INTERACTION OF D-BRANES ON ORBIFOLDS AND MASSLESS PARTICLE EMISSION

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We discuss various D-brane configurations in 4-dimensional orbifold compactifications of type II superstring theory which are point-like 0-branes from the 4-dimensional space-time point of view. We analyze their interactions and compute the amplitude for the emission of a massless NSNS boson from them, in the case where the branes have a non vanishing relative velocity. In the large distance limit, our computation agrees with the expected field theory results.

Talk presented by Claudio A. Scrucca

1 Introduction and summary

We discuss various D-brane configurations in generic orbifold compactifications which are 0-branes from the 4-dimensional space-time point of view, but can have extension in the compact directions. Two cases turn out to be particularly interesting: the 0-brane of type IIA and the 3-brane of type IIB.

The dynamics of these D-branes is determined by a one loop amplitude which can be conveniently evaluated in the boundary state formalism [1,2]. In particular, one can compute the force between two D-branes moving with constant velocity, extending Bachas' result [3] to compactifications breaking some supersymmetry [4].

Analyzing the large distance behavior of the interaction and its velocity dependence, it is possible to read the charges with respect to the massless fields, and relate the various D-brane configurations to known solutions of the 4-dimensional low energy effective supergravity.

Finally, we discuss the emission of massless NSNS states from two interacting D-branes [5]. The correlators that are involved have twisted boundary conditions because of the non zero velocity of the branes, but they can be systematically computed in a natural way using again the boundary state formalism. We then outline the field theory interpretation of the large distance behavior of the amplitude.

2 Interactions on orbifolds

Consider two D-branes moving with velocities $V_1 = \mathrm{tgh}\, v_1$, $V_2 = \mathrm{tgh}\, v_2$ (say along 1) and transverse positions $\vec{Y}_1$, $\vec{Y}_2$ (along 2,3). The potential between these two D-branes is given by the cylinder vacuum amplitude and can be thought either as the Casimir energy stemming from open string vacuum fluctuations or as the interaction energy related to the exchange closed strings between the two branes. The amplitude in the closed string channel

$$\mathcal{A} = \int_0^\infty dl \sum_s < B, V_1, \vec{Y}_1 | e^{-lH} | B, V_2, \vec{Y}_2 >_s$$

is just a tree level propagation between the two boundary states, which are defined to implement the boundary conditions defining the branes.

There are two sectors, RR and NSNS, corresponding to periodicity and antiperiodicity of the fermionic fields around the cylinder, and after the GSO projection there are four spin structures, R± and NS±, corresponding to all the possible periodicities of the fermions on the covering torus.

In the static case, one has Neumann b.c. in time and Dirichlet b.c. in space. The velocity twists the 0-1 directions and gives them rotated b.c. The moving boundary state is most simply obtained by boosting the static one with a negative rapidity $v = v_1 - v_2$ [6].

$$|B, V, \vec{Y} >= e^{-ivJ^{01}} |B, \vec{Y} > \quad .$$

In the large distance limit $b \to \infty$ only world-sheets with $l \to \infty$ will contribute, and momentum or winding in the compact directions can be safely neglected since they correspond to massive subleading components.

The moving boundary states

$$|B, V_1, \vec{Y}_1 >= \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{Y}_1} |B, V_1 > \otimes |k_B > \,,$$
$$|B, V_2, \vec{Y}_2 >= \int \frac{d^3\vec{q}}{(2\pi)^3} e^{i\vec{q}\cdot\vec{Y}_2} |B, V_2 > \otimes |q_B > \,,$$

can thus carry only space-time momentum in the boosted combinations

$$k_B^\mu = (V_1\gamma_1 k^1, \gamma_1 k^1, \vec{k}_T) = (\mathrm{sh}\, v_1 k^1, \mathrm{ch}\, v_1 k^1, \vec{k}_T) \,,$$
$$q_B^\mu = (V_2\gamma_2 q^1, \gamma_2 q^1, \vec{q}_T) = (\mathrm{sh}\, v_2 q^1, \mathrm{ch}\, v_2 q^1, \vec{q}_T) \,.$$

Notice that because of their non zero velocity, the branes can also transfer energy, and not only momentum as in the static case.

Integrating over the bosonic zero modes and taking into account momentum conservation ($k_B^\mu = q_B^\mu$), the amplitude factorizes into a bosonic and a fermionic partition functions:

$$\mathcal{A} = \frac{1}{\operatorname{sh} v}\int_0^\infty dl \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q_B^2}{2}} \sum_s Z_B Z_F^s = \frac{1}{\operatorname{sh} v}\int_0^\infty \frac{dl}{2\pi l} e^{-\frac{b^2}{2l}} \sum_s Z_B Z_F^s$$

with $Z_{B,F} =< B, V_1|e^{-lH}|B, V_2 >_{B,F}^s$ and from now on, $X^\mu \equiv X^\mu_{osc}$.

