ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Handout 1 | Introduction to Communication Systems

Solutions to Homework 1 | September 25, 2008

Problem 1. The plots of various signals are shown in Figures 1,2,3,4,5,6,7:

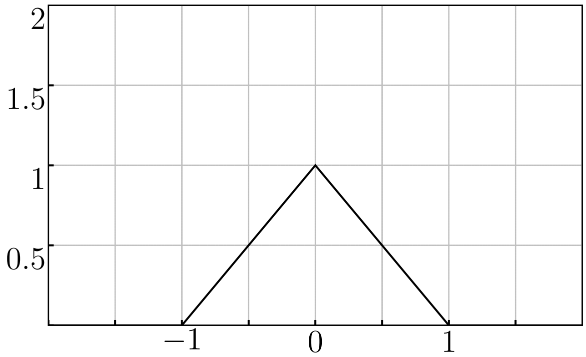

Figure 1: Triangle(t)

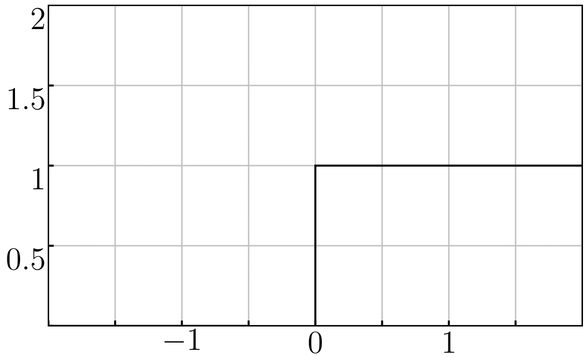

Figure 2: Step(t)

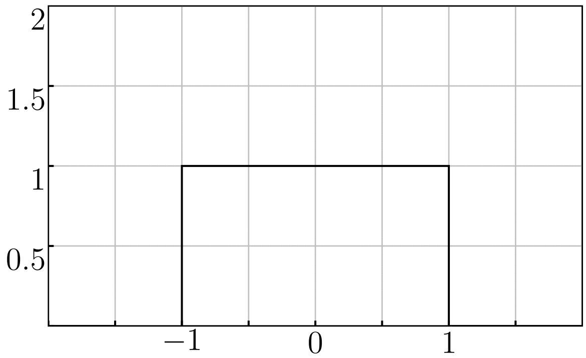

Figure 3: Pulse(t)

Problem 2. 1. Consider the signal, $p(t) = 5\sin(10t + \frac{\pi}{2}) + 2.5\cos(5t)$. From the definition of periodicity, $p(t + T_p) = p(t)$, we have that $10T_p$ and $5T_p$ should both be integer multiples of 2π. Clearly the smallest such T_p is given by $\frac{2\pi}{g} = \frac{2\pi}{5}$, where g is the greatest common divisor of $5, 10$. This is true because if we take any $d > g$ and suppose the period is $\frac{2\pi}{d}$, then it must be that both $\frac{10}{d}$ and $\frac{5}{d}$ are integers, which would contradict the fact that g is the g.c.d.

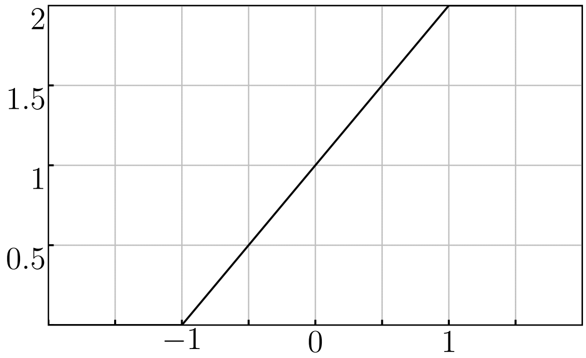

Figure 4: Ramp(t)

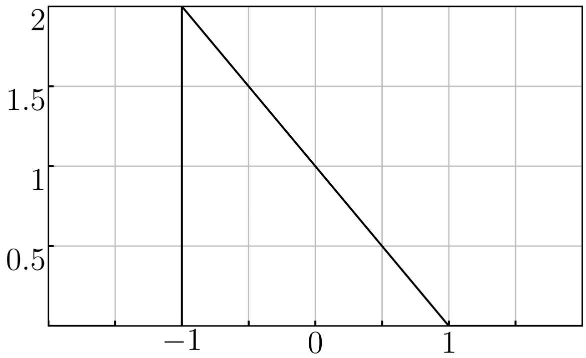

Figure 5: Diff(t)

