ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Principles of Digital Communications:	Assignment date: Feb 24, 2012
Summer Semester 2012	Due date: Feb 29, 2012

Homework 2

Problem 1. *(Conditioning Technique)*

Assume that $X_1, X_2, \ldots, X_n$ are i.i.d. random variables uniformly distributed over $[0,1]$. Let N be an integer valued random variable uniformly distributed over $\{1, 2, \ldots, n\}$. Assume that N and X_i, $i = 1, 2, \ldots, n$ are independent of each other. Let $Y = \sum_{i=1}^{N} X_i$. Hence, Y is the sum of random number of X_i.

1. Compute $E\{Y\}$. **Hint:** Use conditioning on N.
2. Compute $E\{Y^2\}$ and variance of Y. **Hint:** You may find $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ and $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ useful for your calculations.

Problem 2. *(Conditioning Technique)*

We generate two random variables X and Y in the following way. We first pick X randomly from $[0,1]$ then pick Y randomly from $[0, X]$.

1. Find the conditional distribution of Y given $X = x$.
2. Find the marginal distribution of Y and use it to compute the expected value of Y. **Hint:** You may need $\int y \log(y) dy = \frac{y^2}{2}(\log(y) - \frac{1}{2})$.
3. Use the conditioning technique to find the expected value of Y. **Hint:** $E\{E\{Y|X\}\} = E\{Y\}$.

Problem 3. *(Conditioning Technique)*

Assume that X and Y are i.i.d. random variables. Use the symmetry and the conditioning technique to find $E\{X|X+Y\}$.