PROBLEM 1 (PROBLEM 5.6 IN THE BOOK). Consider the following input-output relations and, for each of the underlying systems, determine whether the system is linear, time invariant, BIBO stable, causal or anti-causal. Characterize the eventual LTI systems by their impulse response.

- \( y[n] = x[-n] \).
- \( y[n] = e^{-j\omega n}x[n] \).
- \( y[n] = \sum_{k=n-n_0}^{n+n_0} x[k] \).
- \( y[n] = ny[n-1] + x[n] \) such that if \( x[n] = 0 \) for \( n < n_0 \), then \( y[n] = 0 \) for \( n < n_0 \).

PROBLEM 2 (PROBLEM 5.7 IN THE BOOK). Derive the impulse response of a bandpass filter with center frequency \( \omega_0 \) and passband \( \omega_b \):

\[
H_{bp}(e^{j\omega}) = \begin{cases} 
1 & \omega_0 - \omega_b \leq \omega \leq \omega_0 + \omega_b, \\
1 - \omega_0 - \omega_b & -\omega_0 - \omega_b \geq \omega \geq -\omega_0 + \omega_b, \\
0 & \text{elsewhere.}
\end{cases}
\]

(Hint: Consider the following ingredients: a cosine of frequency \( \omega_0 \), a lowpass filter of bandwidth \( \omega_b \) and the modulation theorem.)

PROBLEM 3. For each of the following sequences \( x[n] \), find the z-transform \( X(z) \) and the corresponding region of convergence, and sketch the pole-zero diagram.

- \( x[n] = \delta[n-3] + \delta[n+3] \)
- \( x[n] = na^n u[n] \)
- \( x[n] = -na^n u[-n-1] \)
- \( x[n] = 2^n u[n] - 3^n u[-n-1] \)
- \( x[n] = e^{n^2 \cos(\frac{\pi}{12} n)} u[n] - e^{n^2 \cos(\frac{\pi}{12} n)} u[n-1] \)

PROBLEM 4. An LTI system is described by the difference equation

\( y[n] = \frac{9}{2} y[n-1] + \frac{5}{2} y[n-2] + x[n-1] + x[n] \)

(i). Find the system response \( H(z) = Y(z)/X(z) \), and plot its pole-zero diagram.

(ii). Find the region of convergence of \( H(z) \) when the system is

- anticausal, unstable.
- causal, unstable.
- causal, stable.
• noncausal, stable.

• Find the impulse response $h[n]$ of each of the above systems.

**Problem 5 (Problem 5.36 in Oppenheim, Schafer, Buck).** A causal linear time-invariant system has the system function

$$H(z) = \frac{(1 - 1.5z^{-1} - z^{-2})(1 + 0.9z^{-1})}{(1 - z^{-1})(1 + 0.7jz^{-1})(1 - 0.7jz^{-1})}$$

1. Write the difference equation that is satisfied by the input and the output of the system.

2. Plot the pole-zero diagram and indicate the region of convergence for the system function.

3. State whether the following are true or false about the system:
   • The system is stable.
   • The impulse response approaches a constant for large $n$.
   • The system has a stable and causal inverse.