Discrete Structures EPFL, Fall 2013

Problem Set 11

Date: 29.11.2013 Not graded

Problem 1. A vending machine dispensing books of stamps accepts only \$1 coins, \$1 bills and \$2 bills. Let a_n denote the number of ways of depositing n dollars in the vending machine, where the order in which the coins and bills are deposited matters.

(a) Find a recurrence relation for a_n and give the necessary initial condition(s).

(b) Find an explicit formula for a_n by solving the recurrence relation in part (a).

Problem 2. Solve the following recurrence relations using the characteristic equation.

i) $a_n = 5a_{n-1} - 4a_{n-2}$, with $a_0 = 1$ and $a_1 = 0$.

ii) $a_n = 5a_{n-1} - 4a_{n-2}$, with $a_0 = 0$ and $a_1 = 1$.

iii) $a_n = -10a_{n-1} - 21a_{n-2}$, with $a_0 = 2$ and $a_1 = 1$.

iv) $a_n = a_{n-2}$, with $a_0 = 2$ and $a_1 = -1$.

v) $a_n = 2a_{n-1} + 2a_{n-2}$, with $a_0 = 0$ and $a_1 = 1$.

vi) $a_n = 2a_{n-1} - a_{n-2}$, with $a_0 = 3$ and $a_1 = 5$.

vii) $a_n = -6a_{n-1} - 11a_{n-2} - 6a_{n-3}$, with $a_0 = 0$, $a_1 = 1$, and $a_2 = 2$.

viii) $a_n = 10a_{n-1} - 37a_{n-2} + 60a_{n-3} - 36a_{n-4}$, with $a_0 = 0$, $a_1 = 0$, $a_2 = 1$, and $a_3 = 0$.

Problem 3. Solve the same recurrence relations of **Problem 2** using generating functions.

Problem 4. Find the coefficient of x^8 in the power series of each of the following functions:

(a) $\dfrac{1}{1-2x}$;

(b) $\dfrac{x^3}{1-3x}$;

(c) $\dfrac{1}{(1-x)^2}$;

(d) $\dfrac{x^2}{(1+2x)^2}$;

(e) $\dfrac{1}{1-3x^2}$;

(f) $x^3 \cdot \dfrac{5+2x-21x^2}{2x^3-x^2-2x+1}$.