ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 31 Signal Processing for Communications
Homework 9 Solution June 20, 2010

PROBLEM 1.

$$X_s(j\Omega) = X(j\Omega) * P(j\Omega) \quad (1)$$

$$= \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} X(\Omega - \frac{k}{T_s}) \quad (2)$$

If $x(t)$ is bandlimited to $\frac{\pi}{T_s}$, then no aliasing occurs in the above sum as $\Omega_N = \frac{\pi}{T_s} \leq \frac{\Omega_s}{2}$

PROBLEM 2. Let $x(t) = e^{j2\pi f_0 t}$, where $f_0 = 10KHz$. Then the sampled version would be $x[n] = e^{j\omega_0 n}$, where $\omega_0 = 2\pi\frac{f_0}{F_s}$ and $F_s = 8KHz$. So in this example $x[n] = e^{j2\pi f_b}$ with $f_b = 2KHz$ and in fact all continuous-time frequencies of the form $f = (2 + 8k) \times 10^3 Hz$ ($f_b = 2000Hz < 4000Hz = \frac{F_s}{2}$) are aliased to the same discrete-time frequency $f_b = 2KHz$ which is thus the perceived frequency of the interpolated sinusoid.

PROBLEM 4. What we know from $x(t)$ is that it is time limited. This says that $x(t)$ is not bandlimited in frequency domain. Now if we sample this signal in any desired sampling frequency F_s, we cannot avoid aliasing due to the non-zero $X(j\Omega)$ in the whole spectrum. (Look at figure 9.12 of the textbook)

PROBLEM 5. The Fourier transform of a continuous-time signal $x(t)$ and its inversion formula are defined as 9.4 and 9.5 in the textbook but their convergence is only assured for functions which satisfy the so-called Dirichlet conditions. In particular, the FT is always well defined for square integrable (finite energy), continuous-time signals. Let's first check if $x_c(t)$ is a finite energy:

$$\int_{-\infty}^{\infty} e^{-2\frac{t}{T_s}} d_t = -\frac{T_s}{2}(e^{-\infty} - e^{\infty}) = \infty \quad (3)$$

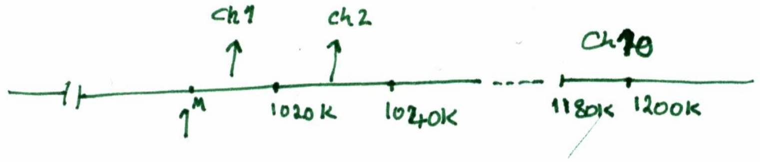
So the following step CANNOT be written and concluded for $x(t) = e^{-\frac{t}{T_s}}$:

$$X(j\Omega) = \left(\frac{\pi}{\Omega_N}\right) rect\left(\frac{\pi}{2\Omega_N}\right) \sum_{n=-\infty}^{\infty} x[n]e^{-j\pi(\Omega/\Omega_N)n} \quad (4)$$

$$= \begin{cases} \frac{\pi}{\Omega_N} X(e^{j\pi\Omega/\Omega_N}) & for|\Omega| \leq \Omega_N \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

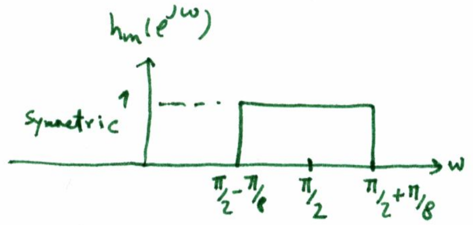
Prob 3.

1)

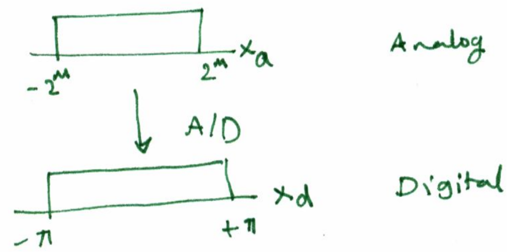


2) $h_m[n] = \cos(w_m n)\, h[n] \Longrightarrow h_m(e^{jw}) = \frac{1}{2}\left[\delta(w - w_m) + \delta(w + w_m)\right] \circledast h(e^{jw})$

$= \frac{1}{2}\left[h(e^{j(w - w_m)}) + h(e^{j(w + w_m)})\right]$



2;3)



$\Longrightarrow x_d(e^{jw}) = x_a(jw \frac{2000}{\pi})$

or

$x_a(jw) = x_d(e^{\frac{jw\pi}{2000}})$

Thus in the digital domain, channel 9 corresponds to the frequency band $[\frac{\pi}{2}, \frac{\pi}{2} + \frac{\pi}{100}]$

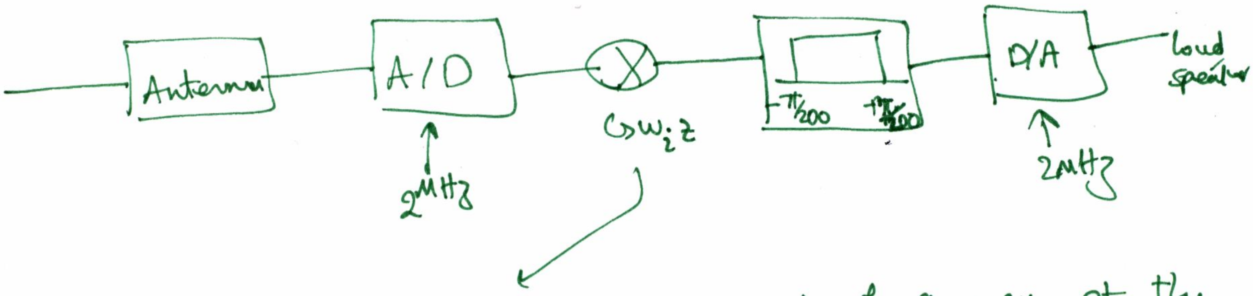
Thus the tunning frequnency should be $\frac{\pi}{2} + \frac{\pi}{200}$. For channel 5 it should be $\frac{\pi}{2} + \frac{\pi}{100} \times 5 - \frac{\pi}{200}$ and for channel 10 it should be

$\frac{\pi}{2} + \frac{\pi}{100} \times 10 - \frac{\pi}{200}$.

Also the cut off frequnecy should be, in the digital domain, equal to $\frac{\pi}{200}$ for the lowpass filter.

However, if we want to have the same filters above in the analogue domain, every thing should be multiplied by a factor $\frac{2000}{\pi}$.

(1)



cos $w_i t$ is the tunning frequnney of the

ith channel.