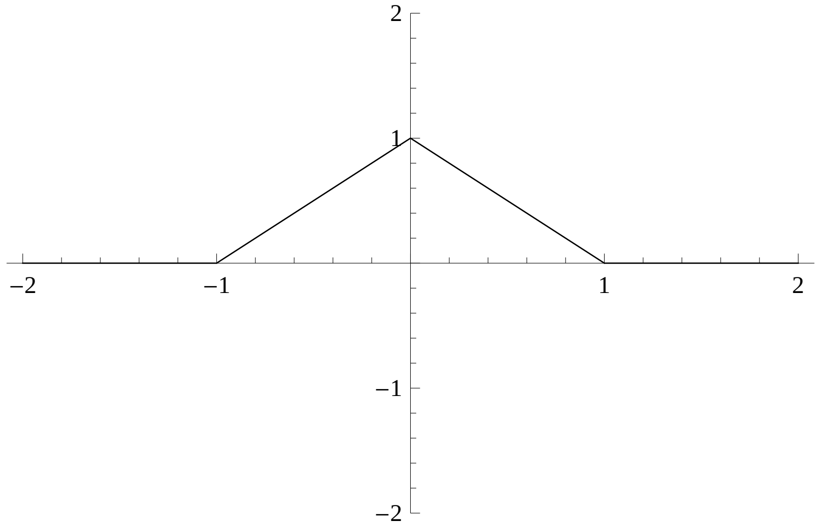
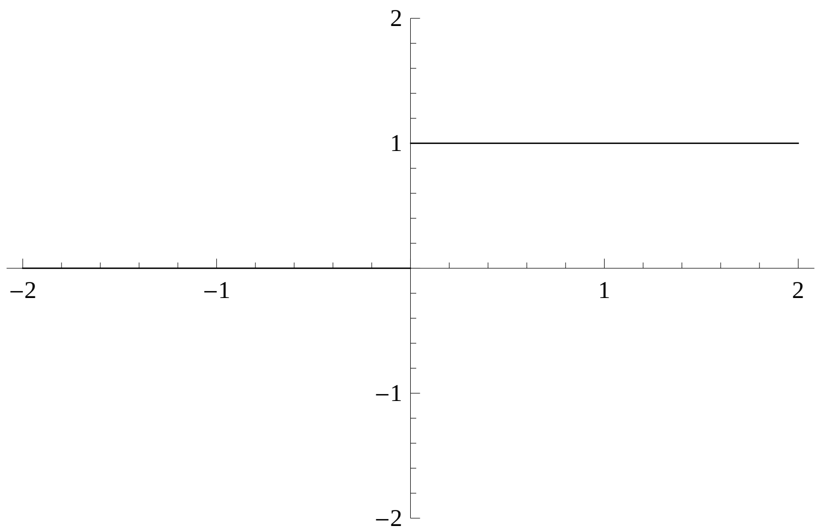
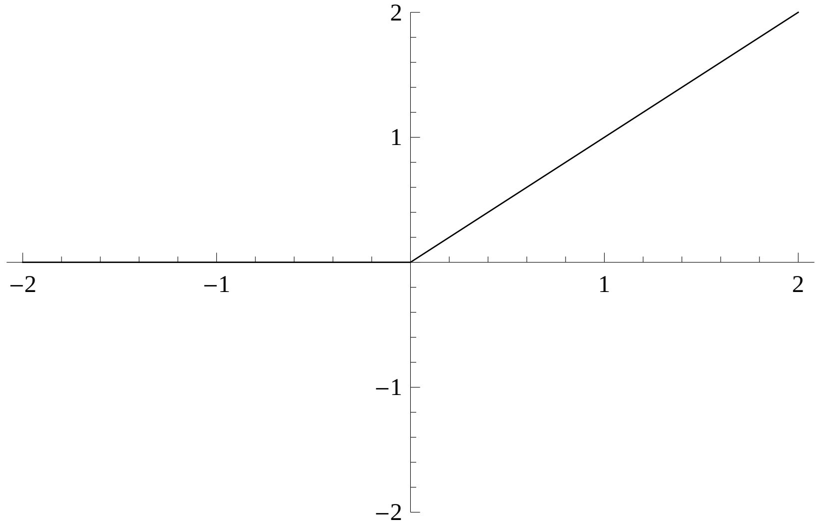
Handout 8 | Introduction to Communication Systems
Solution 4 | October 22, 2009

