ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 24 Signal Processing for Communications
Solutions to Homework 12 June 4, 2009

See next page.

Prob 1 (10.1 in the book)

Let $y[n]$ be the output process, i.e. $y[n] = Q\{x[n]\}$. Clearly $y[n]$ is also a stationary iid random process. we further have:

$$y[n] = \begin{cases} +1 & \text{with probability } \frac{1}{4} \rightarrow 0.5 \le x[n] \le 1 \\ 0 & \text{with probability } \frac{1}{2} \rightarrow -.5 \le x[n] \le 0.5 \\ -1 & \text{with probability } \frac{1}{4} \rightarrow -1 \le x[n] < -0.5 \end{cases} \quad (*)$$

$$\Rightarrow R_y[k] = E[y[n]\, y^*[n+k]] = \begin{cases} E[y[n]y[n+k]] & k \neq 0 \\ E[(y[n])^2] & k = 0 \end{cases}$$

As $y[n]$ is iid, then for $k \neq 0$ we have $E[y[n]y[n+k]] = E[y[n]]E[y[n+k]]$
$= (E[y[n]])^2$

$$\xrightarrow{(*)} \begin{cases} E[y[n]] = -1 \times \frac{1}{4} + 0 \times \frac{1}{2} + +1 \times \frac{1}{4} = 0 \\ E[(y[n])^2] = +1 \times \frac{1}{2} + 0 \times \frac{1}{2} = 0 \end{cases}$$

$$\Rightarrow R_y[n] = \begin{cases} \frac{1}{2} & n = 0 \\ 0 & n \neq 0 \end{cases} \Rightarrow R_y[n] = \frac{1}{2}\delta[n]$$

$P_y(e^{j\omega}) =$ Fourier Transform of $R_y[n] = \frac{1}{2}$

Prob 2 (11.1 in the book)

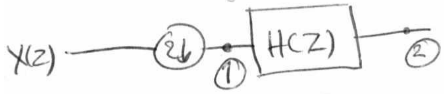
If we down sample $X(z)$ by K ($x[n] \rightarrow (K\downarrow) \rightarrow$), then the Z-Transform of the output would be: $\frac{1}{K}\sum_{i=1}^{K} X(w_K^i z^{\frac{1}{K}})$ where $w_K = e^{j\frac{2\pi}{K}}$.

If we up sample $X(z)$ by K ($x[n] \rightarrow (K\uparrow) \rightarrow$), then the Z-Transform of the output would be: $X(z^K)$

So if we downsample by 2, the output would have a Z-transform as $\frac{1}{2}(X(z^{\frac{1}{2}}) + X(-z^{\frac{1}{2}}))$ and if we upsample by 2, the output would be $X(z^2)$.

page 2

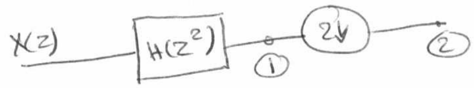
1.



$① \rightarrow \frac{1}{2}\left(X(z^{+1/2}) + X(-z^{+1/2})\right)$

$② \rightarrow \frac{H(z)}{2}\left(X(z^{1/2}) + X(-z^{1/2})\right)$

$\Rightarrow \text{output} = \frac{H(z)}{2}\left(X(z^{1/2}) + X(-z^{1/2})\right)$



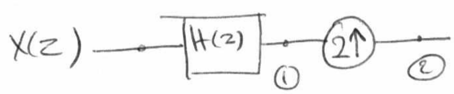
$① \rightarrow X(z)\,H(z^2)$

$② \rightarrow \frac{1}{2}\left(X(z^{1/2})H((z^{1/2})^2) + X(-z^{1/2})H((-z^{1/2})^2)\right)$

$\Rightarrow \text{output} = \frac{1}{2}H(z)\left(X(z^{1/2}) + X(-z^{1/2})\right)$

equal

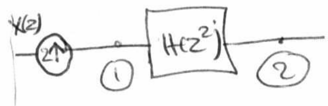
2.



