Problem 1 (Congruences). 1. If \( a \equiv a' \pmod{m} \), show that for any integer \( t \)
\[
at \equiv a't \pmod{m}.
\]

2. If \( ad \equiv a'd \pmod{m} \) and \( d \) and \( m \) are relatively prime, show that
\[
a \equiv a' \pmod{m}.
\]

Does this property still hold if \( d \) and \( m \) are not relatively prime?

Problem 2 (Euler’s Birthday Party). Euler invites a group of \( n \) friends to celebrate
his 251-st birthday. He has ordered a humongous Nusstorte from Spruengli. The cake is
already been cut into 5005 pieces. Euler asks his \( n \) friends to split into subgroups of equal
size. Amazingly, each group gets exactly the same number of pieces. How many possible
choices of \( n \) are there?

Problem 3 (A trip to China Town). (i) Four friends go to eat dim sum at a restaur-
ant. They order \( k \) pieces. After dividing equally they are left with 3 pieces. Since
the food was delicious, the next evening they take along one additional friend and
order again \( k \) pieces. Dividing again fairly, they are left with 2 pieces. One piece costs
5 CHF and a single piece per person is not enough. What is the minimum amount
of money they paid?

(ii) Assume exactly the same situation as above except that on the second evening they
take along two additional friends.

Problem 4 (RSA Encryption). In this problem we perform RSA encryption and de-
cryption. Assume that each letter of the English alphabet is represented by its position,
i.e. \( A = 1, B = 2, \ldots \). For the RSA scheme, we encode using integers modulo 33. Thus
\( m = 11 \cdot 3 \).

- Find a public key, secret key pair \((K,k)\).

- Pair up with the person next to you. Encrypt the plaintext CIPHER. Ask your
neighbor to decrypt.