PROBLEM 1 (PROBLEM 7.1).

1.

$$\begin{aligned} y_0[n] &= y_0[n-1] + x[n] \\ y_1[n] &= y_1[n-1] + x[n-1] \\ y[n] &= y_0[n] - y_1[n]. \end{aligned}$$

From zero initial conditions for $y_0[n]$,

$$\begin{aligned} y_0[0] &= x[0] = 0 \\ y_0[1] &= y_0[0] + x[1] = x[1] + x[0] \\ y_0[2] &= y_0[1] + x[2] = x[2] + x[1] + x[0] \\ y_0[3] &= y_0[2] + x[3] = x[3] + x[2] + x[1] + x[0] \\ y_0[n] &= y_0[n-1] + x[n] = x[n] + x[n-1] + \ldots + x[1] + x[0] = \sum_{l=0}^{n} x[l]. \end{aligned}$$

Similarly, for $y_1[n]$

$$\begin{aligned} y_1[0] &= x[-1] = 0 \\ y_1[1] &= y_1[0] + x[0] = 0 \\ y_1[2] &= y_1[1] + x[1] = x[1] \\ y_1[3] &= y_1[2] + x[2] = x[2] + x[1] \\ y_1[n] &= y_1[n-1] + x[n-1] = x[n-1] + x[n-2] + \ldots + x[1] + x[0] = \sum_{l=0}^{n-1} x[l]. \end{aligned}$$

Then

$$y[n] = \sum_{l=0}^{n} x[l] - \sum_{l=0}^{n-1} x[l] = x[n].$$

2.

$$\begin{aligned} Y_0(z) &= Y_0(z)z^{-1} + X(z) \Rightarrow Y_0(z) = \frac{1}{1-z^{-1}}X(z) \\ Y_1(z) &= Y_1(z)z^{-1} + X(z)z^{-1} \Rightarrow Y_1(z) = \frac{z^{-1}}{1-z^{-1}}X(z) \\ Y(z) &= Y_0(z) - Y_1(z) \Rightarrow Y_1(z) = \frac{1-z^{-1}}{1-z^{-1}}X(z) = X(z). \end{aligned}$$

3. Since $y[n] = x[n]$, the overall system is BIBO stable.

4. The system is not stable internally, both H_0 and H_1 have poles on the unit circle.

$$h_0[n] = u[n] \quad h_1[n] = u[n-1].$$

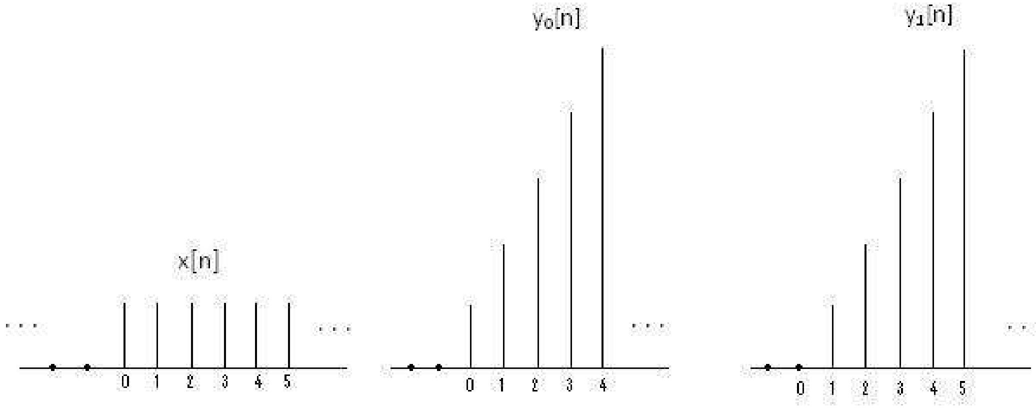

Figure 1: The evolution of $y_0[n]$ and $y_1[n]$

5. The evolution of $y_0[n]$ and $y_1[n]$ are shown in Figure 1.

6. The system is not stable internally. Both $y_0[n]$ and $y_1[n]$ sum up to infinity and can not be represented with finite precision. Then the output can not be determined. The system can not be implemented in practice.

PROBLEM 2 (PROBLEM 7.2). A casual IIR filter with impulse response $g[n]$ and transfer function $G(z)$ is represented as

$$G(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \ldots}{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + \ldots}$$

1. For the inverse system $1/G(z)$ to be stable we should check the poles which corresponds to the zeros of $G(z)$. $G(z)$ is stable then the poles are inside the unit circle but there is no information about the zeros of $G(z)$. Then we can not determine whether the inverse system is stable or not.

2. For the inverse system to be FIR the following equation should be satisfied

$$\frac{1}{G(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + \ldots}{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \ldots} = c_0 + c_1 z^{-1} + c_2 z^{-2} + c_3 z^{-3} + \ldots$$

   Again we should check the zeros of $G(z)$ in order to conclude if the inverse system $1/G(z)$ is FIR or not.The above equation is satisfied if the coefficients $\{a_1, a_2, \ldots\}$ are 0, however we don't have any information about the zeros of $G(z)$. Then we can not determine whether the inverse system is FIR or not.

3. The DTFT of $g[n]$ exists since it is a stable system. The ROC consists of the unit circle so we can compute the DTFT by $G(e^{j\omega}) = G(z)|_{z=e^{j\omega}}$

4. The cascade system is stable since all the poles of the cascaded system will be inside the unit circle and ROC consists unit circle. The cascaded system $G(z)G(z)$ have twice as many poles as the original system $G(z)$.

PROBLEM 3 (PROBLEM 7.3).

1. False. Zeros of $G(z)$ can be anywhere in the complex plane.

2. False. For example, the ROC could be $|z| > 0.6$. Such a system is casual and stable.

3. True. The new system has only an extra pole with respect to the original one and as mentioned in part 1, the zeros do not matter in determining the ROC.

4. False. For example, we could have $G(z) = 1$ which has $h[n] = \delta[n]$ (FIR filter).

PROBLEM 4 (PROBLEM 7.4.). 1. This is a periodic signal with period 4. We have $x[n] = -\cos(\frac{n\pi}{2})$ which gives $x[0] = -1$, $x[1] = 0$, $x[2] = 1$, $x[3] = 0$.

2. Since the signal is periodic (and therefore not finite in length and also not absolutely summable), the DFS would be the most appropriate Fourier representation. The DFS is also a periodic signal with period 4. We have:

$$X_k = \sum_{n=0}^{3} x[n] e^{-j\frac{2\pi n}{4}k}$$

Thus, $X_0 = X_2 = 0$ and $X_1 = X_3 = -2$.

3. We have: $x[n] = -\cos(\frac{n\pi}{2}) = -\frac{e^{\frac{jn\pi}{2}} + e^{\frac{-jn\pi}{2}}}{2}$. Thus the DTFT (in one period) would be: $-\frac{1}{2}(\delta(w - \frac{\pi}{2}) + \delta(w + \frac{\pi}{2}))$.

4. $h[n]$ is a low pass filter with cutoff frequency $\omega_c = 1$. Thus multiplying the DTFTs we get zero as output.