Problem 1. (Equivalent Representations)

A bandpass signal $x(t)$ may be written as $x(t) = \sqrt{2}\Re\{x_E(t)e^{j2\pi f_0t}\}$, where $x_E(t)$ is the baseband equivalent of $x(t)$.

1. Show that a signal $x(t)$ can also be written as $a(t)\cos[2\pi f_0t + \theta(t)]$ and describe $a(t)$ and $\theta(t)$ in terms of $x_E(t)$. Interpret this result.

2. Show that the signal $x(t)$ can also be written as $x_{EI}(t)\cos 2\pi f_0t - x_{EQ}(t)\sin(2\pi f_0t)$, and describe $x_{EI}(t)$ and $x_{EQ}(t)$ in terms of $x_E(t)$. (This shows how you can obtain $x(t)$ without doing complex-valued operations.)

3. Find the baseband equivalent of the signal $x(t) = A(t)\cos(2\pi f_0t + \varphi)$, where $A(t)$ is a real-valued lowpass signal.

Problem 2. (Equivalent Baseband Signal)

1. Consider the waveform $\psi(t) = \text{sinc}\left(\frac{t}{T}\right)\cos(2\pi f_0t)$.

What is the equivalent baseband signal of this waveform.

2. Assume that the signal $\psi(t)$ is passed through the filter with impulse response $h(t)$ where $h(t)$ is specified by its baseband equivalent impulse response $h_E(t) = \frac{1}{\sqrt{2}}\text{sinc}\left(\frac{t}{2T}\right)$.

What is the output signal, both in passband as well as in baseband?

*Hint: The Fourier transform of $\cos(2\pi f_0t)$ is $\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$.

Problem 3. (Bandpass Nyquist Pulses)

Consider a pulse $p(t)$ defined via its Fourier transform $p_F(f)$ as follows:
1. What is the expression for $p(t)$?

2. Determine the constant $c$ so that $\psi(t) = cp(t)$ has unit energy.

3. Assume that $f_0 - \frac{B}{2} = B$ and consider the infinite set of functions $\cdots, \psi(t + T), \psi(t), \psi(t - T), \psi(t - 2T), \cdots$. Do they form an orthonormal set for $T = \frac{1}{2B}$? (Explain).

4. Determine all possible values of $f_0 - \frac{B}{2}$ so that $\cdots, \psi(t + T), \psi(t), \psi(t - T), \psi(t - 2T), \cdots$ forms an orthonormal set for $T = \frac{1}{2B}$. 

\[ \begin{array}{c|c|c|c|c}
& f & & & \\
-\frac{f_0 - B}{2} & -\frac{f_0 + B}{2} & \frac{f_0 - B}{2} & \frac{f_0 + B}{2} & p_F \\
\end{array} \]