Problem 1



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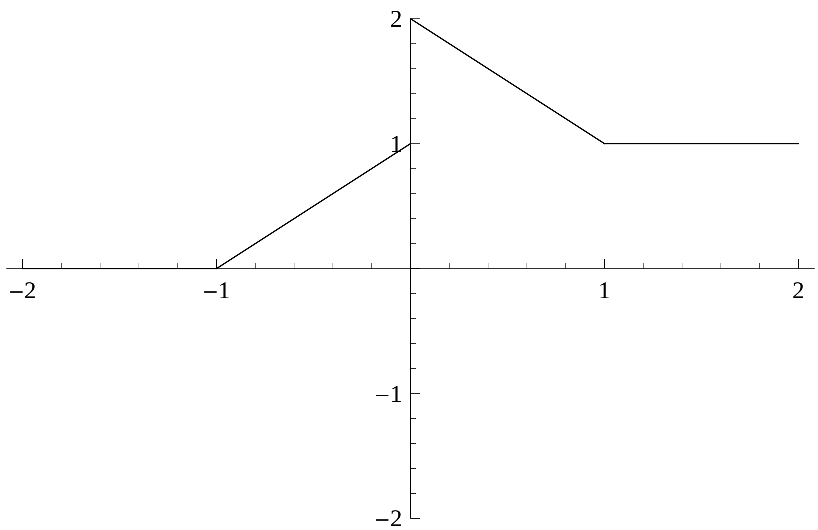
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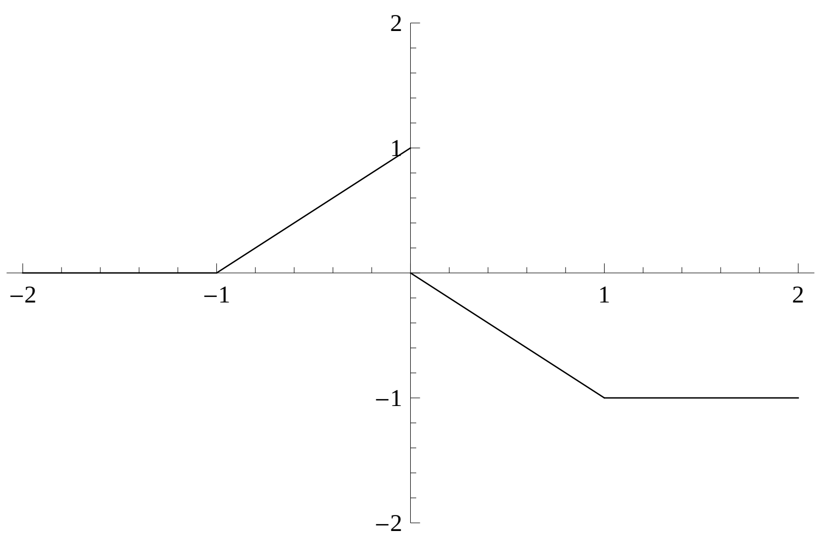
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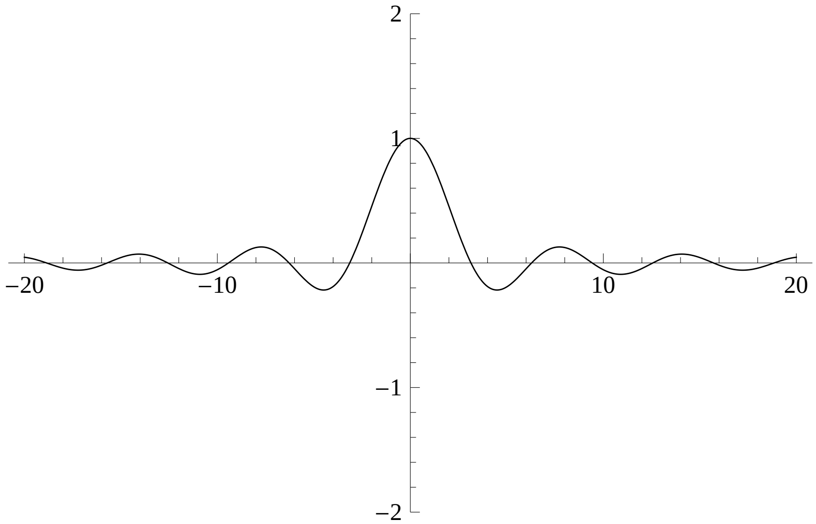


Problem 2

$$A\sin(2\pi v t+\phi)=5\cos\left(10t+\frac{\pi}{2}\right)=5\sin\left(\frac{5}{\pi}(2\pi t)+\pi\right)$$

$A=5$, $T=\pi/5$, $\phi=\pi$.

