ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Principles of Digital Communications:	Assignment date: May 23, 2012
Summer Semester 2012	Due date: May 30, 2012

Homework 14

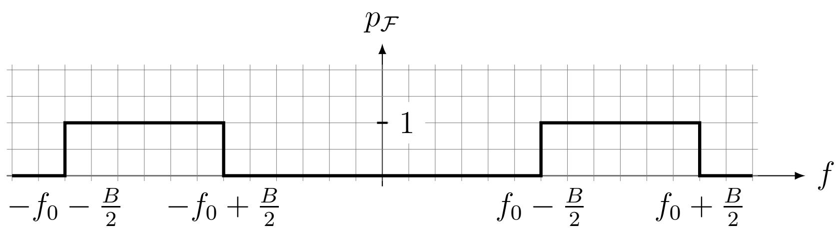
Problem 1. *(Equivalent Representations)*
A bandpass signal $x(t)$ may be written as $x(t) = \sqrt{2}\Re\{x_E(t)e^{j2\pi f_0 t}\}$, where $x_E(t)$ is the baseband equivalent of $x(t)$.

1. Show that a signal $x(t)$ can also be written as $a(t)\cos[2\pi f_0 t + \theta(t)]$ and describe $a(t)$ and $\theta(t)$ in terms of $x_E(t)$. Interpret this result.
2. Show that the signal $x(t)$ can also be written as $x_{EI}(t)\cos 2\pi f_0 t - x_{EQ}(t)\ \sin(2\pi f_0 t)$, and describe $x_{EI}(t)$ and $x_{EQ}(t)$ in terms of $x_E(t)$. (This shows how you can obtain $x(t)$ without doing complex-valued operations.)
3. Find the baseband equivalent of the signal $x(t) = A(t)\cos(2\pi f_0 t + \varphi)$, where $A(t)$ is a real-valued lowpass signal.

Problem 2. *(Equivalent Baseband Signal)*

1. Consider the waveform
$$\psi(t) = \text{sinc}\left(\frac{t}{T}\right)\cos(2\pi f_0 t).$$
What is the equivalent baseband signal of this waveform.
2. Assume that the signal $\psi(t)$ is passed through the filter with impulse response $h(t)$ where $h(t)$ is specified by its baseband equivalent impulse response $h_E(t) = \frac{1}{T\sqrt{2}}\text{sinc}^2\left(\frac{t}{2T}\right)$.
What is the output signal, both in passband as well as in baseband?
Hint: The Fourier transform of cos$(2\pi f_0 t)$ *is* $\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$.

Problem 3. *(Bandpass Nyquist Pulses)*
Consider a pulse $p(t)$ defined via its Fourier transform $p_{\mathcal{F}}(f)$ as follows:



1. What is the expression for $p(t)$?

2. Determine the constant c so that $\psi(t) = cp(t)$ has unit energy.

3. Assume that $f_0 - \frac{B}{2} = B$ and consider the infinite set of functions $\cdots$, $\psi(t+T)$, $\psi(t)$, $\psi(t-T)$, $\psi(t-2T)$, $\cdots$. Do they form an orthonormal set for $T = \frac{1}{2B}$? (Explain).

4. Determine all possible values of $f_0 - \frac{B}{2}$ so that $\cdots$, $\psi(t+T)$, $\psi(t)$, $\psi(t-T)$, $\psi(t-2T)$, $\cdots$ forms an orthonormal set for $T = \frac{1}{2B}$.