ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 3 Introduction to Communication Systems
Homework 2 September 24, 2009

PROBLEM 1. (*carry over from HW1*) For each of the following three codes, say if it is uniquely decodable. If so, is it instantaneous?

	Code 1	Code 2	Code 3
s_1	0	0	0
s_2	1	10	01
s_3	00	110	011
s_4	11	111	111

PROBLEM 2. Suppose we have a source S, which emits 5 symbols a, b, c, e, l.

1. Assume that we use the following code for encoding the source stream. What is the

Symbol	Code
a	00
b	01
c	10
e	110
l	111

encoder output for the input stream $b\ a\ l\ e\ l\ e\ c$?

2. Assume that at the decoder you receive the bit stream {01111000111100}. What is the decoder output ?

3. Is it possible to have a uniquely decodable code for the present source S with length of the codewords restricted to be less than or equal to 2 ?

PROBLEM 3. Consider a code for a source having 5 symbols, with lengths of codewords given by $l(s_1) = 2, l(s_2) = 2, l(s_3) = 2, l(s_4) = 3, l(s_5) = 3$. The code is shown in the table below.

Symbol	Code	Probability
s_1	00	0.25
s_2	10	0.25
s_3	11	0.2
s_4	010	0.15
s_5	011	0.15

1. Is the Kraft's inequality satisfied ?

2. Is the code uniquely decodable ? If yes, explain.

3. What is the average length of the code?

PROBLEM 4. Suppose we have a source S with m symbols given by $\{1, 2, 3, \ldots, m\}$. Assume that the probability of symbol i is p_i.

1. Is it possible to construct a prefix free code with lengths of the codewords given by $l_i = \lceil \log_2(\frac{1}{p_i}) \rceil$?
   Hint: Verify Kraft's inequality.

2. If the answer to the above question is yes, then can you upper bound the average length of this code in terms of the entropy $H(S)$ of the source ?
   Hint: Use $x \leq \lceil x \rceil \leq x + 1$ for $x \geq 0$.