ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 9 — Introduction to Communication Systems
Homework 5 — October 16, 2008

PROBLEM 1. Consider the two probabilitiy distributions $P = \{0.4, 0.35, 0.15, 0.1\}$ and $Q = \{0.25, 0.35, 0.15, 0.25\}$.

1. Compute the two entropies $H(P)$ and $H(Q)$. Which one is larger?
2. Can you answer the above question without computing explicitly $H(P)$ and $H(Q)$?

PROBLEM 2. Consider a random variable s which takes an infinite number of values whith corresponding probabilities $p_i = \frac{1}{2^i}$, $i \in \mathbb{N} = \{1, 2, 3, \dots\}$.

1. Check that it is indeed a probability distribution.
2. What is the entropy of s?

Hint: If $|r| < 1$, $\sum_{i=0}^{\infty}(a + id)r^i = \frac{a}{1-r} + \frac{rd}{(1-r)^2}$.

PROBLEM 3. For each of the following three codes, say if it is uniquely decodable. If so, is it instantaneous?

	Code 1	Code 2	Code 3
s_1	0	0	10
s_2	010	10	00
s_3	01	110	11
s_4	10	111	110

PROBLEM 4. In this exercise, we will prove Gibbs inequality. Consider the two probabilitiy distributions $P = \{p_1, \dots, p_n\}$ and $Q = \{q_1, \dots, q_n\}$. Gibbs inequality states that

$$\sum_{i=1}^{n} p_i \log_2 \frac{1}{p_i} \leq \sum_{i=1}^{n} p_i \log_2 \frac{1}{q_i}, \tag{1}$$

with equality if and only if $p_i = q_i, \forall i$.

1. Show that $\ln(x) \leq x - 1$, if $x \geq 0$.
   Hint: Use the Taylor serie $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
2. Set $x = \frac{q_i}{p_i}$ and prove Gibbs inequality.