ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 17 Advanced Digital Communications
Homework 8 November 29, 2010

PROBLEM 1. Suppose we have a linear time invariant channel, i.e.,

$$Y(D) = Q(D)X(D) + Z(D);$$

with

$$\begin{aligned} S_Z(D) &= Q(D)N_0 \\ Q(D) &= a^*D^{-1} + 1 + aa^* + aD \\ 0 &\leq |a| < 1. \end{aligned}$$

(a) Find the zero forcing and minimum mean square error linear equalizers $W_{\text{ZFE}}(D)$ and $W_{\text{MMSE-LE}}(D)$. Use the variable $b = (q_0)(1 + \frac{1}{\text{SNR}_{\text{MFB}}})$ in your expression for $W_{\text{MMSE-LE}}(D)$.

Hint.

$$\text{SNR}_{\text{MFB}} = \frac{q_0 \mathcal{E}_x}{N_0}$$

(b) By substituting $e^{-j2\pi\theta} = D$ and taking $\mathcal{E}_x/N_0 = 10$, use MATLAB to plot (for different values of θ) $W(e^{j2\pi\theta})$ for both ZFE and MMSE-LE for $a = .5$ and $a = .9$. Discuss the differences between the plots.

(c) Find the roots r_1, r_2 of the polynomial

$$aD^2 + bD + a^*.$$

Show that $b^2 - 4aa^*$ is always a real positive number (for $|a| \neq 1$).

Hint. Consider the case where $\frac{1}{\text{SNR}_{\text{MFB}}} = 0$. Let r_2 be the root for which $|r_2| < |r_1|$. Show that $r_1 r_2^* = 1$.

(d) Use the previous results to show that for the MMSE-LE

$$W(D) = \frac{1}{a} \frac{D}{(D - r_1)(D - r_2)} = \frac{1}{a(r_1 - r_2)} \left(\frac{r_1}{D - r_1} - \frac{r_2}{D - r_2} \right).$$

(e) Show that for the MMSE-LE, $w_0 = \frac{1}{\sqrt{b^2 - 4aa^*}}$. By taking $\frac{1}{\text{SNR}_{\text{MFB}}} = 0$, show that for the ZFE, $w_0 = \frac{1}{1 - aa^*}$.

(f) For $\mathcal{E}_x = 1$ and $\sigma^2 = 0.1$ find expressions for σ^2_{ZFE}, $\sigma^2_{\text{MMSE-LE}}$, γ_{ZFE} and $\gamma_{\text{MMSE-LE}}$

Hint.

$$\gamma_{\text{ZFE}} = 10 \log_{10} \frac{\text{SNR}_{\text{MFB}}}{\text{SNR}_{\text{ZFE}}}$$

and

$$\gamma_{\text{MMSE-LE}} = 10 \log_{10} \frac{\text{SNR}_{\text{MFB}}}{\text{SNR}_{\text{MMSE-LE}}}.$$

(g) Find γ_{ZFE} and $\gamma_{\text{MMSE-LE}}$ in terms of the parameter a and calculate for $a = 0, 0.5, 1$. Sketch γ_{ZFE} and $\gamma_{\text{MMSE-LE}}$for $0 \leq a < 1$.