Homework 1 - part 2

Problem 5
If $\mathbf{Z} = (Z_1, Z_2)$ is a Gaussian random vector, then by definition, $Z_1$ and $Z_2$ are Gaussian random variables. However, the contrary is not true in general. Construct a simple example showing that $Z_1$ and $Z_2$ being Gaussian random variables does not imply that $Z_1$ and $Z_2$ are jointly Gaussian.

Problem 6
Show that $U$ is a proper random variable if and only if

$$\hat{K}_U = 2K_U.$$

Problem 7
A bandpass signal $x(t)$ may be written as $x(t) = \sqrt{2} \Re\{x_E(t)e^{j2\pi f_0 t}\}$, where $x_E(t)$ is the baseband equivalent of $x(t)$.

1. Show that a signal $x(t)$ can also be written as $x(t) = a(t) \cos[2\pi f_0 t + \theta(t)]$ and describe $a(t)$ and $\theta(t)$ in terms of $x_E(t)$. Interpret this result.

2. Show that the signal $x(t)$ can also be written as $x(t) = x_{EI}(t) \cos 2\pi f_0 t - x_{EQ}(t) \sin(2\pi f_0 t)$, and describe $x_{EI}(t)$ and $x_{EQ}(t)$ in terms of $x_E(t)$. (This shows how you can obtain $x(t)$ without doing complex-valued operations.)

3. Find the baseband equivalent of the signal $x(t) = A(t) \cos(2\pi f_0 t + \varphi)$, where $A(t)$ is a real-valued lowpass signal. Hint: You may find it easier to guess an answer and verify that it is correct.