Problem 1.

\[ X_s(j\Omega) = X(j\Omega) * P(j\Omega) \]
\[ = \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} X(\Omega - \frac{k}{T_s}) \]

If \( x(t) \) is bandlimited to \( \frac{\pi}{T_s} \), then no aliasing occurs in the above sum as \( \Omega_N = \frac{\pi}{T_s} \leq \frac{f_s}{2} \).

Problem 2. Let \( x(t) = e^{j2\pi f_0 t} \), where \( f_0 = 10KHz \). Then the sampled version would be \( x[n] = e^{j\omega_0 n} \), where \( \omega_0 = 2\pi f_0 F_s \) and \( F_s = 8KHz \). So in this example \( x[n] = e^{j2\pi f_b} \) with \( f_b = 2KHz \) and in fact all continuous-time frequencies of the form \( f = (2 + 8k) \times 10^3 Hz \) \( (f_b = 2000Hz < 4000Hz = \frac{F_s}{2}) \) are aliased to the same discrete-time frequency \( f_b = 2KHz \) which is thus the perceived frequency of the interpolated sinusoid.

Problem 4. What we know from \( x(t) \) is that it is time limited. This says that \( x(t) \) is not bandlimited in frequency domain. Now if we sample this signal in any desired sampling frequency \( F_s \), we cannot avoid aliasing due to the non-zero \( X(j\Omega) \) in the whole spectrum. (Look at figure 9.12 of the textbook)

Problem 5. The Fourier transform of a continuous-time signal \( x(t) \) and its inversion formula are defined as 9.4 and 9.5 in the textbook but their convergence is only assured for functions which satisfy the so-called Dirichlet conditions. In particular, the FT is always well defined for square integrable (finite energy), continuous-time signals. Let’s first check if \( x_c(t) \) is a finite energy:

\[ \int_{-\infty}^{\infty} e^{-2\pi^2 t^2} dt = -\frac{T_s}{2}(e^{-\infty} - e^{\infty}) = \infty \]

So the following step CANNOT be written and concluded for \( x(t) = e^{-\frac{t}{T_s}} \):

\[ X(j\Omega) = \left( \frac{\pi}{\Omega_N} \right) \text{rect} \left( \frac{\pi}{2\Omega_N} \right) \sum_{n=-\infty}^{\infty} x[n] e^{-j\pi(\Omega/\Omega_N)n} \]
\[ = \left\{ \begin{array}{ll} \frac{\pi}{\Omega_N} X(e^{j\pi\Omega/\Omega_N}) & \text{for } |\Omega| \leq \Omega_N \\ 0 & \text{otherwise} \end{array} \right. \]
Problem 3.

1) \[ \text{Channel 1, Channel 2, Channel 98} \]

2) \[ h_m[n] = e^{j\omega_m n} h[n] \] \[ \Rightarrow h_m(e^{j\omega}) = \frac{1}{2} \left[ 8(w-w_m) + 8(w+w_m) \right] \ast h(e^{j\omega}) \]

\[ h_m(e^{j\omega}) = \frac{1}{2} \left[ h(e^{j(w-w_m)}) + h(e^{j(w+w_m)}) \right] \]

3) Analog \[ \xrightarrow{A/D} \] Digital

\[ \Rightarrow x_d(e^{j\omega}) = x_a(e^{j\omega \frac{2000}{\pi}}) \]

or

\[ x_a(e^{j\omega}) = x_d(e^{-j\omega \frac{2000}{\pi}}) \]

Thus in the digital domain, channel 9 corresponds to the frequency band \[ \left[ \frac{\pi}{2}, \frac{\pi}{2} + \frac{\pi}{100} \right] \]

Thus the tuning frequency should be \[ \frac{\pi}{2} + \frac{\pi}{200} \] for channel 9. It should be \[ \frac{\pi}{2} + \frac{\pi}{100} \] and for channel 10 it should be \[ \frac{\pi}{2} + \frac{\pi}{100} \times 10 - \frac{\pi}{200} \]

Also the cut off frequency should be, in the digital domain, equal to \( \frac{\pi}{200} \) for the lowpass filter.

However, if we want to have the same filters above in the analogue domain, every thing should be multiplied by a factor \( \frac{2000}{\pi} \).
The text written on the diagram is: "an wet is the tuning frequency of the i-th channel."