

Solutions to Homework 3

Problem 1

(a) Orthogonality principle implies,

$$\mathbb{E}[(X - \hat{X})\mathbf{Y}^*] = 0$$

where $\mathbf{Y} = \begin{bmatrix} X + Z_1 \\ Z_2 \end{bmatrix}$. Let $\mathbf{W} = \begin{bmatrix} W_1, & W_2 \end{bmatrix}$, then orthogonality principle implies

$$\mathbb{E}[(X - \mathbf{W}\mathbf{Y})\mathbf{Y}^*] = 0$$

or,

$$\mathbf{W} = \mathbb{E}[X\mathbf{Y}^*](\mathbb{E}[\mathbf{Y}\mathbf{Y}^*])^{-1}$$

Since,

$$\mathbb{E}[\mathbf{Y}\mathbf{Y}^*] = \begin{bmatrix} \mathcal{E}_x + \sigma^2 & \rho\sigma^2 \\ \rho^*\sigma^2 & \sigma^2 \end{bmatrix}$$

and,

$$\mathbb{E}[X\mathbf{Y}^*] = \begin{bmatrix} \mathcal{E}_x, & 0 \end{bmatrix}.$$

We get that,

$$\mathbf{W}_{opt} = \begin{bmatrix} W_{1,opt}, & W_{2,opt} \end{bmatrix} = \frac{\mathcal{E}_x}{\mathcal{E}_x + \sigma^2(1 - |\rho|^2)}[1, \quad -\rho],$$

and

$$\widehat{X} = \frac{\mathcal{E}_x}{\mathcal{E}_x + \sigma^2(1 - |\rho|^2)}(X + Z_1 - \rho Z_2)$$

(b)

$$\sigma^2_{MMSE} = \text{Trace}\left(R_{XX} - W_{\text{opt}} R_{YX}\right).$$

We have,

$$R_{XX} = \mathcal{E}_x, \quad R_{YX} = \begin{bmatrix} \mathcal{E}_x \\ 0 \end{bmatrix}$$

So,

$$\sigma^2_{MMSE} = \frac{\mathcal{E}_x \sigma^2 (1 - |\rho|^2)}{\mathcal{E}_x + \sigma^2(1 - |\rho|^2)}.$$

We get that for $|\rho| = 1, \sigma^2_{MMSE} = 0$. The interpretation is that if $|\rho| = 1$, Z_1 is perfectly predictable from Z_2 and hence can be canceled from $X + Z_1$. This will yield $\sigma^2_{MMSE} = 0$.

(c) The linear estimator for Z_1 from Y_2 is,

$$\hat{Z}_1 = WY_2.$$

Applying orthogonality principle we get that $\hat{Z}_1 = \rho Y_2 = \rho Z_2$. Thus $\hat{Y}_1 = Y_1 - \rho Y_2 = X + Z_1 - \rho Z_2$. Thus the best linear MMSE estimator can be again obtained by applying orthogonality principle.

$$W_{opt}^{(2)} = \frac{\mathcal{E}_x}{\mathcal{E}_x + \sigma^2(1 - |\rho|^2)},$$

and hence

$$\hat{X}^{(2)} = \frac{\mathcal{E}_x}{\mathcal{E}_x + \sigma^2(1 - |\rho|^2)} \hat{Y}_1 = \frac{\mathcal{E}_x}{\mathcal{E}_x + \sigma^2(1 - |\rho|^2)} (Y_1 - \rho Y_2),$$

which is the same as in (a). The interpretation is that it is optimal for the prediction of X to optimally predict Z_1 from Z_2 and then cancel it from Y_1.