$① \rightarrow X(z)\,H(z)$

$② \rightarrow X(z^2)\,H(z^2)$

$\Rightarrow \text{output} = X(z^2)H(z^2)$



$① \rightarrow X(z^2)$

$② \rightarrow X(z^2)H(z^2)$

$\Rightarrow \text{output} = X(z^2)H(z^2)$

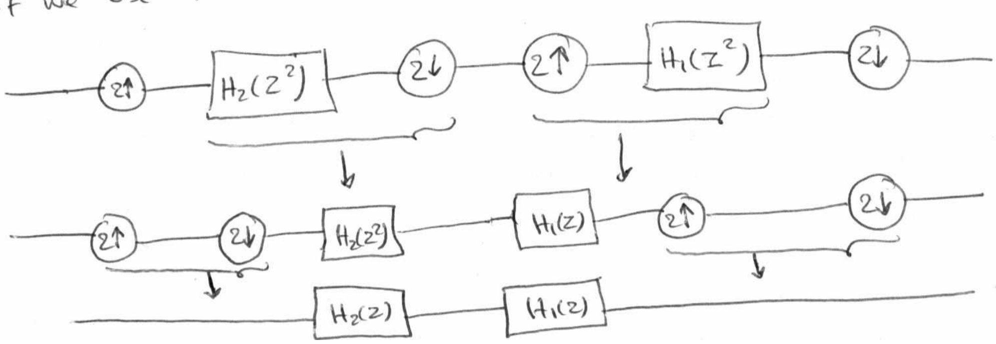
equal

Problem

Prob. 3 (H.2 in the book)

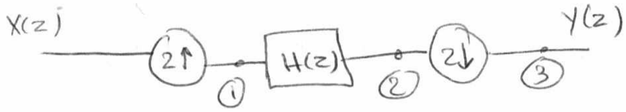
1.

If we use the identities in 11.1, we get:



$\rightarrow y(z) = H_1(z)H_2(z)X(z)$

2.



$① \to X(z^2)$

$② \to X(z^2)H(z)$

$③ \to \frac{1}{2}\left[X(z)H(z^{1/2}) + X(+z)H(-z^{1/2}) \right]$ $(*)$

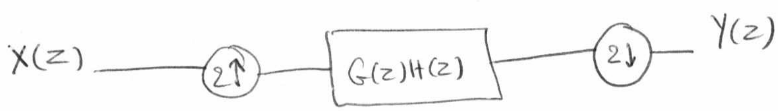
Now. $H(z) = E_0(z^2) + z^{-1}E_1(z^2)$, Thus by replacing $H(z)$ in $(*)$ we have:

$$Y(z) = \frac{1}{2}\left[X(z)\left(E_0(z) + z^{-1/2}E_1(z)\right) + X(z)\left(E_0(z) - z^{-1/2}E_1(z)\right)\right]$$

$$= \frac{1}{2}X(z)\left[E_0(z) + E_0(z) + z^{-1/2}\left(E_1(z) - E_1(z)\right)\right]$$

$$= \frac{1}{2}X(z)\left[2E_0(z)\right] = X(z)E_0(z)$$

3.



If we write $H(z)G(z)$ as $E_0(z^2) + z^{-1}E_1(z^{+2})$, then we have:

Case 1: $H(z)G(z) + H(-z)G(-z) = 2 \Rightarrow E_0(z^2) + z^{-1}E_1(z^2) + E_0(z^2) - z^{-1}E_1(z^2) = 2$

$\Rightarrow 2E_0(z^2) = 2 \Rightarrow \underline{E_0(z^2) = 1}$

$\Rightarrow \underline{E_0(z) = 1}$

According to part 2 (above) we know that the output is:

$Y(z) = X(z)E_0(z) \Rightarrow Y(z) = X(z)$ (unity)

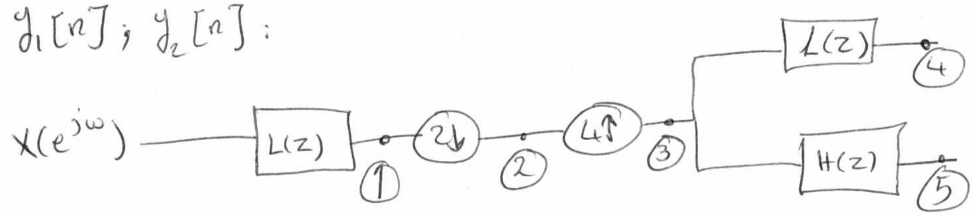
~~Cas~~

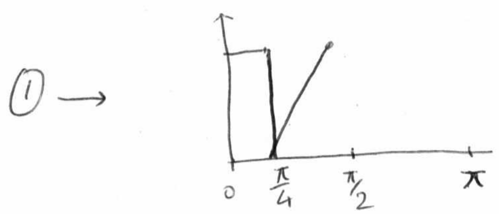
Case 2.

