Problem 1. Problem 2.2 in the notes.

Problem 2. Consider an \((l, r)\)-regular ensemble of LDPC codes of length \(n\) as introduced in class. We have derived in class the expected number of double edges (the number of pairs of edges whose endpoints are the same) in the limit of large \(n\). We can think of such a pair also as a 2-cycle (a cycle of length 2). Let \(C_2(G)\) denote the number of 2-cycles which are present in a given graph \(G\) from this ensemble. We will see next week that the distribution of \(C_2(G)\) converges to a Poisson distribution.

Consider \(C_4(G)\), i.e., consider cycles of length 4.

- Compute the expected value of \(C_4(G)\) in the limit of large \(n\).
- Show that \(C_4(G)\) converges in distribution to a Poisson in the limit of large \(n\). NOTE: Just show how to compute the first two factorial moments.

Problem 3. Consider \(A(G, w)\), the number of codewords of weight \(w\) which are contained in the graph \(G\). Consider an \((l, r)\)-regular ensemble of LDPC codes of length \(n\).

- Compute the expected value of \(A(G, w)\) for \(w = 1, 2, 3\) and \(n\) tending to infinity as a function of \(l\) and \(r\).
- Comment on your results. Are there some special values of \(l\) or \(r\) that give you qualitatively different results than other values?
- Consider the case \(l = 2\). Is it true that \(A(G, w = 2)\) converges in distribution to a Poisson?