Problem 2

(a)

$$\widehat{X}_a = \frac{H_a^* \sigma_x^2}{H_a H_a^* \sigma_x^2 + \sigma_a^2} Y_a, \quad \widehat{X}_b = \frac{H_b^* \sigma_x^2}{H_b H_b^* \sigma_x^2 + \sigma_b^2} Y_b.$$

$$\begin{aligned}
P_a &= \sigma_x^2 - H_a^* \sigma_x^2 \left(H_a H_a^* \sigma_x^2 + \sigma_a^2 \right)^{-1} H_a \sigma_x^2, \\
&= \frac{\sigma_x^2 \sigma_a^2}{|H_a|^2 \sigma_x^2 + \sigma_a^2}, \\
P_b &= \sigma_x^2 - H_b^* \sigma_x^2 \left(H_b H_b^* \sigma_x^2 + \sigma_b^2 \right)^{-1} H_b \sigma_x^2, \\
&= \frac{\sigma_x^2 \sigma_b^2}{|H_b|^2 \sigma_x^2 + \sigma_b^2}.
\end{aligned}$$

(b) Using the identities

$$\begin{aligned}
\widehat{X}_a &= \left(\frac{1}{\sigma_x^2} + \frac{H_a H_a^*}{\sigma_a^2} \right)^{-1} \frac{H_a^*}{\sigma_a^2} Y_a, \\
\Rightarrow \left(\frac{1}{\sigma_x^2} + \frac{H_a H_a^*}{\sigma_a^2} \right) \widehat{X}_a &= \frac{H_a^*}{\sigma_a^2} Y_a, \\
\Rightarrow P_a^{-1} \widehat{X}_a &= \frac{H_a^*}{\sigma_a^2} Y_a.
\end{aligned}$$

Similarly,

$$P_b^{-1} \widehat{X}_b = \frac{H_b^*}{\sigma_b^2} Y_b.$$

(c) Now

$$\widehat{X} = \begin{bmatrix} H_a^* & H_b^* \end{bmatrix} \sigma_x^2 \begin{bmatrix} H_a H_a^* \sigma_x^2 + \sigma_a^2 & H_a H_b^* \sigma_x^2 \\ H_b H_a^* \sigma_x^2 & H_b H_b^* \sigma_x^2 + \sigma_b^2 \end{bmatrix}^{-1} \begin{bmatrix} Y_a \\ Y_b \end{bmatrix}$$

$$P = \mathcal{E}_x - \sigma_x^2 \begin{bmatrix} H_a^* & H_b^* \end{bmatrix} \begin{bmatrix} H_a H_a^* \sigma_x^2 + \sigma_a^2 & H_a H_b^* \sigma_x^2 \\ H_b H_a^* \sigma_x^2 & H_b H_b^* \sigma_x^2 + \sigma_b^2 \end{bmatrix}^{-1} \begin{bmatrix} H_a \\ H_b \end{bmatrix} \sigma_x^2.$$

Using the matrix identities by identifying

$$\mathbf{H} = \begin{bmatrix} H_a \\ H_b \end{bmatrix}, \mathbf{R_v} = \begin{bmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_b^2 \end{bmatrix}, \mathbf{R_x} = \sigma_x^2.$$

We get

$$\begin{aligned}
\widehat{X} &= \left(\frac{1}{\sigma_x^2} + \begin{bmatrix} H_a^* & H_b^* \end{bmatrix} \begin{bmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_b^2 \end{bmatrix}^{-1} \begin{bmatrix} H_a \\ H_b \end{bmatrix} \right)^{-1} \begin{bmatrix} H_a^* & H_b^* \end{bmatrix} \begin{bmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_b^2 \end{bmatrix}^{-1} \begin{bmatrix} Y_a \\ Y_b \end{bmatrix}, \\
&\Rightarrow \quad P^{-1}\widehat{X} = \frac{H_a^*}{\sigma_a^2} Y_a + \frac{H_b^*}{\sigma_b^2} Y_b = P_a^{-1}\widehat{X}_a + P_b^{-1}\widehat{X}_b.
\end{aligned}$$

