ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Principles of Digital Communications: Assignment date: Feb 22, 2012
Summer Semester 2012 Due date: Feb 29, 2012

Homework 1

Reading Assignment for next Wednesday: Chapter 1 and Chapter 2 up to and including Section 2.2.1. (Binary Hypothesis Testing).

Problem 1. *(Probabilities of Basic Events)*

Assume that X_1 and X_2 are independent random variables that are uniformly distributed in the interval $[0, 1]$. Compute the probability of the following events. **Hint:** For each event, identify the corresponding region inside the unit square.

1. $0 \leq X_1 - X_2 \leq \frac{1}{3}$.
2. $X_1^3 \leq X_2 \leq X_1^2$.
3. $X_2 - X_1 = \frac{1}{2}$.
4. $(X_1 - \frac{1}{2})^2 + (X_2 - \frac{1}{2})^2 \leq (\frac{1}{2})^2$.
5. Given that $X_1 \geq \frac{1}{4}$, compute the probability that $(X_1 - \frac{1}{2})^2 + (X_2 - \frac{1}{2})^2 \leq (\frac{1}{2})^2$.

Problem 2. *(Conditional Distribution)*

Assume that X and Y are random variables with probability density function

$$f_{X,Y}(x, y) = \begin{cases} A & 0 \leq x < y \leq 1 \\ 0 & \text{every where else.} \end{cases}$$

1. Are X and Y independent? **Hint**: Argue geometrically.
2. Find the value of A. **Hint**: Argue geometrically.
3. Find the marginal distribution of Y. Do it first by arguing geometrically then compute it formally.

4. Find $\Phi(y) = \mathbb{E}[X|Y=y]$. **Hint**: Argue geometrically.
5. Find $\mathbb{E}[\Phi(Y)]$ using the marginal distribution of Y.
6. Find $\mathbb{E}[X]$ and show that $\mathbb{E}[X] = \mathbb{E}[\mathbb{E}[X|Y]]$.

Problem 3. *(Basic Probabilities)*

Find the following probabilities:

- A box contains m white and n black balls. Suppose k balls are drawn. Find the probability of drawing at least one white ball.
- We have two coins; the first is fair and the second is two-headed. We pick one of the coins at random, we toss it twice and heads shows both times. Find the probability that the coin is fair. **Hint**: Define X as the random variable that takes value 0 when the coin is fair and 1 otherwise.

Problem 4. *(Basic Probabilities)*

We toss a biassed coin until head appears for the first time. Let T be the number of tosses and let p be the probability that head appears.

1. Find the probability mass function of the random variable T.
2. If $n > k \geq 1$ are two ineger valued numbers show that $\mathbb{P}[T = n|T > k] = \mathbb{P}[T = n - k]$ and interpret this result.
3. Argue that $\mathbb{E}[T|T > k] = k + \mathbb{E}[T]$.