ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Handout 9 Signal Processing for Communications

Solutions to homework 4. March 22, 2010

PROBLEM 1. Consider the two signals $x[n]$ and $y[n]$ defined as follows:

$$x[n] = \begin{cases} \sin(\frac{n\pi}{5}) & 0 \leq n \leq 9 \\ 0 & \text{otherwise} \end{cases}$$
$$y[n] = \begin{cases} n & 0 \leq n \leq 9 \\ 0 & \text{otherwise} \end{cases}$$

Use MATLAB to:

1. Plot $z[n] = x[n] + y[n]$.
2. Compute and plot $z[n] = x[n] \star y[n]$.
3. Compute the energy of $x[n]$.
4. Using the MATLAB function **fft**, verify Parseval's identity between $x[n]$ and its DFT.

The plot is shown in figure 1.
Code:

```
% Homework 4
% Problem 1
close all;
clear all;
clc;
n = 0:9;
x = sin(n.*pi/5);
y = n;
% 1)
z = x+y;
subplot(1,2,1);
plot(n,z);
xlim([n(1) n(end)]);
title('z[n]=x[n]+y[n]');

z = conv(x,y);
timeConv = 0:length(z)-1;
subplot(1,2,2);
plot(timeConv,z);
xlim([timeConv(1) timeConv(end)]);
title('z[n]=conv(x[n],y[n])');

energy_x = norm(x)^2 % = 5
fft_x = fft(x);
energy_x_f = (1/length(fft_x))*norm(fft_x)^2 % = 5
```

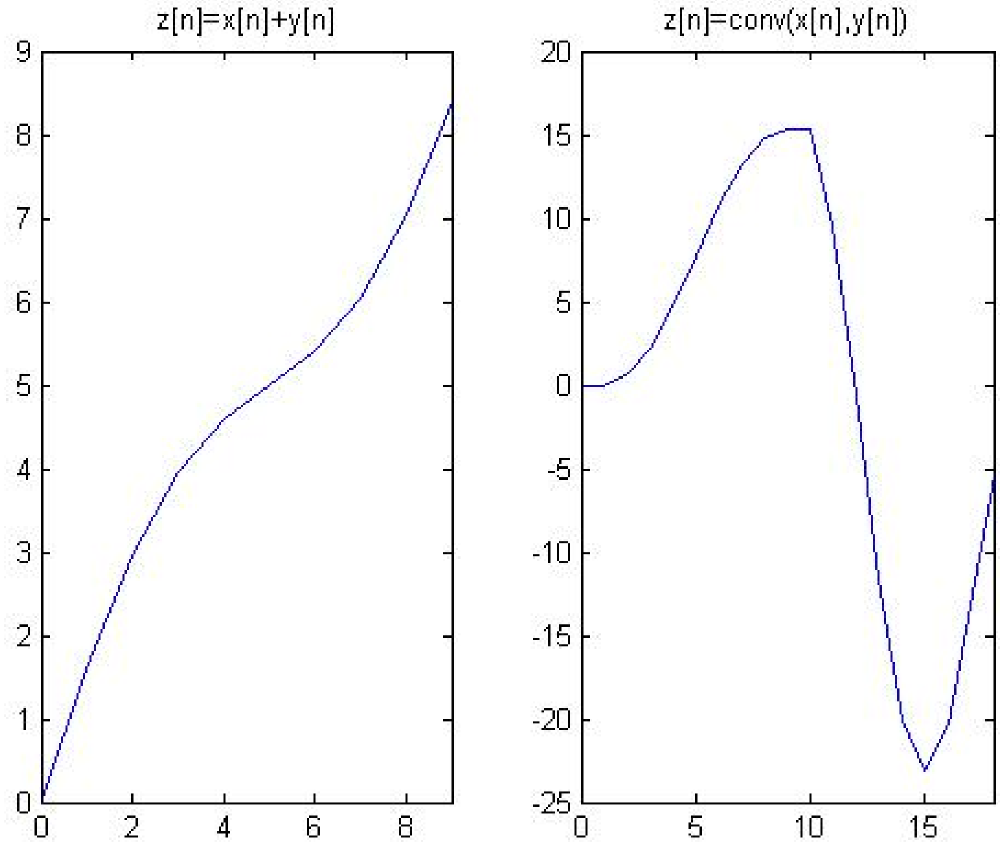

Figure 1: Problem 1

PROBLEM 2. Write a Matlab function that takes as input a sequence $x[n]$ of length N, returns the DFT of $x[n]$, and plots both $x[n]$ and its DFT (magnitude and phase).