Now

$$\begin{aligned}
P^{-1} &= \left(\frac{1}{\sigma_x^2} + \begin{bmatrix} H_a^* & H_b^* \end{bmatrix} \begin{bmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_b^2 \end{bmatrix}^{-1} \begin{bmatrix} H_a \\ H_b \end{bmatrix} \right), \\
&= \left(\frac{1}{\sigma_x^2} + \frac{H_a^* H_a}{\sigma_a^2} + \frac{H_b^* H_b}{\sigma_b^2} \right), \\
&= P_a^{-1} + P_b^{-1} - \frac{1}{\sigma_x^2}.
\end{aligned}$$

Problem 3

- (Precalculations) Before go through the solution, let compute the power spectral density of x and its spectral factorization (Supposing that the Paley-Wiener condition holds).

$$\begin{aligned}
S_x(D) &= \sum_{k=-\infty}^{+\infty} r_x(k) D^k \\
&= \sum_{k=-\infty}^{-1} r_x(k) D^k + r_x(0) + \sum_{1}^{+\infty} r_x(k) D^k \\
&= \sum_{k=-\infty}^{-1} (\frac{2}{3})^{-k} D^k + \frac{23}{28} + \sum_{k=1}^{\infty} (\frac{2}{3})^k D^k \\
&= \sum_{k=1}^{\infty} (\frac{2}{3} D^{-1})^k + \frac{23}{28} + \sum_{k=1}^{\infty} (\frac{2}{3} D)^k \\
&= \frac{\frac{2}{3} D^{-1}}{1 - \frac{2}{3} D^{-1}} + \frac{23}{28} + \frac{\frac{2}{3} D}{1 - \frac{2}{3} D} \\
&= \frac{\frac{2}{3} D - \frac{4}{9} + \frac{2}{3} D^{-1} - \frac{4}{9} + \frac{23}{28} (1 - \frac{2}{3} D)(1 - \frac{2}{3} D^{-1})}{(1 - \frac{2}{3} D)(1 - \frac{2}{3} D^{-1})} \\
&= \frac{\frac{5}{21} (\frac{1}{2} D + \frac{1}{2} D^{-1} + \frac{5}{4})}{(1 - \frac{2}{3} D)(1 - \frac{2}{3} D^{-1})} \\
&= \underbrace{\frac{5}{21}}_{\Gamma} \frac{\overbrace{(1 + \frac{1}{2} D)}^{(i)}}{\underbrace{(1 - \frac{2}{3} D)}_{(iii)}} \frac{\overbrace{(1 + \frac{1}{2} D^{-1})}^{(ii)}}{\underbrace{(1 - \frac{2}{3} D^{-1})}_{(iv)}}
\end{aligned}$$

$S_x(D)$ is then expressed under the form $\Gamma L(D) L^*(D^{-*})$.
We choose $L(D)$ as the minimum-phase part (all zeros and poles are outside the unit circle for the D-Transform, or equivalently, inside the unit circle for the Z-Transform).
The roots of the polynomials (i), (ii), (iii) and (iv) are -2, $-\frac{1}{2}$, $\frac{3}{2}$, $\frac{2}{3}$, respectively. Only polynomials i and iii have their roots outside the unit circle, so $L(D) = \frac{(1+\frac{1}{2}D)}{(1-\frac{2}{3}D)}$. (Note that we have chosen the coefficients such that the result is monic.)

Now lets compute the inverse D-Transform of $L(D)$ and of $\frac{1}{L(D)}$ that will be requested for solving the problem.

