Homework Set # 2

Problem 1

What is the period of the following sequence?

\[ \tilde{x}[n] = 3 + \sin \left( \frac{2\pi}{5} n \right) + \cos \left( \frac{3\pi}{2} n \right) \]

Problem 2 (Discrete Fourier Transform)

Show how to compute the DFT of two even complex length-N sequences \( x_1[n] \) and \( x_2[n] \) performing only one length-N transformation. Follow the steps below:

(a) Build the auxiliary sequence \( y(n) = W_N^n x_1[n] + x_2[n] \) \((W_N = e^{-j\frac{2\pi}{N}})\).

(b) Show that \( Y[k] = X_1[k + 1] + X_2[k] \).

(c) Using symmetry properties of the DFT, show that \( Y[-k - 1] = X_1[k] + X_2[k + 1] \).

(d) Use the results of (b) and (c) to create a recursion to compute \( X_1[k] \) and \( X_2[k] \). Note that \( X[0] = \sum_{n=0}^{N-1} x[n] \).

Problem 3

Compute the DFS coefficients of the periodic sequences below.

(a) \( \tilde{x}[n] = e^{-2(n \mod 20)} \).

(b) \( \tilde{x}[n] = \begin{cases} 1, & \text{for } n \text{ even} \\ -1, & \text{for } n \text{ odd} \end{cases} \)

Problem 4 (MATLAB and FFT)

(a) Read the MATLAB help for the function `fft`.

(b) Compute (analytically) the DFT of the signal

\[ x[n] = \sin \left( \frac{4\pi n}{16} \right), \quad n = 0, 1, \ldots, N. \]

Now compute the DFT of this signal using `fft` and compare the results.
**Problem 5**

Let $\tilde{x}_1[n]$ be periodic with period $N = 50$, where one period is given by

$$
\tilde{x}_1[n] = \begin{cases} 
ne^{-0.3n}, & 0 \leq n \leq 25 \\
0, & 26 \leq n \leq 49 
\end{cases}
$$

and let $\tilde{x}_2[n]$ be periodic with period $N = 100$, where one period is given by

$$
\tilde{x}_2[n] = \begin{cases} 
ne^{-0.3n}, & 0 \leq n \leq 25 \\
0, & 26 \leq n \leq 99 
\end{cases}
$$

These two periodic sequences differ in their periodicity but otherwise have equal nonzero samples.

(a) Find the DFS $\tilde{X}_1[k]$ of $\tilde{x}_1[n]$ (using the `fft` function) and plot samples (using the `stem` function) of its magnitude and angle versus $k$.

(b) Find the DFS $\tilde{X}_2[k]$ of $\tilde{x}_2[n]$ and plot samples of its magnitude and angle versus $k$.

(c) What is the difference between the above two DFS plots?

Consider now the periodic sequence $\tilde{x}_3[n]$ with period 100, obtained by concatenating two periods of $\tilde{x}_1[n]$. Clearly, $\tilde{x}_3[n]$ is different from $\tilde{x}_2[n]$, even though both of them are periodic with period 100.

(d) Find the DFS $\tilde{X}_3[k]$ of $\tilde{x}_3[n]$ and plot samples of its magnitude and angle versus $k$.

(e) What effect does the periodicity doubling have on the DFS?

**Problem 6 (DFT and DTFT)**

Let the infinite sequence $x[n]$ be defined as

$$
x[n] = \begin{cases} 
7 + n, & -6 \leq n \leq -1 \\
6 - n, & 0 \leq n \leq 5 \\
0, & \text{otherwise}, 
\end{cases}
$$

i.e., $x[n] = \{1, 2, \ldots, 6, 6, 5, \ldots, 1\}$ for $n = \{-6, -5, \ldots, 5, 5\}$.

(a) Write a MATLAB function `dtft` to compute the DTFT of an arbitrary sequence. The function should take as arguments a row vector $x$ containing the sequence, a row vector with the set of frequencies $\omega$ on which the DTFT is to be evaluated, and a number indicating the index of the first sample of $x$.

For example, to compute the DTFT of the above sequence $x[n]$, you would write

```matlab
>> X = dtft([1:6, 6:-1:1], linspace(0, 2*pi, 100), -6);
```

This would evaluate the DTFT of $x[n]$ on 100 equally spaced points between 0 and $2\pi$.

(b) Use the function `dtft` to compute $X(e^{j\omega})$, the DTFT of $x[n]$, and plot (using the `plot` function) its magnitude and phase.
(c) Let $y[n]$ be the finite sequence of length 12 obtained by wrapping the “negative” parts of $x[n]$ to the positive axis, i.e., $x[n] = \{6, 5, \ldots, 1, 1, 2, \ldots, 6\}$, for $0 \leq n \leq 11$. Using the \texttt{fft} function, determine the DFT $Y[k]$ of $y[n]$. Plot (using the \texttt{stem} function) the magnitude and phase onto the magnitude and phase plots of (b), respectively (using the \texttt{hold} function), and verify that the DFT is indeed a sampled version of the DTFT.