1. Try your function for the input signal $x[n] = \delta[n-3]$.
2. Use Matlab's **fft** function to verify your answer to part (1).

The plot is shown in figure 2.

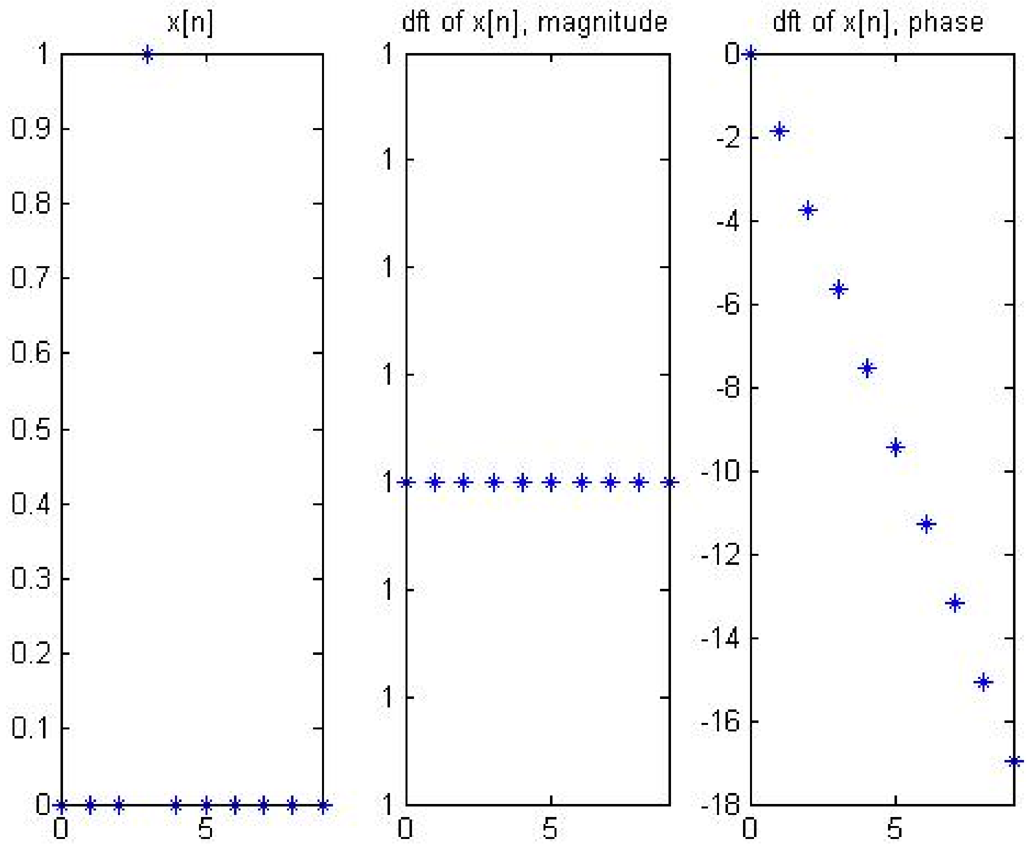

Figure 2: Problem 2

Code:

```
% Homework 4
% Problem 2
function dft_x = problem2_hw4(x)

    close all;
    clc;
    N = length(x);
    n = 0:N-1;
    n = n';
    k = 0:N-1;
    expos = n*k;
    F = exp(-2*pi*1j*expos./N);
    dft_x = F*x';
```

```
    %dft_x = fftshift(fft(x));

    subplot(1,3,1);
    timeX = 0:N-1;
    plot(timeX,x,'*');
    if N~=1
    xlim([timeX(1) timeX(end)]);
    end
    title('x[n]');

    subplot(1,3,2);
    timeDft = 0:length(dft_x)-1;
    plot(timeDft,abs(dft_x),'*');
    if N~=1
    xlim([timeDft(1) timeDft(end)]);
    end
    title('dft of x[n], magnitude');

    subplot(1,3,3);
    plot(timeDft,phase(dft_x),'*');
    if N~=1
    xlim([timeDft(1) timeDft(end)]);
    end
    title('dft of x[n], phase');

end
```

Problem 3.