$$\begin{aligned}
L(D) &= \frac{(1+\frac{1}{2}D)}{(1-\frac{2}{3}D)} = (1+\frac{1}{2}D)\sum_{k=0}^{\infty}(\frac{2}{3}D)^k \\
&= \sum_{k=0}^{\infty}(\frac{2}{3}D)^k + \frac{1}{2}\sum_{k=0}^{\infty}(\frac{2}{3})^k D^{k+1} \\
&= 1 + \sum_{k=1}^{\infty}(\frac{2}{3}D)^k + \frac{1}{2}\sum_{m=1}^{\infty}(\frac{2}{3})^{m-1} D^m \quad \text{(by using } m = k+1 \text{ in the second summation)} \\
&= 1 + \sum_{k=1}^{\infty}\left((\frac{2}{3})^k + \frac{1}{2}(\frac{2}{3})^{k-1}\right) D^k \\
&= 1 + \frac{7}{4}\sum_{k=1}^{\infty}(\frac{2}{3})^k D^k
\end{aligned}$$

so

$$l(k) = \begin{cases} 1 & \text{if} \quad k = 0 \\ \frac{7}{4}(\frac{2}{3})^k & \text{if} \quad k \geq 1 \end{cases}$$

(a) $\hat{x}_{k+2} = \sum_{m=2}^{\infty} a_m x_{k+2-m}$
We have to find $a_{m\,m=2}^{\;\infty}$ such that $\mathbb{E}[|x_{k+2} - \hat{x}_{k+2}|^2]$ is minimized. Using the orthogonality

principle, we have

$$\begin{aligned}
\mathbb{E}[(x_{k+2}-\hat{x}_{k+2})x_{k-n}] &= 0 \quad n=0,1,...\\
\mathbb{E}[(x_{k+2}-\sum_{m=2}^{\infty} a_m x_{k+2-m})x_{k-n}] &= 0\\
\mathbb{E}[x_{k+2}x_{k-n}]-\sum_{m=2}^{\infty} a_m \mathbb{E}[x_{k+2-m}x_{k-n}] &= 0\\
r_x(n+2) &= \sum_{m=2}^{\infty} a_m r_x(n+2-m)\\
r_x(l) &= \sum_{m=2}^{\infty} a_m r_x(l-m)\\
&\quad\quad \text{(using } l=n+2 \text{ for simplicity, } l=2,3,...)\\
g_l &= r_x(l)-\sum_{m=2}^{\infty} a_m r_x(l-m)\\
g_l &= \sum_{m=0}^{\infty} a'_m r_x(l-m)\\
&\quad (*)(\text{with } a'_0=1,\ a'_1=0 \text{ and } a'_m=-a_m \text{ for } m\geq 2)
\end{aligned}$$

Note that $g_l = 0$ for $l = 2, 3, ...$, thus it is neither causal nor anti-causal because it has non-null terms for $l \leq 1$. But it can be transformed to an anti-causal sequence by a shift left of 1, i. e.,

$$\begin{aligned}
G(D) &= g_1 D + g_0 + g_{-1}D^{-1} + g_{-2}D^{-2} + \cdots\\
&= D \cdot \underbrace{\left[g_1 + g_0 D^{-1} + g_{-1}D^{-2} + g_{-2}D^{-3} + \cdots\right]}_{\tilde{G}(D)}
\end{aligned}$$

where $\tilde{G}(D)$ is an anti-causal function. Now, we have

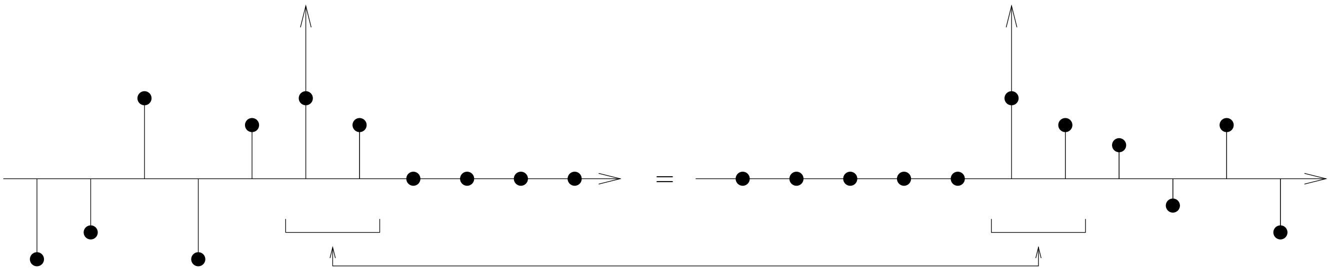
$$\begin{aligned}
G(D) &= A'(D)S_x(D)\\
&= A'(D)\Gamma L(D)L(D^{-1}),
\end{aligned}$$

