Introductory Quiz: Solutions

1. Let P be a probability and A, B be two events. Among the following identities, which are always true?

☐ a) $P(A \cup B) = P(A) + P(B)$.

■ b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

■ c) $P(A|B) = \frac{P(A \cap B)}{P(B)}$, if $P(B) \neq 0$.

☐ d) $P(A \cap B) = P(A) \cdot P(B)$.

2. Gaussian random variables

a) Let X be a random variable such that $P(X \geq 0) = 1$. Can X be a Gaussian random variable? NO, a Gaussian random variables always takes positive and negative values.

b) Let Y be a Gaussian random variable with mean μ. Is it true that $P(Y \geq \mu) = 1/2$? YES.

c) Let Z be a random variable such that $P(Z = 1) = 1/2$. Can Z be a Gaussian random variable? NO, a Gaussian random variable is continuous and cannot therefore take a given value with positive probability.

3. You throw a (fair) coin 1000 times. What is the probability that you end up with exactly 500 tails and 500 heads?

☐ a) 1/2 ☐ b) 0 ■ c) approximately 0.025

4. Name three scientists whose names are associated to distributions of random variables.

a) Gauss b) Poisson c) Bernoulli ...

5. Let X be a random variable such that $P(X = 1) = P(X = 0) = 1/2$.

a) What is the mean of X? 1/2 b) What is variance of X? 1/4

Let now Y be the random variable defined as $Y = 2X - 1$.

c) What is the mean of Y? 0 d) What is variance of Y? 1

6. Which of the following statements is true?

☐ a) If $\mathrm{Cov}(X, Y) = 0$, then X and Y are independent.

■ b) If X and Y are independent, then $\mathrm{Cov}(X, Y) = 0$.

☐ c) X and Y are independent if and only if $\mathrm{Cov}(X, Y) = 0$.

7. Subsidiary question: give the definition of convergence in probability:

$X_n \xrightarrow{P} X$ means $\forall \varepsilon > 0$, $\lim_{n \to \infty} P(|X_n - X| > \varepsilon) = 0$.