It will prove convenient to group the fields into pairs

$$\begin{aligned}
X^\pm = X^0 \pm X^1 &\rightarrow \alpha_n, \beta_n = a_n^0 \pm a_n^1 \ , \\
X^i, X^{i*} = X^i \pm iX^{i+1} &\rightarrow \beta_n^i, \beta_n^{i*} = a_n^i \pm i a_n^{i+1} \ , \quad i = 2,4,6,8 \ , \\
\chi^{A,B} = \psi^0 \pm \psi^1 &\rightarrow \chi_n^{A,B} = \psi_n^0 \pm \psi_n^1 \ , \\
\chi^i, \chi^{i*} = \psi^i \pm i\psi^{i+1} &\rightarrow \chi_n^i, \chi_n^{i*} = \psi_n^i \pm i\psi_n^{i+1} \ , \quad i = 2,4,6,8 \ .
\end{aligned}$$

For the RR zero modes, which satisfy a Clifford algebra and are thus proportional to Γ-matrices, $\psi_o^\mu = i\Gamma^\mu/\sqrt{2}$, $\tilde{\psi}_o^\mu = i\tilde{\Gamma}^\mu/\sqrt{2}$, on can construct similarly the creation-annihilation operators

$$a, a^* = \frac{1}{2}(\Gamma^0 \pm \Gamma^1) \ , \quad b^i, b^{i*} = \frac{1}{2}(-i\Gamma^i \pm \Gamma^{i+1}) \ .$$

In this way, any rotation or boost will reduce to a simple phase transformation on the modes. In fact, for an orbifold rotation ($g_a = e^{2\pi i z_a}$)

$$\begin{aligned}
&\beta_n^a \rightarrow g_a \beta_n^a \ , \quad \chi_n^a \rightarrow g_a \chi_n^a \ , \quad b^a \rightarrow g_a b^a \ , \\
&\beta_n^{a*} \rightarrow g_a^* \beta_n^{a*} \ , \quad \chi_n^{a*} \rightarrow g_a^* \chi_n^{a*} \ , \quad b^{a*} \rightarrow g_a^* b^{a*} \ .
\end{aligned} \tag{1}$$

whereas for a boost of rapidity v,

$$\begin{aligned}
&\alpha_n \rightarrow e^{-v}\alpha_n \ , \quad \chi_n^A \rightarrow e^{-v}\chi_n^A \ , \quad a \rightarrow e^{-v} a \ , \\
&\beta_n \rightarrow e^{v}\beta_n \ , \quad \chi_n^B \rightarrow e^{v}\chi_n^B \ , \quad a^* \rightarrow e^{v} a^* \ .
\end{aligned} \tag{2}$$

The boundary state which solves the b.c. can be factorized into a bosonic and a fermionic parts; the latter can be further splitted into zero mode and oscillator parts, and finally

$$|B >= |B >_B \otimes |B_o >_F \otimes |B_{osc} >_F \ .$$

2.1 *Orbifold construction*

Let us briefly recall the orbifold construction. An orbifold compactification can be obtained by identifying points in the compact part of space-time which are connected by discrete rotations $g = e^{2\pi i \sum_a z_a J_{aa+1}}$ on some of the compact pairs X^a,χ^a, a=4,6,8. In order to preserve at least one supersymmetry, one has to impose the condition $\sum_a z_a = 0$.

We will consider three case: toroidal compactification on T_6 and orbifold compactification on $T_2 \otimes T_4/Z_2$ and T_6/Z_3. The construction is universal, and these three cases can be obtained by explicit choices for the angles z_a:

$$\begin{aligned} T_6/Z_3 \ (N=2 \text{ SUSY}) &: \text{take } z_4, z_6 = \tfrac{1}{3}, \tfrac{2}{3}\ , \quad z_8 = -z_4 - z_6\ , \\ T_2 \otimes T_4/Z_2 \ (N=4 \text{ SUSY}) &: \text{take } z_4 = -z_6 = \tfrac{1}{2}\ , \quad z_8 = 0\ , \\ T_6 \ (N=8 \text{ SUSY}) &: \text{take } z_4 = z_6 = z_8 = 0\ . \end{aligned}$$

The spectrum of the theory now contains additional twisted sectors, in which periodicity is achieved only up to an element of the quotient group Z_N. This leads to fractional moding in the compact directions.

