

PROBLEM 1. 1. Calculate $5^{21} \bmod 8$.

2. Calculate $31^{201} \bmod 33$.

3. What are the last two digits of 9^{30} ? (Hint: Compute $9^{10} \bmod 100$)

PROBLEM 2. In each part, decide if the inverse exists. In the case that inverse exists, compute it using Bezout's algorithm.

1. $5^{-1} \bmod 26$.

2. $11^{-1} \bmod 36$.

3. $14^{-1} \bmod 35$

PROBLEM 3. Let m, p, q be distinct prime numbers.

1. Prove that $\phi(m^4) = m^3(m - 1)$.

2. Show that $\phi(pq) = (p - 1)(q - 1)$.

PROBLEM 4. 1. Find the Euler Totient function, $\phi(42)$.

2. Using the Euler Totient function find $11^{-1} \bmod 42$.

PROBLEM 5. 1. Find x such that $3^x \equiv 5 \pmod{17}$.

2. Find another solution for x .

3. Find x such that $3^x \equiv 5 \pmod{15}$ if such an x exists.