or

$$\underbrace{D\frac{\tilde{G}(D)}{\Gamma L^*(D^{-*})}}_{\text{anti-causal}} = \underbrace{A'(D)L(D)}_{\text{causal}}$$

So far we observe that the left hand side is an anti-causal sequence shifted to the right by 1 and the right hand side is a causal sequence. The equality lead to the conclusion that all terms are null except in $k = 0$ and $k = 1$.

Then we can write the following: $A'(D)L(D) = \gamma_0 + \gamma_1 D$. Using (*) and the fact that L(D) is monic (division of two monic polynomials remains monic) developing and identifying we get the following: $(1 + a'_2 D^2 + a'_3 D^3 + ...)(1 + l_1 D + l_2 D^2 + ...) = \gamma_0 + \gamma_1 D$,



thus $\gamma_0 = 1$ and $\gamma_1 = l_1 = \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$
Putting all together we obtain $A'(D)$ and by an inverse D-Transform and still using (*) we can find $A(D)$.

$$\begin{aligned}
A'(D) &= \frac{\gamma_0 + \gamma_1 D}{L(D)} = \frac{1 + \frac{7}{6}D}{L(D)} \\
&= (1 + \frac{7}{6}D)(1 - \frac{2}{3}D)\frac{1}{(1 + \frac{1}{2}D)} \\
&= (1 + \frac{1}{2}D - \frac{7}{9}D^2)\sum_{k=0}^{\infty}(-\frac{1}{2}D)^k \\
&= \sum_{k=0}^{\infty}(-\frac{1}{2}D)^k + \frac{1}{2}\sum_{k=0}^{\infty}(-\frac{1}{2})^k D^{k+1} - \frac{7}{9}\sum_{k=0}^{\infty}(-\frac{1}{2})^k D^{k+2} \\
&= 1 - \frac{1}{2}D + \sum_{k=2}^{\infty}(-\frac{1}{2})^k D^k + \frac{1}{2}D + \frac{1}{2}\sum_{k=1}^{\infty}(-\frac{1}{2})^k D^{k+1} - \frac{7}{9}\sum_{k=0}^{\infty}(-\frac{1}{2})^k D^{k+2} \\
&= 1 - \frac{1}{2}D + \sum_{k=2}^{\infty}(-\frac{1}{2})^k D^k + \frac{1}{2}D + \frac{1}{2}\sum_{k=2}^{\infty}(-\frac{1}{2})^{k-1} D^{k} - \frac{7}{9}\sum_{k=2}^{\infty}(-\frac{1}{2})^{k-2} D^{k} \\
A'(D) &= 1 + \sum_{k=2}^{\infty}\left((-\frac{1}{2})^k + \frac{1}{2}(-\frac{1}{2})^{k-1} - \frac{7}{9}(-\frac{1}{2})^{k-2}\right)D^k \\
&= 1 - \frac{28}{9}\sum_{k=2}^{\infty}(-\frac{1}{2})^k D^k \\
\Longrightarrow a'_k &= \begin{cases} 1 & \text{if } \quad = 0 \\ 0 & \text{if } \quad k = 1 \\ -\frac{28}{9}(-\frac{1}{2})^k - & \text{if } \quad k \geq 2 \end{cases}
\end{aligned}$$

or

$$a_m = \frac{28}{9}(-\frac{1}{2})^m \quad \text{for } m \geq 2$$

(b) $\hat{x}_{k+1} = \sum_{m=1}^{\infty} b_m x_{k+1-m}$:
This case is the same as in the lecture note: section 5.1.3 (One-step linear prediction).
We founded that $B'(D) = \frac{1}{L(D)}$ where $b'_0 = 1$ and $b'_m = -b_m$

