Introductory Quiz

SURNAME: ...................... FIRST NAME: ...................... SECTION: ......................

1. Let $P$ be a probability and $A$, $B$ be two events. Among the following identities, which are always true?

- \( a) \quad P(A \cup B) = P(A) + P(B). \)
- \( b) \quad P(A \cup B) = P(A) + P(B) - P(A \cap B). \)
- \( c) \quad P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ if } P(B) \neq 0. \)
- \( d) \quad P(A \cap B) = P(A) \cdot P(B). \)

2. Gaussian random variables

a) Let $X$ be a random variable such that $P(X \geq 0) = 1$. Can $X$ be a Gaussian random variable?

b) Let $Y$ be a Gaussian random variable with mean $\mu$. Is it true that $P(Y \geq \mu) = 1/2$?

c) Let $Z$ be a random variable such that $P(Z = 1) = 1/2$. Can $Z$ be a Gaussian random variable?

3. You throw a (fair) coin 1000 times. What is the probability that you end up with exactly 500 tails and 500 heads?

- \( a) \quad 1/2 \)
- \( b) \quad 0 \)
- \( c) \quad \text{approximately } 0.025 \)

4. Name three scientists whose names are associated to distributions of random variables.

a) .................................. b) .................................. c) ............................

5. Let $X$ be a random variable such that $P(X = 1) = P(X = 0) = 1/2$.

a) What is the mean of $X$? ...........  

b) What is variance of $X$? ...........

Let now $Y$ be the random variable defined as $Y = 2X - 1$.

c) What is the mean of $Y$? ...........  

d) What is variance of $Y$? ...........

6. Which of the following statements is true?

- \( a) \quad \text{If } \text{Cov}(X,Y) = 0, \text{ then } X \text{ and } Y \text{ are independent.} \)
- \( b) \quad \text{If } X \text{ and } Y \text{ are independent, then } \text{Cov}(X,Y) = 0. \)
- \( c) \quad X \text{ and } Y \text{ are independent if and only if } \text{Cov}(X,Y) = 0. \)

7. Subsidiary question: give the definition of convergence in probability:

$X_n \xrightarrow{P} X$ means ............................................................

Stochastic Calculus I  
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