PROBLEM 1. • If \( a \equiv a' \pmod{m} \), show that for any integer \( t \), \( a^t \equiv a'^t \pmod{m} \).

• Is the converse true? (i.e. if \( a^t \equiv a'^t \pmod{m} \) for some \( t \geq 2 \), can we always conclude that \( a \equiv a' \pmod{m} \))

PROBLEM 2. For which positive integer numbers \( a \), is \( a^3 + 3 \) divisible by \( a + 3 \)? (Hint: \( 3 = 27 - 24 \))

PROBLEM 3. Prove that if \( n \) is an odd integer number then:

• \( n^2 - 1 \) is divisible by 8

• \( n^8 - 1 \) is divisible by 32

PROBLEM 4. Find all the integer numbers \( n \) such that \( 7n + 5 \equiv 0 \pmod{2009} \).