PROBLEM 1. 1. Since $a \equiv a'; (\bmod m)$, it implies that $a - a' = km$ for some integer k. On the other hand we have:

$$a^t - a'^t = (a - a')(a^{t-1} + a^{t-2}a' + \cdots + a'^{t-1}) = mk(a^{t-1} + a^{t-2}a' + \cdots + a'^{t-1})$$

Therefore, $a^t - a'^t$ is divisible by m, which implies that $a^t \equiv a'^t; (\bmod m)$.

2. The converse is not necessarily true. For example $(-1)^2 \equiv (+1)^2; (\bmod 5)$ but $(-1) \neq (+1) (\bmod 5)$.

PROBLEM 2. We have

$$a^3 + 3 = a^3 + 27 - 24 = a^3 + 3^3 - 24 = (a + 3)(a^2 - 3a + 9) - 24$$

So, if $a^3 + 3$ is devisible by $a + 3$, $(a + 3)(a^2 - 3a + 9) - 24$ should be also divisible by $a + 3$. Therefore 24 is devisible by $a + 3$. But we know that the only factors of 24 are : $1, 2, 3, 4, 6, 8, 12, 24$. This means that $a + 3$ should be one of these numbers. Since a is assumed to be a positive integer number, the only possibilities for a are $a = 1, 3, 5, 9,$ or 21.

PROBLEM 3. Since n is assumed to be an odd number, we can write $n = 2k + 1$

1. We have:

$$n^2 - 1 = (n - 1)(n + 1) = (2k + 1 - 1)(2k + 1 + 1) = (2k)(2k + 2) = 4k(k + 1).$$

Notice that k and $k + 1$ are two consecutive numbers. Therefore one of them is even and the other one is odd. In both cases, the product of them is an even number. So, $k(k + 1) = 2m$. Therefore:

$$n^2 - 1 = 4k(k + 1) = 4(2m) = 8m.$$

This shows that $n^2 - 1$ is a mutiple of 8.

2. We will factor $n^8 - 1$ as:

$$n^8 - 1 = (n^4 - 1)(n^4 + 1) = (n^2 - 1)(n^2 + 1)(n^4 + 1).$$

From the first part of the question we know that $n^2 - 1$ is divisible by 8. So, $n^2 - 1 = 8m$. Also, we know that the other factors are even numbers, since n is an odd number. Therefore we have:

$$n^8 - 1 = (n^2 - 1)(n^2 + 1)(n^4 + 1) = 8m \times 2l \times 2p = 32mlp.$$

This shows that $n^8 - 1$ is divisible by 32.

PROBLEM 4. First we write the congruence equation as

$$7n \equiv -5; (\bmod 2009)$$

Now, we need to find $7^{-1} (\bmod 2009)$. Using Euclid's algorithm, we can show that $7^{-1} \equiv 287; (\bmod 2009)$. Now, if we multiply both sides of the equation with 287 we have:

$$287 \times 7n \equiv 287 \times (-5); (\bmod 2009)$$

Therefore :

$$n \equiv -1435 \equiv 574; (\bmod 2009).$$