1. Write a Matlab function that takes as input N and plots the following signal for $N = 5, 8$.

$$x_N[n] = \begin{cases} 1 & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$$

   The plot is shown in figure 3.
   Code:

```
% Homework 4
% Problem 3
function [x,dft_x] = problem3_hw4(N)

    close all;
    clc;
    x = [ones(1,N) zeros(1,N)];
    twoN = length(x);
    n = 0:twoN-1;
    n = n';
    k = 0:twoN-1;
    expos = n*k;
    F = exp(-2*pi*1j*expos./(twoN));
```

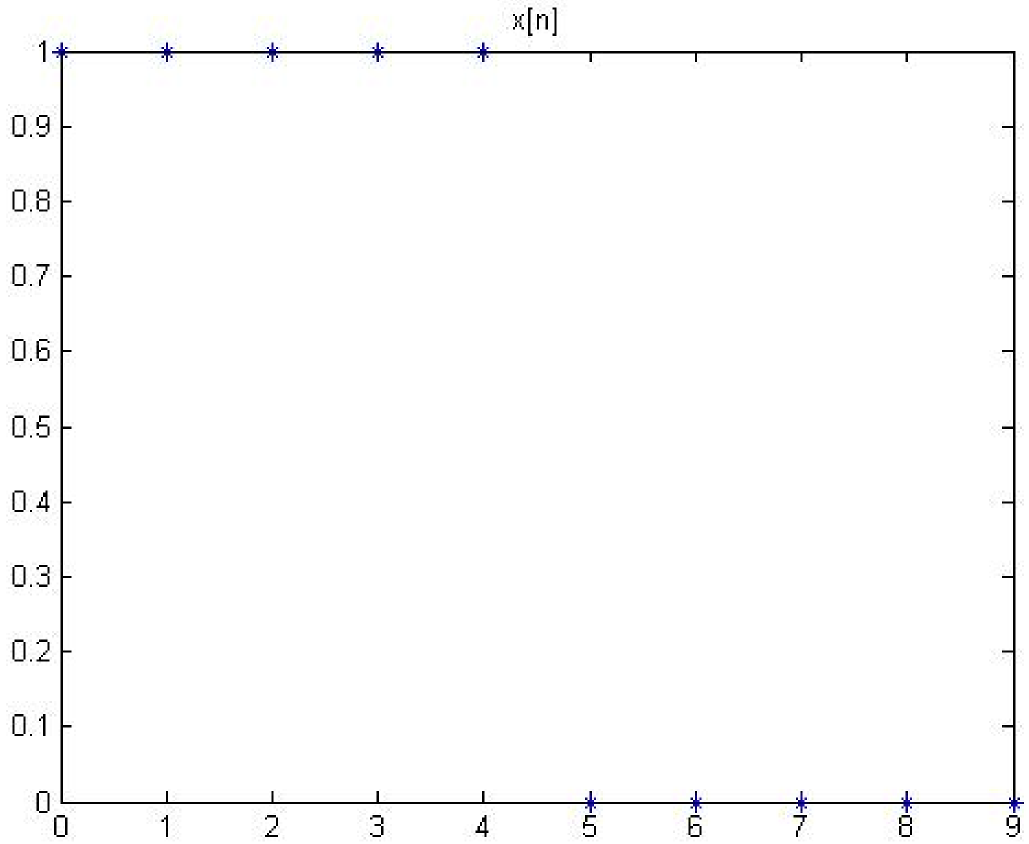

Figure 3: Problem 3.1

```
    dft_x = F*x';
    %dft_x = fftshift(fft(x));

    timeX = 0:twoN-1;
    plot(timeX,x,'*');
    title('x[n]');
    xlim([timeX(1) timeX(end)]);

end
```

2. Derive analytically the $2N$ point DFT of the above defined step function for an arbitary N and plot it for $N = 5, 8$.
   $X[k] = \sum_{n=0}^{2N-1} x[n]e^{-j2\pi\frac{nk}{2N}} = \sum_{n=0}^{N-1} e^{-j2\pi\frac{nk}{2N}}$.

3. Modify the DFT function you wrote for problem 2 to compute and plot (both the phases and the magnitudes of) the $2N$ point DFT of $x_N(n)$ for $N = 5, 8$.
   Use the unswer of 3.1.

