ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE

School of Computer and Communication Sciences

Handout 27 | Signal Processing for Communications

Homework 11 | May 16, 2011, INF 213 - 10:15-12:00

Problem 1 (Multirate Identities)**.** Prove the following two identities:

(i) Downsampling by 2 followed by filtering by $H(z)$ is equivalent to filtering by $H(z^2)$ followed by downsampling by 2.

(ii) Filtering by $H(z)$ followed by upsampling by 2 is equivalent to upsampling by 2 followed by filtering by $H(z^2)$.

Problem 2 (Polyphase Implementation of Downsampling)**.** Consider the downsampling system given in Fig. 1, where $H(z)$ is an arbitrary filter with impulse response $h[n]$.

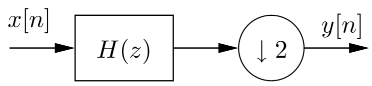

Figure 1: Downsampling system for Problem 1.

We define

$$e_0[n] = h[2n], \text{ and } e_1[n] = h[2n+1].$$

Prove that the system of Fig. 2 is equivalent to the one given in Fig. 1.

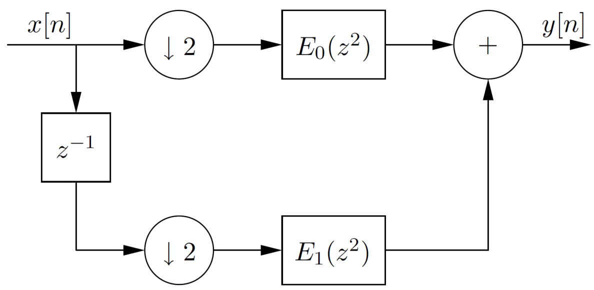

Figure 2: Equivalent system.

Problem 3 (Quatization Error)**.** Consider the quantizer $\mathcal{Q}\{\}$ which takes X that has a value in the interval $[0,1]$ and outputs $\hat{X}$, the first r bits of its binary expansion. Assume we feed X with the following probability density function into the quantizer:

$$f_X(x) = \frac{1}{2} + bx.$$

(i) Find b.

(ii) Compute $\mathbb{P}\left(\hat{X} = \hat{x}\right)$.

(iii) Compute the power of the quantization error.

Problem 4 (Minimize Quantization Error)**.** Consider a stationary i.i.d. random process $X[n]$ whose samples have normal distribution $N(0,1)$. The process is quantized with a 3 points quantizer $\mathcal{Q}\{\}$ with the following characteristic:

$$\mathcal{Q}\{x\} = \begin{cases} +a & x \geq t, \\ 0 & -t < x < t, \\ -a & x \leq -t. \end{cases}$$

Find the proper values for a and t in order for the power of the quantization error to be minimized.