Handout 22 Signal Processing for Communications
Solutions to Homework 10 June 4, 2009

Problem 1 (7.9.). a.

$$\begin{aligned} H(z) &= \frac{1+z^{-1}}{1-1.6\cos(2\pi/7)z^{-1}+0.64z^{-2}} \\ &= \frac{1+z^{-1}}{1-1.6\frac{e^{j\frac{2\pi}{7}}+e^{-j\frac{2\pi}{7}}}{2}z^{-1}+0.64z^{-2}} \\ &= \frac{1+z^{-1}}{(1-0.8e^{j\frac{2\pi}{7}}z^{-1})(1-0.8e^{-j\frac{2\pi}{7}}z^{-1})} \end{aligned}$$

Therefore, the zero of $H(z)$ is $z=-1$, whereas the poles are $z=0.8e^{j\frac{2\pi}{7}}$ and $z=0.8e^{-j\frac{2\pi}{7}}$. The pole-zero plot is given in Figure 3. Since the filter is causal, we have $ROC = \{z : |z| > 0.8\}$.

b. The magnitude of the filter's frequency response is given in Figure 4.

c. Figures 1 and 2 show two implementions of the filter.

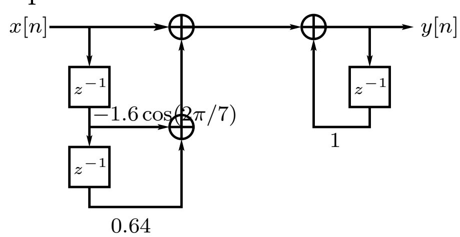

Figure 1:

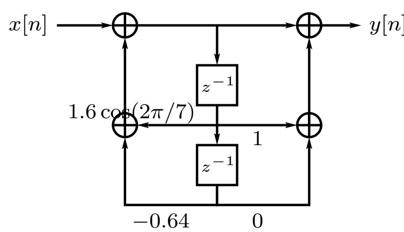

Figure 2:

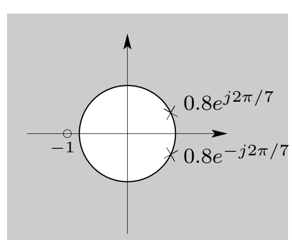

Figure 3:

Problem 2 (7.11.). The channel output is given by

$$y[n] = x[n] - \alpha x[n-D].$$

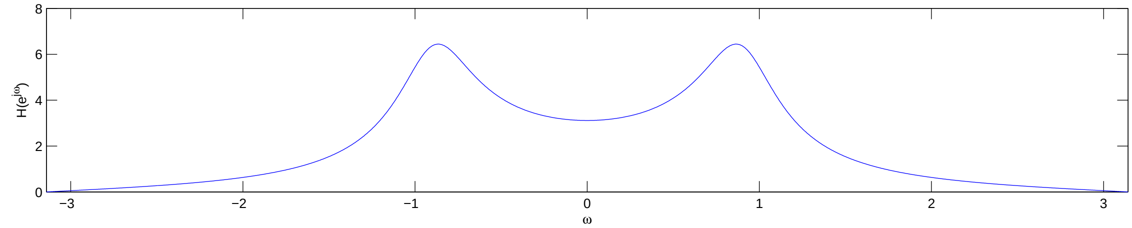

Figure 4:

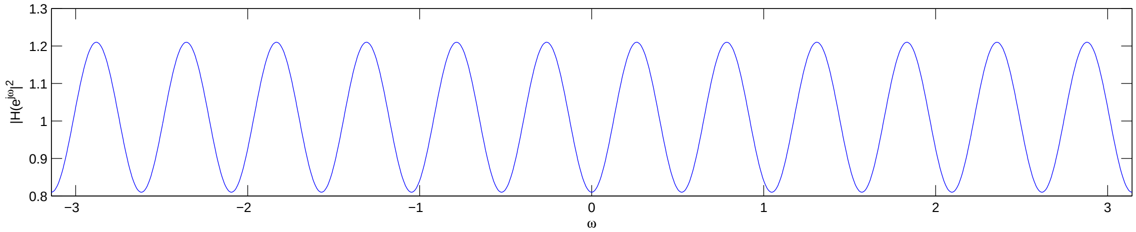

Figure 5:

a. $H(z) = 1 - \alpha z^{-D}$.

