each value of p, repeat your random experiment $M = 50$ times and plot the average of the normalized size of the largest connected component (size of the largest connected component divided by n) as a function of p. What do you observe?