

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Exercise 2

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Graph Theory Applications

Spring 2013

Problem 1. Give an example of an undirected graph (with no multiple edges) that (i) has a Hamiltonian cycle but no Eulerian trail, (ii) has an Eulerian circuit but no Hamiltonian cycle.

Problem 2. Consider a directed graph $G = (V, E)$ and assume that it is weakly connected. This means, the underlying undirected graph is connected. Prove that G is Eulerian if and only if $d^+(v) = d^-(v)$ for every vertex $v \in V$, where $d^+(v)$ denotes the outdegree and $d^-(v)$ the indegree of a vertex.

Problem 3. A tournament is a directed graph in which every two vertices are connected by exactly one directed edge in either of the two possible directions. Prove that every tournament has a Hamiltonian path. (*Hint*: Use induction on the number of vertices.)

Problem 4. Let G be a graph with all n vertices of degree greater than or equal to k for some integer $k > 1$. Prove that the circumference $G(G) \geq k + 1$.