ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 6 Introduction to Communication Systems
Solutions to Homework 3 October 9, 2008

PROBLEM 1. Let f_p be the perceived frequency. We have that

$$\cos(\pi n/4) = \cos(2\pi f_p nT).$$

Thus f_p satisfies for all n,

$$\pi n/4 = 2\pi f_p nT + m2\pi$$

where $m \in \mathcal{Z}$. Thus f_p must satisfy $f_p = \frac{f_s}{8} + Nf_s$ for $N \in \mathcal{Z}$. So two values of f_p which could have given the signal are $f_p = f_s/8 = 125$Hz or $f_p = f_s/8 + f_s = 1125$Hz. The ideal interpolator will give a signal whose maximum frequency is less than $f_s/2 = 500$Hz. Thus the reconstructed signal would be $\cos(250\pi t)$.

PROBLEM 2. Since the sampling period is 1s we have

1. $y(t) = \texttt{rect}(t) + 0.8\texttt{rect}(t-1) + 2\texttt{rect}(t-2) + 4\texttt{rect}(t-3)$
2. $y(t) = \texttt{triangle}(t) + 0.8\texttt{triangle}(t-1) + 2\texttt{triangle}(t-2) + 4\texttt{triangle}(t-3)$

PROBLEM 3.

1. Sampling frequency should be twice the bandwidth. In this case, the bandwidth or the maximum frequncy of the signal is 25Hz (because $5o\pi = 2\pi f$ gives $f = 25$). Thus the sampling frequency should be greater than 50Hz to avoid aliasing. If we sample at 40Hz, the frequency 12.5Hz (of the sinusoid) is unaffected and will be reconstructed exactly since it less than $f_s/2 = 20$Hz. The other frequency perceived will be $f_p = f + mf_s$ where $m \in \mathcal{Z}$. The reconstructed frequency will be less than $f_s/2$, hence for $m = -1$ we get $f_p = -15$Hz which has absolute value less than 20Hz. Thus the reconstructed signal will be $\sin(25\pi t) + \cos(-30\pi t) = \sin(25\pi t) + \cos(30\pi t)$.
2. In this case the sampling frequency should be greater than 50Hz.
3. The signal in this case can be written as $\sin(25\pi t)\cos(50\pi t) = \frac{1}{2}[\sin(75\pi t) - \sin(25\pi t)]$. Thus the maximum frequency is at 75/2Hz which implies that we should sample at frequency greater than 75Hz.

PROBLEM 4. Note that linear velocity $v = r\omega$, where r is the radius and ω is the angular speed in radians per second.

1. The angular speed of the wheel in this case is 3 revolutions per second (or 6π radians per second). Since the wheel has 4 spokes, and we are sampling at 12 frames per second, the wheel moves 1 spoke in one frame. Since all the spokes are identical, we observe that the wheel is standstill.
2. The angular speed in this case is 9/4 revolutions per second. Thus if we sample at 12 frames per second, the wheel moves $\frac{9}{4} \times \frac{4}{12}$ spokes per frame. Thus the wheel moves $\frac{3\pi}{8}$ radians per frame (or 3/4 spoke per frame). Thus the motion that we perceive is that the wheel is moving in the backward direction with speed 1/4 spoke per frame or angular speed $12 \times \frac{1}{4}\frac{\pi}{2} = \frac{3\pi}{2}$ radians per second which gives a linear speed of $\frac{3\pi}{4}$ meters per second in the backward direction.