ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

**Handout 26** Signal Processing for Communications
Homework 11 May 10, 2010

PROBLEM 1. Consider a zero-mean white random process $X[n]$ with autocorrelation function $r_x[m] = \sigma^2\delta[m]$. The process is filtered with a 2−tap FIR filter whose impulse response is $h[0] = h[1] = 1$. Compute the values of the autocorrelation for the output random process $Y[n] = X[n] * h[n]$ from $n = -3$ to $n = 3$.

PROBLEM 2. Consider the stochastic process defined by

$$Y[n] = X[n] + \beta X[n-1],$$

where $\beta \in \mathbb{R}$ and $X[n]$ is a zero-mean wide-sense stationary process with autocorrelation function given by

$$R_x[k] = \delta^2 a^{|k|}$$

for $|a| < 1$.

(a) Compute the power spectral density $P_Y(e^{jw})$ of $Y[n]$.

(b) For which values of $\beta$ does $Y[n]$ corresponds to a white noise? Explain.

PROBLEM 3. Consider a stationary i.i.d. random process $x[n]$ whose samples are uniformly distributed over the $[-1, 2]$ interval. The process is quantized with a 1−bit quantizer $\mathcal{Q}\{\}$ with the following characteristic:

$$\mathcal{Q}\{x\} = \begin{cases} -1 & \text{if } x < 0 \\ +1 & \text{if } x \geq 0 \end{cases}$$

Compute the signal to noise ratio at the output of the quantizer.

PROBLEM 4. Consider the quantizer $\mathcal{Q}$ which takes $X$ that has a value in the interval $[0, 1]$ and outputs $\hat{X}$, the first $r$ bits of its binary expansion. Assume we feed $X$ with the following density function into the quantizer:

$$f_X(x) = \frac{1}{2} + bx.$$

(a) Find $b$.

(b) Find $Pr(\hat{X} = \hat{x})$.

(c) Around a point $y$, i.e., in a small interval $[y, y + \delta]$ what fraction of points out of the $K = 2^r$ should there be in this interval approximately; in particular how does this relate to $f(x)$.