Homework 9
Quantum Information and Computation

Exercice 1

Let $V$ a $2 \times 2$ unitary matrix. The "double control-$V$" gate denoted CCV is defined by the circuit

1a) Show that for all $2 \times 2$ unitary $U$ :

1b) Find $U$ that realizes the quantum Toffoli gate CCNOT? Give the explicit matrix $U$.

Exercice 2

Let $U$ a unitary matrix and $|u\rangle$ an eigen-vector : $U |u\rangle = \exp(2\pi i \phi) |u\rangle$. Consider the circuit :

3a) Calculate the output for the initial state $|0\rangle \otimes |u\rangle$.

3b) Calculate the probability to observe the first bit in the state $|0\rangle$ (at the output). Same question for the probability to observe it in the state $|1\rangle$. Same question for the probabilities to observe $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$, $\frac{|0\rangle - |1\rangle}{\sqrt{2}}$, $\frac{|0\rangle + i|1\rangle}{\sqrt{2}}$ et $\frac{|0\rangle - i|1\rangle}{\sqrt{2}}$ at the output.

3c) Suppose we replace $U$ by $U^k$, $k$ integer, in the circuit above. Let $\phi = 0, \phi_1\phi_2...\phi_t$ the binary expansion of $0 < \phi < 1$. How does one have to choose $k$ in order to determine the least significant bit $\phi_t$ with just one measurement?