Problem 1. Let $x(t)$ and $y(t) \in \mathbb{R}$ be complex signals with Fourier transforms $X(f)$ and $Y(f)$. Prove the following properties:

1) $x(t - \tau) \xrightarrow{\text{FT}} e^{-j2\pi\tau f} X(f)$.  
2) $X(t) \xrightarrow{\text{FT}} x(-f)$.  
3) $x(at) \xrightarrow{\text{FT}} \frac{1}{a} X\left(\frac{f}{a}\right)$.  
4) $(x * y)(t) \xrightarrow{\text{FT}} X(f)Y(f)$.

Problem 2. One of the standard ways of describing the sampling operation relies on the concept of modulation by a pulse train. Choose a sampling interval $T_s$ and define a continuous-time pulse train $p(t)$ as $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$.

The Fourier Transform of the pulse train is $P(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \delta(f - k/T_s)$.

This is tricky to show, so just take the result as is. The sampled signal is simply the modulation of an arbitrary-continuous time signal $x(t)$ by the pulse train: $x_s(t) = p(t)x(t)$. Note that, now, this sampled signal is still continuous time but, by the properties of the delta function, is non-zero only at multiples of $T_s$; in a sense, $x_s(t)$ is a discrete-time signal brutally embedded in the continuous time world.

Here is the question: derive the Fourier transform of $x_s(t)$ and show that if $x(t)$ is bandlimited to $\pi/T_s$ then we can reconstruct $x(t)$ from $x_s(t)$.

Problem 3. Assume $x(t)$ is a continuous-time pure sinusoid at 10KHz. It is sampled with a sampler at 8KHz and then interpolated back to a continuous-time signal with an interpolator at 8KHz. What is the perceived frequency of the interpolated sinusoid?

Problem 4. 1) Consider a real continuous-time signal $x(t)$. All you know about the signal is that $x(t) = 0$ for $|t| > t_0$. Can you determine a sampling frequency $F_s$ so that when you sample $x(t)$, there is no aliasing? Explain.

2) We have seen that any discrete-time sequence can be sinc-interpolated into a continuous-time signal which is $f_N$-bandlimited; $f_N$ depends on the interpolation interval $T_s$ via the relation $f_N = \frac{1}{2T_s}$. Consider the continuous-time signal $x_c(t) = e^{-\frac{t}{T_s}}$ and the discrete-time sequence $x[n] = e^{-n}$. Clearly, $x_c(nT_s) = x[n]$; but, can we also say that $x_c(t)$ is the signal we obtain if we apply sinc interpolation to the sequence $x[n] = e^{-n}$ with interpolation interval $T_s$? Explain in detail.