ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 3 Signal Processing for Communications
Homework 2 March 1, 2010

PROBLEM 1. 1. Show that the set of all ordered n-tuples $[a_1, a_2, \ldots, a_n]$ with the natural definition for the sum:

$$[a_1, a_2, \ldots, a_n] + [b_1, b_2, \ldots, b_n] = [a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n]$$

and the multiplication by a scalar:

$$\alpha[a_1, a_2, \ldots, a_n] = [\alpha a_1, \alpha a_2, \ldots, \alpha a_n]$$

form a vector space. Give its dimension and find a basis.

2. Show that the set of signals of the form $y(x) = a\cos(x) + b\sin(x)$ (for arbitrary a, b), with the usual addition and multiplication by a scalar, form a vector space. Give its dimension and find a basis.

3. Are the four diagonals of a cube orthogonal?

4. Express the discrete-time impulse $\delta[n]$ in terms of the discrete-time unit step $u[n]$ and conversely.

5. Show that any function $f(t)$ can be written as the sum of an odd and an even function, i.e. $f(t) = f_o(t) + f_e(t)$ where $f_o(-t) = -f_o(t)$ and $f_e(-t) = f_e(t)$.

PROBLEM 2. Let $\{x(k)\}$, $k = 0, \ldots, N-1$, be a basis for a space S. Prove that any vector $z \in S$ is uniquely represented in this basis.

Hint. Prove by contradiction.

PROBLEM 3. Assume v and w are two vectors in the vector space. Prove the triangular inequality for each v and w.

$$||v + w|| \leq ||v|| + ||w||.$$

Hint. Expand $||v + w||^2$ and use Cauchy-Schwarz inequality.

PROBLEM 4. Consider the following signal

$$x[n] = (5 - |n|).(u[n+5] - u[n-6]).$$

Draw the following signals:

- $x[n]$.
- $x[-2n+3]$.
- $\sum_{k=-\infty}^{n} x[2k]$.

PROBLEM 5. Find the inverse z-transform of following series.

(a) $X(z) = \frac{1}{(1-1/4z^{-1})(1-1/2z^{-1})}$, $|z| > 1/2$.

(b) $X(z) = \frac{1}{(1-1/5z^{-1})(1+3z^{-1})}$, $3 > |z| > 1/5$.