ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 11 | Introduction to Communication Systems
Homework 6 | October 29, 2009

This is your second graded homework. This time we have some new rules.

- **You have the right to remain silent.**
- **Only the things you write can be used for/against your evaluation.**
- **You have two weeks to hand in your solutions.**
- **Each single problem should be answered in one piece of paper, otherwise your answers will not be graded.**

Problem 1

Consider the following signal

$$x(n) = \begin{cases} 2^{n+1} - 3n^2 & \text{if } -1 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Sketch

a) $x(n)$

b) $x(n-2)$

c) $x(2-n)$

d) $x(n)\,\delta(n-1)$

e) $x(n) * \delta(n)$

f) $x(n-1) * \delta(n-2)$

Problem 2

We consider a system with the following impulse response

$$h(n) = \begin{cases} \frac{1}{2^n} & \text{if } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

a) Sketch $h(n)$

b) Is the system causal?

c) Show that for all $r \in \mathbb{R}$

$$\sum_{k=0}^{n} r^k = \frac{1 - r^{n+1}}{1 - r}$$

hint: compute the quantity $(1-r)\sum_{k=0}^{n} r^k$

d) Is the systeme stable?

e) Show that

$$\sum_{k=0}^{n} k\, r^k = r\frac{d}{dr}\left(\frac{1-r^{n+1}}{1-r}\right)$$

hint: study the quantity $\frac{d}{dr}\left(\sum_{k=0}^{n} r^k\right)$ and use the linearity of the derivative.

f) Compute the outpute signal of the input signal $x(n) = n$.

Problem 3

We have a system from which we only know a linear time-invariant recurence relation between the input $x(n)$ and the output $y(n)$, namely

$$y(n+1) = y(n) + x(n)$$
$$\lim_{m\to-\infty} y(m) = 0$$

a) Write $y(4)$ in function of $y(0)$ and x using only this relation.

b) If we know that $y(m)$ for $m < n$ then $y(n) = y(m) + \sum_{k=m}^{n-1} x(k)$. Consedering this relation, find the output solution $y(n)$. What is the action of the system?

c) Find the impulse response using only the recurence equation.

d) What is output, basing on the impulse response find in *c*)? Compare the solution with *b*).

e) Is the system stable? If not, give an example of a signal which not remain stable.

Problem 4

Consider the signal $\sin(10\pi t) + \sin(30\pi t)$

a) What are the sampling frequencies avoiding the aliasing problem?

b) If we use the sampling frequency $f_s = 20Hz$, what the sampled signal would be?

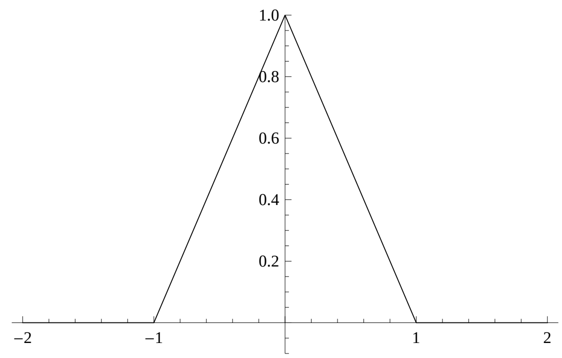
c) Same question than *b*), but we use an ideal antialiasing filter before the sampling.

d) The sampling frequency is $f_s = 25Hz$ and we use an ideal interpolator. What the reconstructed signal would be?

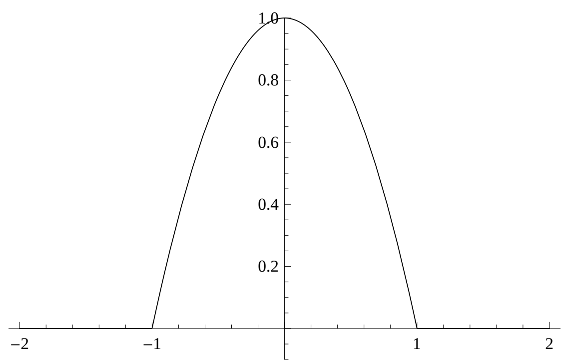
Problem 5

We have a continuous signal $x(t)$ with a missing part between $t = 1$ and $t = -1$. The signal is zero everywhere and the only information we know in the missing part is $x(0) = 1$. We want to interpolate this signal.

a) Find the linear interpolation between $\{x(-1), x(0)\}$ and $\{x(0), x(1)\}$, i.e. find the coefficient of the two linear functions $at + b$. For example the first linear function verify $-1 \cdot a + b = x(-1)$ and $0 \cdot a + b = x(0)$.



b) Find the quadratic interpolation between $\{x(-1), x(0), x(1)\}$, i.e. for interpolation $at^2 + bt + c$.



c) In order to have a smooth interpolation in the part $\{x(-1), x(0)\}$, find the cubic interpolation,$at^3+bt+ct+d$, with the condition that the derivative of the interpolation vanish at $t = -1$ and $t = 0$.