Homework 2

Problem 1. (Conditioning Technique)
Assume that $X_1, X_2, \ldots, X_n$ are i.i.d. random variables uniformly distributed over $[0, 1]$. Let $N$ be an integer valued random variable uniformly distributed over $\{1, 2, \ldots, n\}$. Assume that $N$ and $X_i$, $i = 1, 2, \ldots, n$ are independent of each other. Let $Y = \sum_{i=1}^{N} X_i$. Hence, $Y$ is the sum of random number of $X_i$.

1. Compute $E\{Y\}$. **Hint:** Use conditioning on $N$.

2. Compute $E\{Y^2\}$ and variance of $Y$. **Hint:** You may find $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ and $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ useful for your calculations.

Problem 2. (Conditioning Technique)
We generate two random variables $X$ and $Y$ in the following way. We first pick $X$ randomly from $[0, 1]$ then pick $Y$ randomly from $[0, X]$.

1. Find the conditional distribution of $Y$ given $X = x$.

2. Find the marginal distribution of $Y$ and use it to compute the expected value of $Y$. **Hint:** You may need $\int y \log(y) dy = \frac{y^2}{2} (\log(y) - \frac{1}{2})$.

3. Use the conditioning technique to find the expected value of $Y$. **Hint:** $E\{E\{Y|X\}\} = E\{Y\}$.

Problem 3. (Conditioning Technique)
Assume that $X$ and $Y$ are i.i.d. random variables. Use the symmetry and the conditioning technique to find $E\{X|X + Y\}$. 