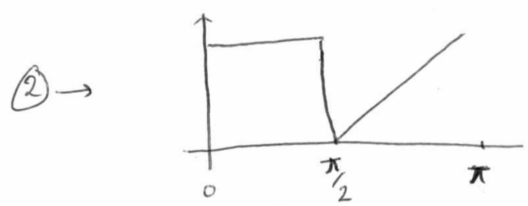
Writing $H(z)G(z)$ as $E_0(z^2)+z^{-1}E_1(z^2)$, by a similar way as Case 1. we get $E_0(z) \equiv 0$, thus as $Y(z)=E_0(z)X(z)$, then the output would be Zero.

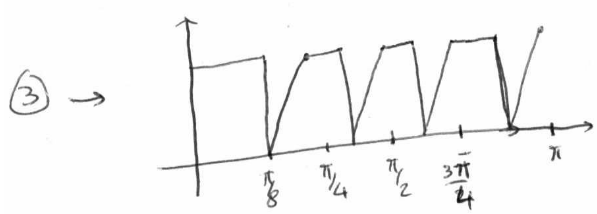
Prob. 4 (11.4 in the book)

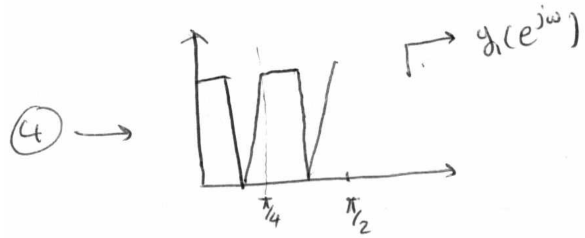
$y_1[n]$; $y_2[n]$:

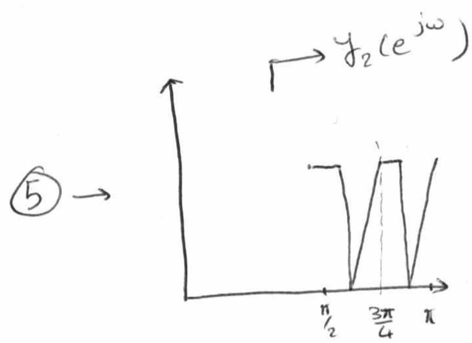




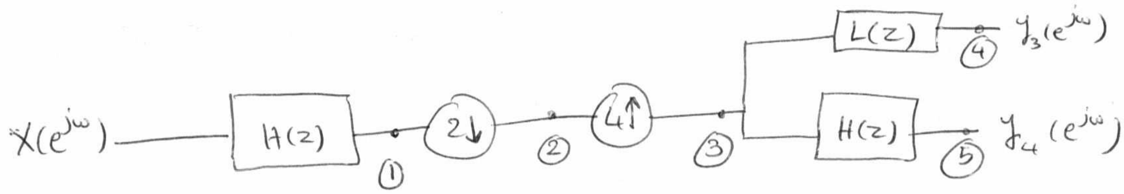




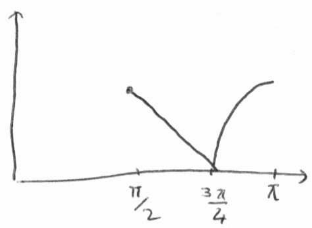




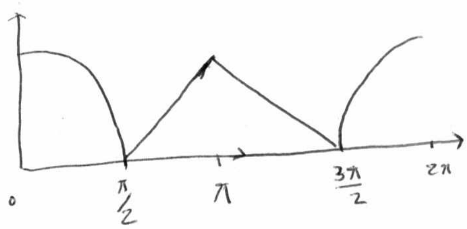
$y_3[n]$, $y_4[n]$



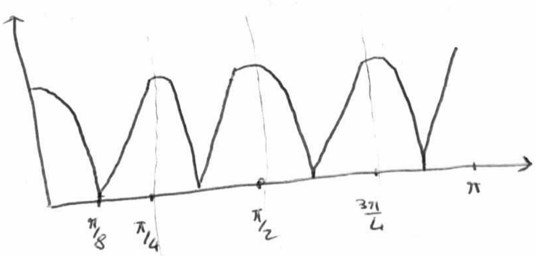
1



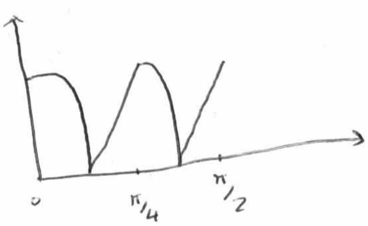
2 →



3 →



4 →



→ $Y_3(e^{j\omega})$

5 →

→ $Y_4(e^{j\omega})$