ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Principles of Digital Communications: Assignment date: Apr 25, 2012
Summer Semester 2012 Due date: May 2, 2012

Homework 9

Reading Part for next Wednesday: From Section 5.3 (Power Spectral Density) till the end of Chapter 5.

Problem 1. *(Plan of the Project)*
Let us get back to our project. Now that we have learned the basic principles of transmission, it is time to come up with a plan of how you want to accomplish the transmission.

By next Wednesday we want from each group a short description of the transmission scheme as well as the receiver design. Specify as far as possible all parameters which you want to use. What is the basic transmission scheme. What are the signal constellations, what are the actually transmitted signals, how will you accomplish synchronization between transmitter and receiver, how will you detect the signals and make the decisions. What error probabilities are you expecting and on what parameters does the error probability depend. How do you model the channel? In the following weeks we will still learn some further techniques which might be useful (e.g., coding). So you might want to glance at these chapters and plan your system accordingly so that you can incorporate such schemes if they are needed.

You can use block diagrams, formulas, or just describe things in words. Please send in your proposal latest on Wednesday May 2nd, by midnight. If you hand it in, we will have a look and, should you have questions, we will give you feedback. If you do not hand it in, you are on your own. :-)

Problem 2. *(Sampling And Reconstruction)*
In this problem we investigate different practical methods used for the reconstruction of band-limited signals that have been sampled. We will see that the Picket-Fence is a useful tool for this problem. (Before solving this problem you are advised to review Appendices 5.C and 5.D).

Suppose that the Fourier transform of the signal $x(t)$ has components only in $f \in [-B, B]$. Assume also that we have an ideal sampler which takes a sample of the signal every T

seconds where $T < \frac{1}{2B}$. The sampling times are $t_n = nT, \quad -\infty < n < \infty$. In order to reconstruct the signal we use different practical interpolation methods including zero-order and first-order interpolator, followed by a lowpass filter.

The zero-order interpolator uses the signal samples to generate $x_0(t) = \sum_{-\infty}^{\infty} x(nT)p_0(t-nT)$ where

$$p_0(t) = \begin{cases} 1 & 0 \le t < T \\ 0 & \text{otherwise.} \end{cases}$$

1. Show that $x_0(t) = [x(t)E_T(t)] \star p_0(t)$ where $E_T(t) = \sum_n \delta(t - nT)$.
2. The signal $x_0(t)$ is then lowpass filtered to obtain a more or less accurate reconstruction. In the proof of the sampling theorem, the reconstruction is done by lowpass filtering $x|(t) = x(t)\sum_n \delta(t - nT)$. Compare (qualitatively) the two results. *(Hint: Work in the frequency domain.)*
3. How does B, T, and the quality of the lowpass filter affect the result?

In the first-order interpolator we connect adjacent points of the signal by a straight line. Let $x_1(t)$ be the resulting signal.

1. Show that $x_1(t) = [x(t)E_T(t)] \star p_1(t)$ where $p_1(t)$ is the triangular shape waveform

$$p_1(t) = \begin{cases} \frac{t+T}{T} & -T \le t < 0 \\ \frac{T-t}{T} & 0 \le t \le T \\ 0 & \text{otherwise.} \end{cases}$$

2. Also in this case the reconstructed signal is what we obtain after lowpass filtering $x_1(t)$. Discuss the relative pros and cons between the two methods.

Problem 3. *(DC-to-DC Converter)*
A unipolar DC-to-DC Converter circuit is an analog circuit with input $x(t)$ and output $y(t)$ defined by $y(t) = x(t)p_\tau(t)$ where $p_\tau(t) = \mathbb{1}_{[-\frac{\tau}{2},\frac{\tau}{2}]}(t) \star E_T(t)$. We are assuming that the relationship between the bandwidth of $x(t)$ and T are such as to satisfy the sampling theorem.

1. Use the Picket-Fence formula to find an expression for the Fourier transform of $y(t)$ as a function of the Fourier transform of $x(t)$.
2. What is the effect of the DC-to-DC Converter circuit on the frequency spectrum of the signal?
3. What happens when τ goes to zero and when τ goes to T?

4. Why do we call this circuit DC-to-DC Converter?

Problem 4. *(Communication Link Design)*
In this problem we want to design a suitable digital communication system between two entities which are 5 Km apart from each other. As a communication link we use a special kind of twisted pair cable that has a power attenuation factor of 16 dB/Km. The allowed bandwidth for the communication is 10 MHz in the baseband ($f \in [-5 \text{ MHz}, 5 \text{ MHz}]$) and the noise of the channel is AWGN with $N_0 = 4.2 \times 10^{-21}$ W/Hz. The necessary quality of the service requires a bit-rate of 40 Mbps (megabits per second) and a bit error probability less than 10^{-5}.

1. Draw the block diagram of the transmitter and the receiver that you propose to achieve the desired bit rate. Be specific about the form of the signal constellation used in the encoder but allow for a scaling factor that you will adjust in the next part to achieve a desired probability of error. Be also specific about the waveform generator and the the corresponding blocks of the receiver.

2. Adjust the scaling factor of the encoder to meet the required error probability. *(Hint: Use the block error probability as an upper bound to the bit error probability.)*

3. Determine the energy per bit, $\mathcal{E}_b$, of the transmitted signal and the corresponding power?