Homework Set # 1

Mathematical Prerequisites

Problem 1 (Geometric Series)

(a) Consider the sequence \( x[n] = a \cdot r^n \) for some real \( r \). Let \( S[n] = \sum_{k=0}^{n} x[k] \). You have seen in class that \( S[n] = (1 - r^{n+1})/(1 - r) \). Find a closed-form expression for \( \tilde{S}[n] = \sum_{k=0}^{n} akr^k \).

(Hint: Write \( \tilde{S}[n] \) as derivative of \( S[n] \).)

(b) Give a closed-form expression for \( \sum_{k=0}^{m} \exp(j2\pi k/n) \). What if \( m = ln - 1, \ l \in \mathbb{N} \)?

(c) Compute \( \sum_{k=0}^{\infty} t[k] \), where \( t[k] = 1/4^k + (1/3)j^k \).

(d) Compute \( |\sum_{k=0}^{n-1} \exp(j\theta k)| \) and express the result without using exponentials.

Problem 2 (Complex Numbers)

(a) Express \( |e^z| \) in terms of \( x \) and \( y \), the real and imaginary parts of \( z \).

(b) Find the real and imaginary parts of \( \sin z \) and \( \cos z \). Express your answers in terms of regular and hyperbolic trigonometric functions.

(c) Complex logarithm. Express \( \log z \) in terms of the modulus and the argument of \( z \). (Hint: Consider all solutions to the equation \( e^{a+jb} = z \).)

Problem 3 (Linear Algebra)

(a) Compute the determinant of the following matrix.

\[
A = \begin{bmatrix}
3 & 0 & -2 & 1 \\
1 & 0 & 1 & 2 \\
0 & 2 & 0 & 1 \\
-1 & -3 & 2 & 0
\end{bmatrix}
\]

(b) Show that the determinant of a matrix equals the product of the eigenvalues. (Hint: Imagine that the characteristic equation is factored into

\[
\det(A - \lambda I) = (\lambda_1 - \lambda) \cdots (\lambda_n - \lambda)
\]

and make a clever choice of \( \lambda \).)

(c) The trace of a matrix is defined as the sum of the diagonal elements. Show that the trace equals the sum of the eigenvalues, in two steps.

1. Find the coefficient of \( (-\lambda)^{n-1} \) on the right-hand side of (1).

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2. Look for all the terms in

$$\text{det}(A - \lambda I) = \text{det} \begin{bmatrix}
  a_{11} - \lambda & a_{12} & \cdots & a_{1n} \\
  a_{21} & a_{22} - \lambda & \cdots & a_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{n1} & a_{n2} & \cdots & a_{nn} - \lambda
\end{bmatrix}$$

(2)

which involve \((-\lambda)^{n-1}\). Explain why they all come from the product down the main diagonal, and find the coefficient of \((-\lambda)^{n-1}\). Compare.

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**Introduction to MATLAB**

**Problem 4 (MATLAB)**

The remaining exercises have to be done in MATLAB. The data/input files required to do the exercises can be found on the course website.

If you have not worked with MATLAB before, you can use the material in handout #4 to get a good introduction.

(a) Go through the following sections of the handout:

- 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 18 (planar plots only).

For this exercise you do not have to hand in anything :-).

Knowing the very basics, it is easy to learn about new functions in MATLAB. The excellent documentation can be accessed through the help menu, by executing the command `helpbrowser` or by using the `help` command for specific commands.

**Problem 5 (Operators)**

Suppose we have a matrix $A$ given by

$$
\begin{bmatrix}
  a + ib & c + id \\
  e + if & g + ih
\end{bmatrix}
$$

(3)

What will be the MATLAB output of the following commands: i) $A'$ ii) $A.$, iii) `fliplr(A)`, iv) `sum(A)`, v) `sum(A, 2)`, vi) $A*A$, vii) $A.*A$. Hint: Have a look in the documentation for arithmetic operations.

