Problem Set 2

Date: 27.02.2014 Not graded

Problem 1. Let G be a graph and let $u, v \in V(G)$. Suppose that there is a walk from u to v. Show that there is a path from u to v.

Problem 2. Recall that A denotes the adjacency matrix of the graph G. Prove that the number of walks from v_i to v_j of length k in G is the (i, j)-th entry of A^k.

Problem 3. Show that any two longest paths in a connected graph have a vertex in common.

Problem 4. We say that a path is *Hamiltonian* if it visits each vertex exactly once. A tournament is a directed graph in which every two vertices are connected by exactly one directed edge in either of the two possible directions. Prove that every tournament has a Hamiltonian path. *Hint*: Use induction on the number of vertices.

Problem 5. We say that the circumference of a graph G, namely $\mathrm{circ}(G)$, is the length of any longest cycle in a graph. Let G be a graph with all n vertices of degree greater than or equal to k for some integer $k > 1$. Prove that $\mathrm{circ}(G) \geq k + 1$.