ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 1 Signal Processing for Communications
Homework 1 February 22, 2010

PROBLEM 1. Decide whether the following signals are periodic, and if so, find the period.

(a) $x[n] = e^{j\frac{\pi}{\sqrt{2}}n}$.

(b) $x[n] = \frac{\sin(\pi n)}{\pi n}$.

(c) $x[n] = \sin(n)$.

(d) $x[n] = 1 + \sin^2(\pi n)$.

(e) $x[n] = e^{j\frac{5\pi}{7}n} + e^{j\frac{3\pi}{4}n}$.

PROBLEM 2. Compute the following sums.

(a) $\sum_{n=i}^{j} a^n$.

(b) $\sum_{n=1}^{\infty}(\frac{1}{2} + j\frac{\sqrt{3}}{2})^n$.

(c) $\sum_{k=1}^{n} \sin(2\pi\frac{k}{N}), \qquad n < N$.

(d) $\sum_{n=1}^{\infty} e^{(1/2+j3/4)n}$.

Hint. Just a simple explanation is enough. Remember to check first if the sum is finite and then try to compute it.

PROBLEM 3. Compute the following integrals.

(a) $\int_0^{\infty} \frac{1}{1+x^4}\,dx$.

(b) $\int_{-\infty}^{\infty} \frac{\cos sx}{k^2+x^2}\,dx$.

Hint. Relate this integral to $\oint_C \frac{e^{jsz}}{k^2+z^2}\,dz$, and find the proper C.

(c) $\int_0^{2\pi} \frac{\sin\theta}{34-16\sin\theta}\,d\theta$.

Hint. Euler once said "$\sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$". Relate this integral to $\oint_C f(z)\,\frac{dz}{jz}$, so that as θ ranges from 0 to 2π, the variable $z = e^{j\theta}$ ranges counterclockwise once around the unit circule $|z| = 1$.

Hint. Use the residue integration method.

`http://en.wikipedia.org/wiki/Methods_of_contour_integration`

PROBLEM 4. Find the z-transform OF following series.

(a) $x[n] = a^n u[n]$.

(b) $x[n] = na^n u[n]$.

Hint. Do not forget to provide the region of convergence.

PROBLEM 5. Find the inverse z-transform of following series.

(a) $X(z) = \frac{1}{(1-1/4z^{-1})(1-1/2z^{-1})}$, $|z| > 1/2$.

(b) $X(z) = \frac{1}{(1-1/5z^{-1})(1+3z^{-1})}$, $3 > |z| > 1/5$.