These twisted states exist at fixed points of the orbifold. They thus occur only for the 0-brane of type IIA, which corresponds to Dirichlet b.c. in all the compact directions and can thus be localized at a fixed point.

Finally, in all sectors, one has to project onto invariant states to get the physical spectrum of the theory which is invariant under orbifold rotations.

2.2 *0-brane: untwisted sector*

Consider a static configuration with Neumann b.c. for time and Dirichlet b.c. for all other directions (i=2,4,6,8 and a=2,4,6).

The boundary state is easily constructed [4] as a Bogolubov transformation from a spinor vacuum $|0> \otimes|\tilde{0}>$ defined such that $a|0> = \tilde{a}|\tilde{0}> = b^i|0> = \tilde{b}^{i*}|\tilde{0}> = 0$. After applying the boost eq. (2), under which the spinor vacuum picks up an imaginary phase, $|0> \otimes|\tilde{0}> \rightarrow e^{-v}|0> \otimes|\tilde{0}>$, the result is

$$\begin{aligned} &|B,V>_B = \exp \frac{1}{2} \sum_{n>0} [e^{-2v} \alpha_{-n} \tilde{\alpha}_{-n} + e^{2v} \beta_{-n} \tilde{\beta}_{-n} + 2\,\mathrm{Re}\,(\beta^i_{-n} \tilde{\beta}^{i*}_{-n})]|0>\ , \\ &|B_{osc},V,\eta>_F = \exp \frac{i\eta}{2} \sum_{n>0} [e^{-2v} \chi^A_{-n} \tilde{\chi}^A_{-n} + e^{2v} \chi^B_{-n} \tilde{\chi}^B_{-n} - 2\,\mathrm{Re}\,(\chi^i_{-n} \tilde{\chi}^{i*}_{-n})]|0>\ , \\ &|B_o,V,\eta>_{RR} = e^{-v} \exp -i\eta [e^{2v} a^* \tilde{a}^* - b^{i*} \tilde{b}^i]|0> \otimes|\tilde{0}>\ . \end{aligned}$$

The complete boosted boundary state is already invariant under orbifold rotations eq. (1). This comes from the fact that the Z_N action rotates pairs of fields with the same b.c. and is thus irrelevant.

In both sectors, the fermion number operator reverses the sign of the parameter η and the GSO-projected boundary state is

$$|B,V>=\frac{1}{2}(|B,V,+>-|B,V,->)\ .$$

The partition function can then be computed carrying out some simple oscillator algebra; the ghosts cancel one untwisted pair, say 2-3, and the result is the product of the contributions of the 0-1 pair and the 3 compact pairs.

After the GSO projection, only the three even spin structures R+ and NS± contribute. Defining $q=e^{-2\pi l}$, the total bosonic (zero-point energy $q^{-\frac{2}{3}}$) and fermionic (zero-point energy $q^{-\frac{1}{3}}$ for NSNS and $q^{\frac{2}{3}}$ for RR) partition functions can be written as

$$\begin{aligned}
Z_B &= 16\pi^3 i\,\mathrm{sh}\,v q^{\frac{1}{3}} f(q^2)^4 \frac{1}{\vartheta_1(i\frac{v}{\pi}|2il)\vartheta_1'(0|2il)^3}\ ,\\
Z_F &= q^{-\frac{1}{3}} f(q^2)^{-4}\Big\{\vartheta_2(i\frac{v}{\pi}|2il)\vartheta_2(0|2il)^3\\
&\qquad -\vartheta_3(i\frac{v}{\pi}|2il)\vartheta_3(0|2il)^3+\vartheta_4(i\frac{v}{\pi}|2il)\vartheta_4(0|2il)^3\Big\}\\
&\sim V^4\ ,
\end{aligned} \tag{3}$$

corresponding to the cancellation of the force between two BPS states[7,3]. Thus, the untwisted sector for the 0-brane gives the same result as the uncompactified theory for every compactification scheme.

2.3 0-brane: twisted sector

Consider now the twisted sector, which has to be included when the 0-brane is at an orbifold fixed point. In this case, the boundary state is similar to the one of the untwisted sector, with fractional moding in the compact directions.