4. Use the Matlab function **subplot** to display your answer to parts (2) and (3) in one window.

The plot is shown in figure 4.

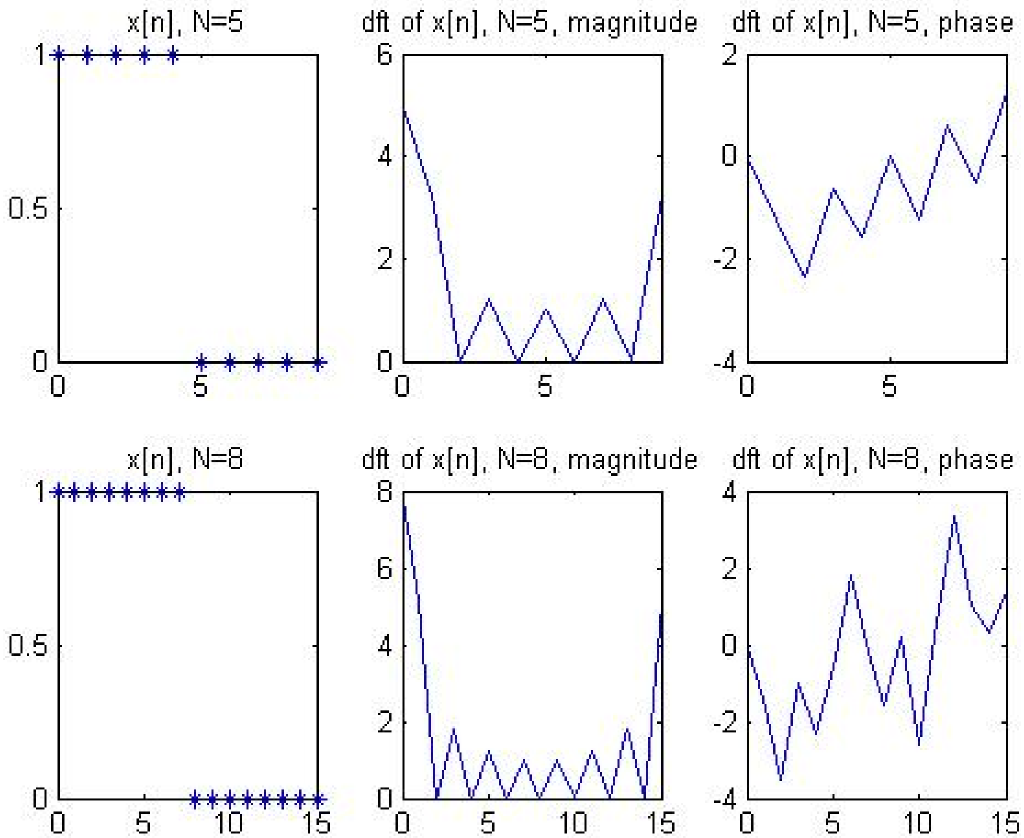

Figure 4: Problem 3.4

Code:

```
[x5,dft_x5] = problem3_hw4(5);
[x8,dft_x8] = problem3_hw4(8);

twoN1 = length(x5);
twoN2 = length(x8);
timeX1 = 0:twoN1-1;
timeX2 = 0:twoN2-1;

subplot(2,3,1);
plot(timeX1,x5,'*');
title('x[n], N=5');
xlim([timeX1(1) timeX1(end)]);

subplot(2,3,2);
timeDft1 = 0:length(dft_x5)-1;
plot(timeDft1,abs(dft_x5));
title('dft of x[n], N=5, magnitude');
xlim([timeX1(1) timeX1(end)]);

subplot(2,3,3);
plot(timeDft1,phase(dft_x5));
title('dft of x[n], N=5, phase');
xlim([timeX1(1) timeX1(end)]);

subplot(2,3,4);
plot(timeX2,x8,'*');
title('x[n], N=8');
xlim([timeX2(1) timeX2(end)]);

subplot(2,3,5);
timeDft2 = 0:length(dft_x8)-1;
plot(timeDft2,abs(dft_x8));
title('dft of x[n], N=8, magnitude');
xlim([timeX2(1) timeX2(end)]);

subplot(2,3,6);
plot(timeDft2,phase(dft_x8));
title('dft of x[n], N=8, phase');
xlim([timeX2(1) timeX2(end)]);
```