Problem 1. 1. Show that the set of all ordered \( n \)-tuples \([a_1, a_2, \ldots, a_n]\) with the natural definition for the sum:
\[
[a_1, a_2, \ldots, a_n] + [b_1, b_2, \ldots, b_n] = [a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n]
\]
and the multiplication by a scalar:
\[
\alpha[a_1, a_2, \ldots, a_n] = [\alpha a_1, \alpha a_2, \ldots, \alpha a_n]
\]
form a vector space. Give its dimension and find a basis.

2. Show that the set of signals of the form \( y(x) = a \cos(x) + b \sin(x) \) (for arbitrary \( a, b \)), with the usual addition and multiplication by a scalar, form a vector space. Give its dimension and find a basis.

3. Are the four diagonals of a cube orthogonal?

4. Express the discrete-time impulse \( \delta[n] \) in terms of the discrete-time unit step \( u[n] \) and conversely.

5. Show that any function \( f(t) \) can be written as the sum of an odd and an even function, i.e. \( f(t) = f_o(t) + f_e(t) \) where \( f_o(-t) = -f_o(t) \) and \( f_e(-t) = f_e(t) \).

Problem 2. Let \( \{x(k)\}, k = 0, \ldots, N-1 \), be a basis for a space \( S \). Prove that any vector \( z \in S \) is uniquely represented in this basis.

Hint. Prove by contradiction.

Problem 3. Assume \( v \) and \( w \) are two vectors in the vector space. Prove the triangular inequality for each \( v \) and \( w \).
\[
||v + w|| \leq ||v|| + ||w||.
\]

Hint. Expand \( ||v + w||^2 \) and use Cauchy-Schwarz inequality.

Problem 4. Consider the following signal
\[
x[n] = (5 - |n|). (u[n + 5] - u[n - 6]).
\]

Draw the following signals:
- \( x[n] \).
- \( x[-2n + 3] \).
- \( \sum_{k=-\infty}^{n} x[2k] \).

Problem 5. Find the inverse \( z \)-transform of following series.
(a) \( X(z) = \frac{1}{(1-1/4z^{-1})(1-1/2z^{-1})}, \ |z| > 1/2. \)
(b) \( X(z) = \frac{1}{(1-1/5z^{-1})(1+3z^{-1})}, \ 3 > |z| > 1/5. \)