ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 10 Signal Processing for Communications
Homework 6 March 29, 2010

PROBLEM 1. (a) What does the following Program do?

```
% Homework 6
% Problem 1
    close all;
    clc;
    N = 2^20;
    tic;
    x=fft(rand(1,N));
    toc
```

(b) Write a function *MYDFT.m*, which takes a vector as its input and returns its DFT. Use *for* command to implement the DFT.

(c) Write a function *MYmodDFT.m*, which takes a vector as its input and returns its DFT. This time use the butterfly operation once, i.e. divide the signal into two parts, use MYDFT to compute the DFT of each part and then properly mix the output to form the DFT of the original signal.

(d) Run the following program. What does it do? *Hint.* It may take a while.

```
% Homework 6
% Problem 1
    close all;
    clc;
    N= 13:17 ;
    t=zeros(3,5);

    for i = 13:17
    2^i
    x=rand(2^i,1);
    tic;
    x=fft(rand(1,N));
    t(1,i-12) = toc
    tic;
    x=MYDFT(rand(1,N));
    t(2,i-12) = toc
    tic;
    x=MymodDFT(rand(1,N));
    t(3,i-12) = toc
    end
    hold on;
    plot(N,x(1,:),'r')
    plot(N,x(2,:),'b')
    plot(N,x(3,:),'g')
```

PROBLEM 2. Consider an L-point input sequence $x[n] = \text{rand}(1, L)$ and a P-point impulse response

$$h[n] = \begin{cases} \frac{100}{n+13} & 0 \le n \le P-1 \\ 0 & \text{otherwise} \end{cases}$$

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(a) Use MATLAB to compute $y[n] = x[n] \star h[n]$ for $L = 100000$ and $P = 20$. How long does this computation take?

(b) Segment $x[n]$ to sections of length $B = 50$ as follows:

$$x[n] = \sum_{r=0}^{\infty} x_r[n - rB],$$

where

$$x_r[n] = \begin{cases} x[n + rB] & 0 \le n \le B-1, \\ 0 & \text{otherwise} \end{cases}$$

and show theoretically that $y[n] = \sum_{r=0}^{\infty} y_r[n - rB]$ where $y_r[n] = x_r[n] \star h[n]$.

(c) Use part (b) to compute $y[n]$. How long does it take?

(d) Verify that if a $B-$point sequence is circularly convolved with a $P < B$-point sequence $(P < L)$, then the first $P - 1$ points of the result are the only points different from what would be obtained had we implemented a linear convolution.

(e) Again divide $x[n]$ into sections of length B so that each input section $x_r[n]$ overlaps the preceeding section by $P-1$ points. Call the circular convolution of each segment with $h[n]$, $y_{rp}[n]$. Write $y[n] = x[n] \star h[n]$ in terms of $y_{rp}[n]$. *Hint:* $x_r[n] = x[n + r(B - P + 1) - P + 1], \quad 0 \le n \le B - 1.$

(f) Use part (e) to compute $y[n]$ for $L = 100000, P = 20, B = 50$. How long does this take?