ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 1 Introduction to Communication Systems
Homework 1 September 18, 2008

PROBLEM 1. Sketch the following signals :

$$\text{Triangle}(t) = \begin{cases} 0 & \text{if} \quad |t| > 1 \\ 1 - |t| & \text{if} \quad |t| \leq 1 \end{cases}$$

$$\text{Step}(t) = \begin{cases} 0 & \text{if} \quad t < 0 \\ 1 & \text{if} \quad t \geq 0 \end{cases}$$

$$\text{Pulse}(t) = \begin{cases} 0 & \text{if} \quad |t| > 1 \\ 1 & \text{if} \quad |t| \leq 1 \end{cases}$$

$$\text{Ramp}(t) = \int_{-\infty}^{t} \text{Pulse}(\tau)\, d\tau$$

$$\text{Diff}(t) = 2\text{Step}(t+1) - \text{Ramp}(t)$$

$$\text{Sum}(t) = \text{Step}(2t) - \text{Step}(t)$$

$$\text{Sinc}(t) = \begin{cases} 1 & \text{if} \quad t = 0 \\ \frac{\sin(\pi t)}{\pi t} & \text{if} \quad t \neq 0 \end{cases}$$

PROBLEM 2. We have seen that the sinusoid $p(t)$ is a periodic signal, i.e. There exists a quantity $T_P \in \mathbb{R}^+$, which is smallest, such that $p(t + T_P) = p(t)$ for any $t \in \mathbb{R}$. In this exercise, we are going to study the periodicity of the sum of a sinusoid and a cosinusoid.

1. Sketch $p(t) = 5\sin(10t + \frac{\pi}{2}) + 2.5\cos(5t)$. What is the period?

Now let $p(t)$ be the signal

$$p(t) = P_0 \sin(2\pi f_0 t + \phi_0) + P_1 \cos(2\pi f_1 t + \phi_1)$$

where P_0, P_1, ϕ_0, ϕ_1 are arbitrary values and f_0, f_1 are specified in the following.

2. Consider the case of f_0 and f_1 multiple of the same frequency $\bar{f}$, i.e. $f_0 = N\bar{f}$, $f_1 = M\bar{f}$. We assume that $\bar{f}$ is chosen so that N and M are coprime (i.e. they don't have common divisors). Show that the signal $p(t)$ is periodic and compute the period.

3. The previous case corresponds to say that the frequencies f_0 and f_1 have a rational ratio, i.e. f_1/f_0 can be written as the ratio of two integers (this is N/M). Consider now the case that the ratio is irrational, for example $f_1/f_0 = \sqrt{2}$, and show that in such a case the signal p(t) is not periodic.

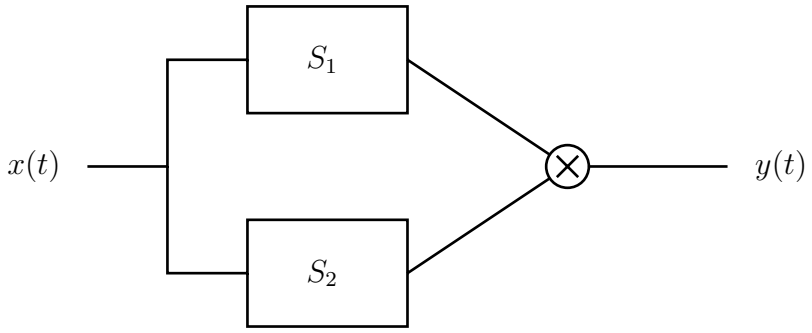
PROBLEM 3. We want to backup Shakespeare's Hamlet on a hard disk using as small a space as possible.

1. Given that the English language has 26 different letters, how many bits do we need to store a particular letter?

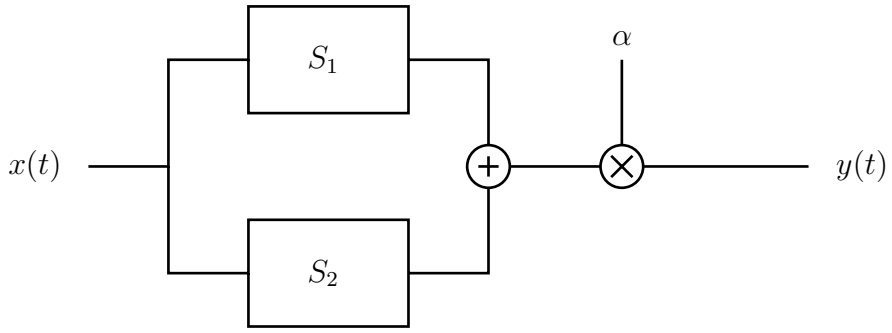
2. Assume that we know that Hamlet contains 170000 characters. How many bits do we need to store the Hamlet?

3. Assume that we know that Hamlet contains 33000 words amongst which 4000 are distinct, but we do not know a priori which 4000 words. Assume further that the English language has 500000 words. Can you think of a scheme to reduce the storage space on the disk? If yes, how much space do you save?

PROBLEM 4. Suppose that the systems S_1 and S_2 are linear. We connect them together, as shown in the following figures, in order to build more complicate systems. In each figure the input signal is called $x(t)$ and the output signal is called $y(t)$. Check in each case if the resulting system is linear or not.



(a)



(b)