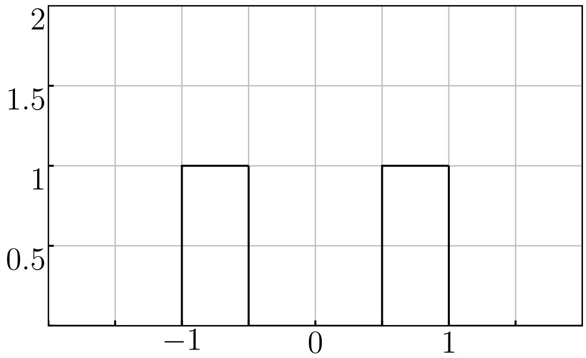

Figure 6: Sum(t)

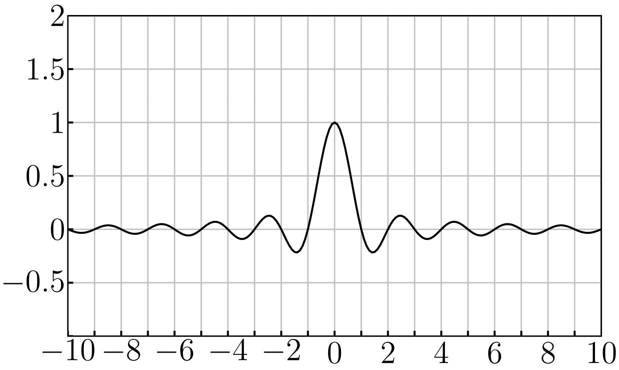

Figure 7: Sinc(t)

2. Again from the definition of the periodicity, we must have that $N\bar{f}T_p$ and $M\bar{f}T_p$ are both integers. The period must be given by $\frac{1}{\text{g.c.d}(N,M)\bar{f}} = \frac{1}{\bar{f}}$.

3. Since we want $f_0 T_p$ and $f_1 T_p$ to be both integers, necessarily we must have that $\frac{f_0}{f_1}$ is a rational number. Thus for f_0, f_1 such that $\frac{f_0}{f_1}$ is irrational, the signal is not periodic.

Problem 3. 1. $\log_2 26$ bits to store each letter.

2. $170000 \log_2 26 = 799075$ bits to store the Hamlet.

3. We need $\log_2 \left[\binom{500000}{4000}\right]$ bits to specify which 4000 words the Hamlet has and further $33000 \log_2 4000$ to specify the entire Hamlet. Thus total bits required are 428474. Thus we can save space by looking at words rather than each character.

Problem 4. Let us denote the output of a linear system S by $S[x(t)]$.

1. The system is not linear. Indeed let $y_1(t) = S_1[x_1(t)]S_2[x_1(t)]$ and let $y_2(t) = S_1[x_2(t)]S_2[x_2(t)]$. Consider the output, $y_{12}(t)$ of the system for the input $x_1(t)+x_2(t)$

$$\begin{aligned}
y_{12}(t) &= S_1[x_1(t)+x_2(t)]S_2[x_1(t)+x_2(t)] \\
&= (S_1[x_1(t)] + S_1[x_2(t)])(S_2[x_1(t)] + S_2[x_2(t)]) \\
&= S_1[x_1(t)]S_2[x_1(t)] + S_1[x_1(t)]S_2[x_2(t)] + S_1[x_2(t)]S_2[x_1(t)] + S_1[x_2(t)]S_2[x_2(t)]
\end{aligned}$$

Clearly

$$y_1(t) + y_2(t) = S_1[x_1(t)]S_2[x_1(t)] + S_1[x_2(t)]S_2[x_2(t)] \neq y_{12}(t)$$

implying that the system is not linear.

2. The system is linear. Indeed let $y_1(t) = \alpha(S_1[x_1(t)] + S_2[x_1(t)])$ and let $y_2(t) = \alpha(S_1[x_2(t)] + S_2[x_2(t)])$. Consider the output $y_{12}(t)$ for the input $\beta x_1(t) + \gamma x_2(t)$. We have

$$\begin{aligned}
y_{12}(t) &= \alpha\big(S_1[\beta x_1(t) + \gamma x_2(t)] + S_2[\beta x_1(t) + \gamma x_2(t)]\big) \\
&= \alpha\big(\beta S_1[x_1(t)] + \gamma S_1[x_2(t)] + \beta S_2[x_1(t)] + \gamma S_2[x_2(t)]\big) \\
&= \beta(\alpha(S_1[x_1(t)] + S_2[x_1(t)])) + \gamma(\alpha(S_1[x_2(t)] + S_2[x_2(t)])) \\
&= \beta y_1(t) + \gamma y_2(t)
\end{aligned}$$

where the second equality follows from the linearity of the systems S_1, S_2.