**Problem 6**

Let $a(x) = \sum_{n=0}^{N-1} a_n x^n$ and $b(x) = \sum_{m=0}^{M-1} b_m x^m$ be two polynomials, and suppose they are represented in MATLAB as row vectors $\mathbf{a} = [a_0, \ldots, a_{N-1}]$ and $\mathbf{b} = [b_0, \ldots, b_{M-1}]$. Find an easy way to compute the coefficients of the polynomial $c(x) = a(x)b(x)$. (Hint: Write down the general formula for the coefficient $c_n$ and conclude.)
Problem 7 (Sequences)

When plotting sequences in MATLAB, the basic plot command is not always the best thing to use. The problem is that by default it will create a continuous function based on the points you pass it. The way to solve this is using an extra argument to the plot function. To plot a sequence \( a[n], n = 1, \ldots, N \), you can use for example

\[
\text{>> plot([1:N],a,'o');}
\]

A plotting command intended specifically for sequences is stem. Have a look in the MATLAB documentation for its use.

(a) Create a sequence \( a[n] = \cos(2\pi n/15) \), \( n = 1, \ldots, 45 \). Give a stem-plot of the sequence.

(b) Construct a new sequence from \( a[n] \) by selecting every fifth sample, provide a stem-plot.

(c) Complete the M-file cshiftright.m, such that it implements a function that takes a sequence \( a[n] \) and a number \( N \) as arguments and that shifts \( a[n] \) \( N \) samples to the right, circularly filling in the samples from the end of the sequence on the left. Provide the completed M-file.

(d) Circularly shift the sequence \( a[n] \) by 8 samples to right. Provide a stem-plot.

Problem 8 (Audio)

MATLAB has builtin commands for handling audio. Take a look at the commands: wavread and soundsc.

(a) Load the audio file handel.wav using

\[
\text{>> [data,fs] = wavread('handel.wav');}
\]

To listen to audio it is important to provide the sampling frequency used to record it. In this case it is provided as \( fs \) by the wavread command.

(b) Provide a stem-plot of samples 1 till 100.

(c) Create a new sequence by reversing the order of the audio sequence. Use soundsc to listen to the reversed sequence. Provide a stem-plot of samples 1 till 100 of the reversed sequence.

The soundsc will ‘play’ any sequence. The sequence does not necessarily have to be recorded audio.

(d) Create three sinusoidal sequences \( a[n] \), \( c[n] \) and \( e[n] \), using \( \sin(f \ n/2\ e3) \), with \( f \) equal to 440 Hz, 523.25 Hz and 659.26 Hz respectively.

(e) Listen to the sequences \( a[n] \), \( a[n] + e[n] \) and \( a[n] + c[n] + e[n] \). Use soundsc(x,2e3).

(f) Give a plot of the first 300 samples of \( a[n] + c[n] + e[n] \).

(g) Listen to the reversed sequences.
Problem 9 (Images)

MATLAB has built-in commands for loading and displaying images. Look in the documentation for `imread` and `imagesc`.

(a) The file `lena.jpg` is a 256 by 256 pixel grayscale image. Load and display `lena.jpg`. Hint: to display a grayscale image use `colormap(gray(256))`.

(b) Extract the top-left to bottom-right diagonal of the image. Provide a stem-plot of the extracted sequence.

By displaying a data sequence as an image we give an interpretation to the sequence. It makes sense to interpret the data in `lena.jpg` as an image. We might, however, just as well interpret this data as audio.

(c) Use `soundsc` to 'listen' to `lena.jpg`. Hint: To have all entries from a matrix \( A \) in a vector you can use e.g. `reshape(A,1,prod(size(A)))`, or simply \( A(1:end) \).

The above exercise demonstrates that not all interpretations of a signal make sense.

In a similar way we can interpret some audio data as an image.

(d) Load `handel.wav`. Reshape the first 4096 samples of the data vector into a square matrix, row first. Display the absolute value of the matrix as a grayscale image. Provide a printout.

Also here we see that we have to choose the right interpretation in order to get results that make sense.