Problem 1. Consider the two probability distributions \( P = \{0.4, 0.35, 0.15, 0.1\} \) and \( Q = \{0.25, 0.35, 0.15, 0.25\} \).

1. Compute the two entropies \( H(P) \) and \( H(Q) \). Which one is larger?

2. Can you answer the above question without computing explicitly \( H(P) \) and \( H(Q) \)?

Problem 2. Consider a random variable \( s \) which takes an infinite number of values with corresponding probabilities \( p_i = \frac{1}{2^i} \), \( i \in \mathbb{N} = \{1, 2, 3, \ldots\} \).

1. Check that it is indeed a probability distribution.

2. What is the entropy of \( s \)?

Hint: If \( |r| < 1 \), \( \sum_{i=0}^{\infty} (a + id)r^i = \frac{a}{1-r} + \frac{rd}{(1-r)^2} \).

Problem 3. For each of the following three codes, say if it is uniquely decodable. If so, is it instantaneous?

<table>
<thead>
<tr>
<th>Code 1</th>
<th>Code 2</th>
<th>Code 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_1 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>010</td>
<td>10</td>
</tr>
<tr>
<td>( s_3 )</td>
<td>01</td>
<td>110</td>
</tr>
<tr>
<td>( s_4 )</td>
<td>10</td>
<td>111</td>
</tr>
</tbody>
</table>

Problem 4. In this exercise, we will prove Gibbs inequality. Consider the two probability distributions \( P = \{p_1, \ldots, p_n\} \) and \( Q = \{q_1, \ldots, q_n\} \). Gibbs inequality states that

\[
\sum_{i=1}^{n} p_i \log_2 \frac{1}{p_i} \leq \sum_{i=1}^{n} p_i \log_2 \frac{1}{q_i},
\]

with equality if and only if \( p_i = q_i, \forall i \).

1. Show that \( \ln(x) \leq x - 1 \), if \( x \geq 0 \).
   
   Hint: Use the Taylor series \( e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \)

2. Set \( x = \frac{a}{p_i} \) and prove Gibbs inequality.