Solutions 1

1. a) $\mathcal{F} = \{\emptyset, \{2\}, \{5\}, \{1,3\}, \{2,5\}, \{4,6\}, \{1,2,3\}, \{1,3,5\}, \{2,4,6\}, \{4,5,6\}, \{1,2,3,5\}, \{1,3,4,6\}, \{2,4,5,6\}, \{1,2,3,4,6\}, \{1,3,4,5,6\}, \{1,2,3,4,5,6\}\}$ ($16 = 2^4$ elements)
b) atoms of $\mathcal{F}$: $\{1,3\}, \{2\}, \{4,6\}, \{5\}$. Notice that one also has $\mathcal{F} = \sigma(\{1,3\}, \{2\}, \{4,6\}, \{5\})$, as already mentioned in the problem set.

2. The set Ω (representing the set of possible values of weights for the object) is $]0, 100]$. We describe below the σ-fields $\mathcal{F}_A$, $\mathcal{F}_B$ et $\mathcal{F}_C$ using their respective atoms:

$$\begin{aligned}
\mathcal{F}_A &= \sigma(]0,20],]20,40],]40,50],]50,60],]60,70],]70,80],]80,90],]90,100]),\\
\mathcal{F}_B &= \sigma(]0,20],]20,40],]40,60],]60,80],]80,100]),\\
\mathcal{F}_C &= \sigma(]0,10],]10,20],]20,30],]30,40],]40,50],]50,60],]60,70],]70,80],]80,90],]90,100]).
\end{aligned}$$

We therefore have $\mathcal{F}_B \subset \mathcal{F}_A \subset \mathcal{F}_C$, that is, B has the least information on the weight, and C has the most. Only A and C are able to tell whether the weight is between 40 and 50g.

Remark: $]0,10] \subset]0,20]$, but notice that $\mathcal{F}_B \subset \mathcal{F}_C$!

3. a) $\Omega = \emptyset^c$.
b) use de Morgan's formula: $(\cap_{n=1}^{\infty} A_n)^c = (\cup_{n=1}^{\infty} A_n^c)$.
c) $B \backslash A = B \cap A^c$.

4. atoms of $\mathcal{F}$: $[-1, -\frac{3}{4}[\cup [\frac{3}{4}, 1[, [-\frac{3}{4}, -\frac{1}{2}[, [\frac{1}{2}, -\frac{3}{4}[, [-\frac{1}{2}, -\frac{1}{4}[\cup [\frac{1}{4}, \frac{1}{2}[, [-\frac{1}{4}, 0[, [0, \frac{1}{4}[$.