Problem 3

a) We have

$$0=A_1\left(\sin(2\pi v_1 t+\phi_1)-\sin(2\pi v_1(t+T)+\phi_1)\right)+A_2\left(\sin(2\pi v_2 t+\phi_2)-\sin(2\pi v_2(t+T)+\phi_2)\right)$$

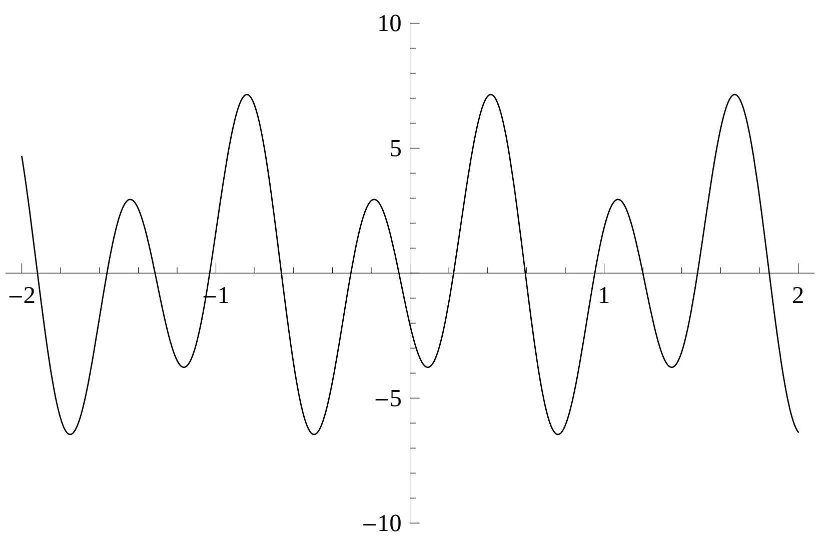
Then

$$T=\frac{p}{v_1}\text{ and }T=\frac{q}{v_2}$$

where $p,q\in\mathbb{N}$ and thus leads to the following condition

$$\frac{v_1}{v_2}=\frac{p}{q}\in\mathbb{Q}$$

b) $v_1=\frac{10}{2\pi}$, $v_2=\frac{5}{2\pi}$, $\frac{v_1}{v_2}=\frac{p}{q}=\frac{2}{1}$ then $T=\frac{p}{v_1}=\frac{2\pi}{5}$.



Problem 4

a) By change of integration variable $\tau = t - s$ we get

$$\begin{aligned}(x * y)(t) &= \int_{-\infty}^{\infty} x(s)\ \ y(t-s)\,ds \\ &= \int_{\infty}^{-\infty} x(t-\tau)\ \ y(\tau)\,(-d\tau) \\ &= \int_{-\infty}^{\infty} x(t-\tau)\ \ y(\tau)\,d\tau \\ &= (y * x)(t)\end{aligned}$$

b)

$$\begin{aligned}\sum_{t=-\infty}^{\infty} z_t &= \sum_{t=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} x_s\ y_{t-s} \\ &= \sum_{s=-\infty}^{\infty}\sum_{t=-\infty}^{\infty} x_s\ y_{t-s} \\ &= \sum_{s=-\infty}^{\infty} x_s \sum_{t=-\infty}^{\infty}\ y_{t-s} \\ &= \sum_{s=-\infty}^{\infty} x_s \sum_{\tau=-\infty}^{\infty}\ y_{\tau}\end{aligned}$$

Thus

$$\begin{aligned}0 &= \sum_{t=-\infty}^{\infty} z_t - \sum_{s=-\infty}^{\infty} x_s \\ &= \sum_{s=-\infty}^{\infty} x_s \sum_{\tau=-\infty}^{\infty}\ y_\tau - \sum_{s=-\infty}^{\infty} x_s \\ &= \sum_{s=-\infty}^{\infty} x_s \left(\sum_{\tau=-\infty}^{\infty}\ y_\tau - 1\right)\end{aligned}$$

and

$$\sum_{\tau=-\infty}^{\infty}\ y_\tau = 1$$