

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Handout 7
Homework 7

Statistical Physics for Communication and Computer Science
April 10, 2013

Problem 1 (Density Evolution via Population Dynamics). In class we have seen density evolution (DE) for transmission over the BEC. This was relatively easy since in this case the “densities” are in fact numbers (erasure probabilities). For general channels, DE is more involved since it really involves the evolution of densities. These are the densities of messages which you would see at the various iterations if you implemented the BP message-passing decoder on an infinite ensemble for a fixed number of iterations.

A quick and dirty way of implementing DE for general channels is by means of a population dynamics approach. Here is how this works. Assume that transmission takes place over a given BMS channel and that we are using the (l, r) -regular Gallager ensemble. Pick a population size N . The larger N the more accurate will be your result but the slower it will be.

- (i) Pick an *initial* population, call it \mathcal{V}_0 . This set consists of N iid log-likelihoods associated to the given BMS channel, assuming that the transmitted bit is 1 (we are using spin notation here). More precisely, each sample is created in the following way. Sample Y according to $p(y | x = 1)$. Compute the corresponding log-likelihood value, call it L .
- (ii) Starting with $\ell = 1$, where ℓ denotes the iteration number, compute now the densities corresponding to the ℓ -th iteration in the following way.
- (iii) To compute \mathcal{C}_ℓ proceed as follows. Create N samples iid in the following way. For each sample, call it Y , pick $r - 1$ samples from $\mathcal{V}_{\ell-1}$ with repetitions. Let these samples be named X_1, \dots, X_{r-1} . Compute $Y = 2 \tanh^{-1}(\prod_{i=1}^{r-1} \tanh(X_i/2))$. Note, these are exactly the message-passing rules at a check node.
- (iv) To compute \mathcal{V}_ℓ proceed as follows. Create N samples iid in the following way. For each sample, call it Y , pick $l - 1$ samples from \mathcal{C}_ℓ with repetitions. Let these samples be named X_1, \dots, X_{l-1} . Further, pick a sample from \mathcal{V}_0 , call it C . Compute $Y = C + \sum_{i=1}^{l-1} X_i$. Note, these are exactly the message-passing rules at a variable node.

We think now of each set \mathcal{V}_ℓ and \mathcal{C}_ℓ as a sample of the corresponding distribution. E.g., in order to construct this distribution approximately we might use a histogram applied to the set. Recall, that we assume here the all-zero codeword assumption. Hence, in order to see whether this experiment corresponds to a successful decoding, we need to check whether in \mathcal{V}_ℓ all samples have positive sign and magnitude which converges (in ℓ) to infinity.

Implement the population dynamics approach for transmission over the BAWGNC(σ) channel using the $(3, 6)$ -regular Gallager ensemble. Estimate the threshold using this method. Plot the threshold on the same plot as the simulation results which you performed for your last homework. Hopefully this vertical line, indicating the threshold, is somewhere around where the error probability curves show a sharp drop-off.