Problem 1

Consider the system function \( H(z) = \frac{1+2z^{-1}}{(1+\frac{1}{2}z^{-1}-2z^{-2})(1-2z^{-1})} \). Find the possible corresponding impulse responses \( h[n] \).

**Solution:** The poles are \( p_1 = 2, p_2 = -4, p_3 = \frac{1}{2} \). After the partial fraction expansion, the system function is

\[
H(z) = \frac{\frac{8}{27}}{1+4z^{-1}} + \frac{-\frac{5}{27}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{8}{9}}{1-2z^{-1}}.
\]

There is a causal system with ROC \( |z| > 4 \) and impulse response

\[
h[n] = \frac{8}{27}(-4)^nu[n] + \frac{-5}{27}\left(\frac{1}{2}\right)^nu[n] + \frac{8}{9}2^nu[n].
\]

There is an anticausal system with ROC \( |z| < \frac{1}{2} \) and impulse response

\[
h[n] = \frac{-8}{27}(-4)^u[-n-1] + \frac{5}{27}\left(\frac{1}{2}\right)^u[-n-1] + \frac{-8}{9}2^u[-n-1].
\]

There is a two-sided system with ROC \( 2 < |z| < 4 \) and impulse response

\[
h[n] = \frac{-8}{27}(-4)^u[-n-1] + \frac{-5}{27}\left(\frac{1}{2}\right)^u[n] + \frac{-8}{9}2^u[-n-1].
\]

There is a two-sided and BIBO-stable system with ROC \( \frac{1}{2} < |z| < 2 \) and impulse response

\[
h[n] = \frac{-8}{27}(-4)^u[-n-1] + \frac{-5}{27}\left(\frac{1}{2}\right)^u[n] + \frac{-8}{9}2^u[-n-1].
\]

Problem 2

Consider the system function \( H(z) = \frac{1+z^{-1}}{1-2z^{-1}+2z^{-2}} \). Find the impulse response of a causal system that corresponds to this system function.

**Solution:** The poles are \( p_1 = 1+j \) and \( p_2 = 1-j \). After the partial fraction expansion, the system function is

\[
H(z) = \frac{\frac{1}{2} - j}{1-(1+j)z^{-1}} + \frac{\frac{1}{2} + j}{1-(1-j)z^{-1}}.
\]

The impulse response is

\[
h[n] = \left(\frac{1}{2} - j\right)(1+j)^nu[n] + \left(\frac{1}{2} + j\right)(1-j)^nu[n].
\]