b. With $\alpha = 0.1$ and $D = 12$ we have $H(z) = 1 - 0.1z^{-12}$. This transfer function has no poles, and its 12 zeros are at $z_k = 0.862e^{j\frac{2\pi}{12}k}, \quad k = 0, \ldots, 11$.

c. The frequency response of the channel is $H(e^{j\omega}) = 1 - 0.1e^{-j\omega 12}$. The squared magnitude response is given in Figure 5.

d. We want $y[n] * g[n] = x[n]$, i.e., $G(z) = 1/H(z) = 1/(1 - \alpha z^{-D})$.

e. $G(z)$'s zeros and poles are $H(z)$'s poles and zeros, respectively. That is, $G(z)$ has no zeros, and 12 poles at $z_k = 0.862e^{j\frac{2\pi}{12}k}, \quad k = 0, \ldots, 11$. Since the system is causal, the ROC is the set $\{z : |z| > 0.862\}$.

PROBLEM 3 (9.1.). a. $x_0 = \sum_{n=-\infty}^{\infty} x[n]\text{rect}(t - n)$.

$$
\begin{aligned}
X_0(j\Omega) &= \int_{-\infty}^{\infty} \sum_n x[n]\text{rect}(t-n)e^{-j\Omega t}dt \\
&= \sum_n x[n] \int_{-\infty}^{\infty} \text{rect}(t-n)e^{-j\Omega t}dt \\
&= \sum_n x[n]\text{sinc}(\Omega/2)e^{-j\Omega n}dt \\
&= \frac{1}{2\pi}\text{sinc}\left(\frac{\Omega}{2\pi}\right) \sum_n x[n]e^{-j\Omega n}dt \\
&= \frac{1}{2\pi}\text{sinc}\left(\frac{\Omega}{2\pi}\right) X(e^{j\Omega}).
\end{aligned}
$$

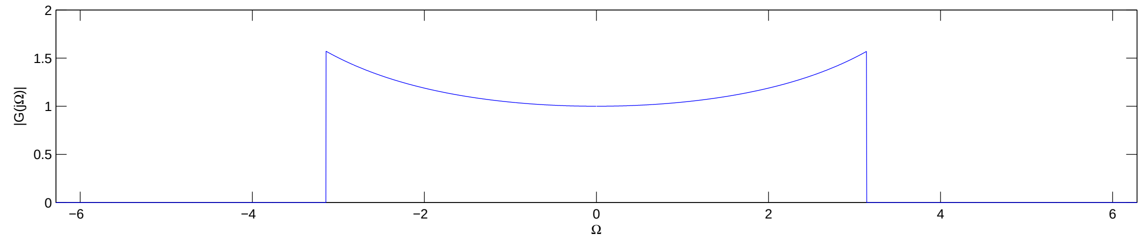

Figure 6:

b.

$$X(j\Omega) = \int_{-\infty}^{\infty} \sum_n x[n]\text{sinc}(t-n)e^{-j\Omega t}dt$$
$$= \sum_n x[n] \int_{-\infty}^{\infty} \text{sinc}(t-n)e^{-j\Omega t}dt$$
$$= \sum_n x[n]\text{sinc}(\Omega/2)e^{-j\Omega n}dt$$
$$= \frac{1}{2\pi}\text{rect}\left(\frac{\Omega}{2\pi}\right) X(e^{j\Omega}).$$

As it is noted in the problem statement, the zero-order hold introduces a distortion in the interpolated signal with respect to the sinc interpolaion in the region $\pi \leq \Omega \leq \pi$. Furthermore, it makes the signal non-bandlimited.

c. The signal $x(t)$ can be obtained from the zero-order hold interpolation $x_0(t)$ as $x(t) = x_0(t) * g(t)$, with

$$G(j\Omega) = \frac{\text{rect}\left(\frac{\Omega}{2\pi}\right)}{\text{sinc}\left(\frac{\Omega}{2\pi}\right)}.$$

$G(j\Omega)$ is plotted in Figure 6.