$$
\begin{aligned}
B'(D) &= \frac{1}{L(D)} = (1-\frac{2}{3}D)\frac{1}{(1+\frac{1}{2}D)} \\
&= (1-\frac{2}{3}D)\sum_{k=0}^{\infty}(-\frac{1}{2}D)^k \\
&= \sum_{k=0}^{\infty}(-\frac{1}{2})^k D^k - \frac{2}{3}\sum_{k=0}^{\infty}(-\frac{1}{2})^k D^{k+1} \\
&= 1+\sum_{k=1}^{\infty}(-\frac{1}{2})^k D^k - \frac{2}{3}\sum_{k=1}^{\infty}(-\frac{1}{2})^{k-1} D^{k} \\
&= 1+\sum_{k=1}^{\infty}\left((-\frac{1}{2})^k - \frac{2}{3}(-\frac{1}{2})^{k-1}\right) D^k \\
&= 1+\frac{7}{3}\sum_{k=1}^{\infty}(-\frac{1}{2})^k D^k.
\end{aligned}
$$

So,

$$
b_m = -\frac{7}{3}(-\frac{1}{2})^m \quad \text{for } m \geq 1
$$

(c) We suppose that $\hat{x}_{k+1}$ is perfect and then $y_{k+1} = x_{k+1}$. Note that this assumption is not mentioned in the question. But without this assumption the problem cannot be solved. The problem become a one-step prediction as used in question (b) but using $S_y(D)$ instead of $S_x(D)$.
Now we will show that they are equal:

$$
\begin{aligned}
r_y(k+1-n) &= \mathbb{E}[y_{k+1}y_n] \\
&= \mathbb{E}[\hat{x}_{k+1}y_n] \quad (n \leq k) \\
&= \sum_{m=1}^{\infty} b_m \mathbb{E}[x_{k+1-m}x_n] \\
&= \sum_{m=1}^{\infty} b_m r_x(k-m-n+1) \\
&= r_x(k-n+1) \quad \text{(Follows from the OP for one-step prediction)}
\end{aligned}
$$

$$
\Longrightarrow S_y(D) = S_x(D)
$$

Using the OP we have

$$
\hat{y}_{k+2} = \sum_{m=1}^{\infty} c_m y_{k+2-m} \perp y_{k+2-n} \quad n = 1,2,3,\ldots
$$

Thus

$$
h_n = r_y(n) - \sum_{m=1}^{\infty} c_m r_y(l-m) = 0 \quad n = 1,2,\ldots
$$

Now again we can write $C'(D) = \frac{1}{L(D)}$, where $c'_0 = 1$ and $c'_m = -c_m$ for $m \geq 1$. Therefore,

$$
c_m = -\frac{7}{3}(-\frac{1}{2})^m \quad \text{for } m \geq 1
$$

(d) Now we just replace y_{k+1} in part (c) by the result of part (b).

$$\begin{aligned}
\hat{y}_{k+2} &= \sum_{m=1}^{\infty} c_m y_{k+2-m} \\
&= c_1 y_{k+1} + \sum_{m=2}^{\infty} c_m y_{k+2-m} \\
&= c_1 \sum_{n=1}^{\infty} b_n x_{k+1-n} + \sum_{m=2}^{\infty} c_m x_{k+2-m} \qquad \text{(because } y_n = x_n \text{ for } n \leq k) \\
&= c_1 \sum_{n=2}^{\infty} b_{n-1} x_{k+2-n} + \sum_{m=2}^{\infty} c_m x_{k+2-m} \\
&= \sum_{m=2}^{\infty} (c_1 b_{m-1} + c_m) x_{k+2-m} \\
\Rightarrow & \; d_m = c_1 b_{m-1} + c_m \qquad \text{for } m \geq 2
\end{aligned}$$

