École Polytechnique Fédérale de Lausanne

School of Computer and Communication Sciences

Handout 5 | Signal Processing for Communications

Homework 3 | Due: March 15, 2010

This is a graded exercise.

Problem 1. Evaluate the following integral,

$$\int_{-\pi}^{\pi} \sin(x)\delta(-2x-\pi)\,dx.$$

Problem 2. Do the following vectors form a basis in $\mathcal{R}^4$?

$(3.14, 2, 1, -2)$, $(1.73, 2, -0.5, -2.7)$, $(0.3, 0.3, -0.5, -0.73)$, $(1, 0.3, -0.5, -0.73)$, $(1, 1, 1, 1)$

Problem 3. Consider a length-N signal $x[n]$, $n = 0, \ldots, N-1$; what is the length-N signal $y[n]$ obtained as

$$y[n] = \text{DFT}\{\text{DFT}\{x[n]\}\}?$$

Problem 4. Derive the formula for the DFT of the length-N signal $x[n] = \cos(\frac{2\pi}{N}Ln + \phi)$.

Problem 5. Compute the DFT of the length-4 signal $x[n] = \{a, b, c, d\}$. For which values of a, b, c, d is the DFT real?

Problem 6. Consider the FT pair of $x[n]$ and $X(e^{j\omega})$. Find $Y(e^{j\omega})$ in terms $X(e^{j\omega})$ for the following signals:

(a) $y[n] = nx[n]$

(b) $y[n] = x[n-n_d]$, n_d is an integer.

(c) $y[n] = -x[n]$.

(d) $y[n] = -3x[4n-7]$.

(e) $y[n] = nx[-n+1]$.

Problem 7. Consider the discrete signal $x[n]$ with support N, take the DTFT of $x[n]$ to obtain $X(e^{j\omega})$. Instead of computing the inverse DTFT of $X(e^{j\omega})$ to obtain $x[n]$, we first sample the $X(e^{j\omega})$ in $[0, 2\pi]$ at N points and we call it $\tilde{Y}[n]$ and then take the inverse of N-point DFT of the $\tilde{Y}[n]$ to obtain $y[n]$. Relate $y[n]$ to $x[n]$.

We haven't covered the LTI systems in the class, but you have seen it in previous courses. In the following problems, we will review LTI systems.

Problem 8. For each of following systems determine whether the system is (1) stable, (2) causal, (3) linear, (4) time invariant and (5) memoryless.

(a) $T(x[n]) = x[n]g[n]$, with $g[n]$ given.

(b) $T(x[n]) = x[Mn]$, with M a positive integer.

(c) $T(x[n]) = \sum_{k=n_0}^{n} x[k]$.

(d) $T(x[n]) = \sum_{k=n-n_0}^{n+n_0} x[k]$.

(e) $T(x[n]) = e^{x[n]}$.

(f) $T(x[n]) = x[n] + 3u[n+1]$.

(g) $T(x[n]) = x[-n]$.