See next page.
Prob 2 (1.1 in the book)

Let \( y[n] \) be the output process, i.e., \( y[n] = Q\{x[n]\} \). Clearly \( y[n] \) is also a stationary \( i.i.d. \) random process. We further have:

\[
\begin{align*}
    y[n] &= \left\{ \begin{array}{ll}
        +1 & \text{with probability } \frac{1}{4} \to 0.5 \leq x[n] \leq 1 \\
        0 & \text{with probability } \frac{1}{2} \to -0.5 \leq x[n] \leq 0.5 \\
        -1 & \text{with probability } \frac{1}{4} \to -1 \leq x[n] < -0.5
    \end{array} \right. \\
    \Rightarrow \quad R_y[k] &= E[y[n]y[n+k]] = \left\{ \begin{array}{ll}
        E[y[n]y[n+k]] & k \neq 0 \\
        E[y[n]^2] & k = 0
    \end{array} \right.
\end{align*}
\]

As \( y[n] \) is \( i.i.d. \), then for \( k \neq 0 \) we have

\[
E[y[n]y[n+k]] = E[y[n]]E[y[n+k]] = (E[y[n]])^2
\]

\[
\Rightarrow \quad \{E[y[n]]\} = -1 \times \frac{1}{4} + 0 \times \frac{1}{2} + 1 \times \frac{1}{4} = 0
\]

\[
\{E[y[n]]^2\} = +1 \times \frac{1}{2} + 0 \times \frac{1}{2} = 0
\]

\[
\Rightarrow \quad R_y[n] = \left\{ \begin{array}{ll}
        \frac{1}{2} & k = 0 \\
        0 & k \neq 0
    \end{array} \right. \Rightarrow R_y[n] = \frac{1}{2} S[n]
\]

\[
P_y(e^{j\omega}) = \text{Fourier Transform of } R_y[n] = \frac{1}{2}
\]

Prob 2 (1.1 in the book)

If we down sample \( X(z) \) by \( K \) (\( X[n] \rightarrow \frac{1}{K}X[k] \)), then the Z-Transform of the output would be:

\[
X(z) \rightarrow \frac{1}{K}X(z^{1/K})
\]

If we up sample \( X(z) \) by \( K \) (\( X[n] \rightarrow X[Kn] \)), then the Z-Transform of the output would be:

\[
X(z^K)
\]

So if we downsample by \( 2 \), the output would have a Z-transform of \( \frac{1}{2}(X(z^{1/2})+X(-z^{1/2})) \) and if we upsample by \( 2 \), the output would be \( X(z^2) \).
Problem

Prob. 3 (H.2 in the book)
1. If we use the identities in 11.1, we get:

\[ y(z) = H(z) H(z^2) X(z) \]
2.

\[ X(z) \xrightarrow{2} H(z) \xrightarrow{2} Y(z) \]

(1) \[ X(z^2) \]

(2) \[ Y(z^2) H(z) \]

(3) \[ \frac{1}{2} \left[ X(z) H(z^2) + X(z+1) H(-z^2) \right] \]

Now, \[ H(z) = \frac{e_0(z^2) + z^{-1} e_1(z^2)}{2} \], thus by replacing \[ H(z) \] in (3) we have:

\[ Y(z) = \frac{1}{2} \left[ X(z) \left( e_0(z) + z^{-1/2} e_1(z) \right) + X(z) \left( e_0(z) - z^{-1/2} e_1(z) \right) \right] \]

\[ = \frac{1}{2} X(z) \left[ e_0(z) + e_0(z) + z^{-1/2} (e_1(z) - e_1(z)) \right] \]

\[ = \frac{1}{2} X(z) \left[ 2 e_0(z) \right] = X(z) e_0(z) \]

3.

\[ X(z) \xrightarrow{2} G(z) H(z) \xrightarrow{2} Y(z) \]

If we write \[ H(z) G(z) \] as \[ e_0(z^2) + z^{-1} e_1(z^2) \], then we have:

Case 1: \[ H(z) G(z) + H(-z) G(-z) = 2 \Rightarrow e_0(z^2) + z^{-1} e_1(z^2) + e_0(z^2) - z^{-1} e_1(z^2) = 2 \]

\[ \Rightarrow 2 e_0(z^2) = 2 \Rightarrow e_0(z^2) = 1 \]

\[ \Rightarrow e_0(z) = 1 \]

According to part 2 (above), we know that the output is:

\[ Y(z) = X(z) G(z) \Rightarrow Y(z) = X(z) (\text{unity}) \]
Case 2.

Writing $H(z)G(z) = G_0(z^2) + z^2 E(z^2)$, by a similar way as Case 1, we get $G_0(z) = 0$, thus as $Y(z) = E(z)X(z)$, then the output would be zero.

**Prob 41 (11.4 in the book)**

$y_1[n]; y_2[n]:$

$X(e^{j\omega}) \xrightarrow{L(z)} Y_1(z) \xrightarrow{U^n} Y_2(z) \xrightarrow{H(z)} Y_2(z)$

\[ y_1[n] \xrightarrow{} \]

\[ y_2[n] \xrightarrow{} \]

\[ y_3[n] \xrightarrow{} \]

\[ y_4[n] \xrightarrow{} \]

\[ y_5[n] \xrightarrow{} \]

\[ y_2(e^{j\omega}) \]

\[ y_2(e^{j\omega}) \]
$y_3(z^n), y_4(z^n)$

$\chi(e^{j\omega}) \rightarrow H(z) \rightarrow 2 \downarrow \rightarrow 4 \uparrow \rightarrow L(z) \rightarrow y_3(e^{j\omega})$

$\chi(e^{j\omega}) \rightarrow H(z) \rightarrow y_4(e^{j\omega})$