$$\begin{aligned}
d_m &= c_1 b_{m-1} + c_m \\
&= -\frac{7}{3}(-\frac{1}{2})^1 \cdot (-\frac{7}{3}(-\frac{1}{2})^{m-1}) + (-\frac{7}{3}(-\frac{1}{2})^m) \\
&= -\frac{7}{3}(-\frac{1}{2})^m(-\frac{7}{3} + 1) \\
&= \frac{28}{9}(-\frac{1}{2})^m = a_m \quad m \geq 2
\end{aligned}$$

As it was expectable, we see that the result of two-step prediction is absolutely the same as the result of the combination of two times using one-setp prediction. The reason is we are using the same observation in the both methods ($\{x_n\}_{n=-\infty}^{n=k}$), and our objective function (cost of the prediction, i. e., $\mathbb{E}[|x_{k+2} - \hat{x}_{k+2}|^2]$) is also the same, and so we will obtain the same results.

Problem 4

(i) In this case the sequence given $\{U_{1k}\}$, $\{U_{2k}\}$ is irrelevant. Let us try to compute the W_{opt} using both the sequences. Let the estimate be given by $\hat{X}_k = \sum_{i=-\infty}^{\infty} W_{1i} U_{1k-i} + W_{2i} U_{2k-i}$ From the orthogonality principle, we have

$$\mathbb{E}[(X_k - \hat{X}_k)U_{1j}] = 0, \quad \forall j$$
$$\mathbb{E}[(X_k - \hat{X}_k)U_{2j}] = 0, \quad \forall j$$

Fromt he orthogonality principle we get

$$R_{XU_1}(D) = W_1(D)R_{U_1U_1}(D) + W_2(D)R_{U_2U_1}(D)$$
$$R_{XU_2}(D) = W_1(D)R_{U_1U_2}(D) + W_2(D)R_{U_2U_2}(D)$$

Since the noise is independent from X, this implies

$$\begin{aligned}
R_{U_1U_1}(D) =& R_{YY}(D) + \sigma_1^2 \\
R_{U_1U_2}(D) =& R_{U_2U_1}(D) = R_{YY}(D) \\
R_{U_2U_2}(D) =& \sigma_2^2 + R_{YY}(D)
\end{aligned}$$

Therefore we have

$$\begin{aligned} R_{XY}(D) =& W_1(D)R_{YY}(D) + W_2(D)R_{YY}(D) \\ R_{XY}(D) =& W_1(D)R_{YY}(D) + W_2(D)R_{YY}(D) + 4W_2(D) \end{aligned}$$

Solving this we get, $W_1(D) = R_{XY}(D)/R_{YY}(D)$, $W_2(D) = 0$. Therefore the sequence $U_2(D)$ was irrelevant.

(ii) In this case

$$\begin{aligned} R_{XY}(D) =& W_1(D)R_{YY}(D) + W_1(D) + W_2(D)R_{YY}(D) \\ R_{XY}(D) =& W_1(D)R_{YY}(D) + W_2(D)R_{YY}(D) + 4W_2(D) \end{aligned}$$

Solving this we get

$$\begin{aligned} W_1(D) =& \frac{4R_{XY}(D)}{4 + 5R_{YY}(D)} \\ W_2(D) =& \frac{R_{XY}(D)}{4 + 5R_{YY}(D)} \end{aligned}$$

(iii) In the part (i) given $\{U_{1k}\}$ the sequence $\{U_{2k}\}$ is irrelevant. This is because the error in the estimate $W_1(D)U_1(D)$ is orthogonal to $\{U_{1k}\}$ and the noise $\{Z_{2k}\}$ is orthogonal to U_{1k}, making the error orthogonal to $\{U_{2k}\}$ also. Hence we do not need $\{U_{2k}\}$ to estimate. But in the part (ii) this is not true and both the sequences are relevant.