In the Z_3 case the total partition functions after the GSO projection are

$$Z_B = 2i\,\mathrm{sh}\,v f(q^2)^4 \frac{1}{\vartheta_1(i\frac{v}{\pi}|2il)\vartheta_1(-\frac{2}{3}il|2il)^3}\ , \tag{4}$$

$$\begin{aligned}
Z_F &= f(q^2)^{-4}\Big\{\vartheta_2(i\frac{v}{\pi}|2il)\vartheta_2(-\frac{2}{3}il|2il)^3\\
&\qquad -\vartheta_3(i\frac{v}{\pi}|2il)\vartheta_3(-\frac{2}{3}il|2il)^3-\vartheta_4(i\frac{v}{\pi}|2il)\vartheta_4(-\frac{2}{3}il|2il)^3\Big\}\\
&\sim V^2\ .
\end{aligned} \tag{5}$$

In the Z_2 case, the analysis is similar and the results behave qualitatively as before, with one twist set to zero. In particular, one has again $Z_F \sim V^2$.

2.4 3-brane

Consider now a particular 3-brane configuration with Neumann b.c. for time, Dirichlet b.c. for space and mixed b.c. for each pair of compact directions, say Neumann for the a directions and Dirichlet for the a+1 directions.

Defining a new spinor vacuum $|0> \otimes|\tilde{0}>$ such that $b^a|0>= \tilde{b}^a|\tilde{0}>= 0$ the compact part of the boundary state is similar to that for the 0-brane, but is not invariant under orbifold rotations, under which the modes of the fields transform as in eq. (1) and the spinor vacuum as $|0> \otimes|\tilde{0}> \rightarrow g_a|0> \otimes|\tilde{0}>$. This was expected since a Z_N rotation now mixes two directions with different b.c., and thus the corresponding closed string state is not Z_N-invariant.

The compact part of the twisted boundary state is found to be

$$
\begin{aligned}
|B, V, g_a>_B &= \exp -\frac{1}{2}\sum_{n>0}[g_a^2\beta^a_{-n}\tilde{\beta}^a_{-n} + g_a^{*2}\beta^{a*}_{-n}\tilde{\beta}^{a*}_{-n}]|0> , \\
|B_{osc}, V, g_a, \eta>_F &= \exp \frac{i\eta}{2}\sum_{n>0}[g_a^2\chi^a_{-n}\tilde{\chi}^a_{-n} + g_a^{*2}\chi^{a*}_{-n}\tilde{\chi}^{a*}_{-n}]|0> , \\
|B_o, V, g_a, \eta>_{RR} &= g_a \exp -i\eta g_a^{*2} b^{a*}\tilde{b}^{a*}|0> \otimes|\tilde{0}> \ .
\end{aligned}
$$

After the GSO projection, the partition functions for a relative twist w_a are

$$
\begin{aligned}
Z_B &= 16i \,\mathrm{sh}\, v q^{\frac{1}{3}} f(q^2)^4 \frac{1}{\vartheta_1(i\frac{v}{\pi}|2il)} \prod_a \frac{\sin \pi w_a}{\vartheta_1(w_a|2il)} , \\
Z_F &= q^{-\frac{1}{3}} f(q^2)^{-4} \Big\{ \vartheta_2(i\frac{v}{\pi}|2il) \prod_a \vartheta_2(w_a|2il) \\
&\qquad - \vartheta_3(i\frac{v}{\pi}|2il) \prod_a \vartheta_3(w_a|2il) + \vartheta_4(i\frac{v}{\pi}|2il) \prod_a \vartheta_4(w_a|2il) \Big\} \\
&\sim \begin{cases} V^4 \ , & w_a = 0 \\ V^2 \ , & w_a \neq 0 \end{cases} \ .
\end{aligned} \tag{6}
$$

Recall that to obtain the invariant amplitude, one has to average over all possible twists w_a. Finally, for this 3-brane configuration there is no twisted sector, as already explained.

2.5 Large distance limit and field theory interpretation

In the large distance limit $l \to \infty$, explicit results with their exact dependence on the rapidity can be obtained and compared to a field theory computation. The behaviors that one finds are the following:

0-brane

a) Untwisted sector

$$\mathcal{A} \sim 4\,\mathrm{ch}\,v - \mathrm{ch}\,2v - 3 \sim V^4 \ . \tag{7}$$

b) Twisted sector

$$\mathcal{A} \sim \mathrm{ch}\,v - 1 \sim V^2 \ . \tag{8}$$

3-brane

$$\begin{aligned} &\mathcal{A}(w_a) \sim 4 \prod_a \cos \pi w_a \,\mathrm{ch}\,v - \mathrm{ch}\,2v - \sum_a \cos 2\pi w_a \ , \\ &\mathcal{A} \sim \begin{cases} \mathrm{ch}\,v - \mathrm{ch}\,2v \sim V^2 \ , \ T_6/Z_3 \\ 4\,\mathrm{ch}\,v - \mathrm{ch}\,2v - 3 \sim V^4 \ , \ T_2 \otimes T_4/Z_2 \ , \ T_6 \end{cases} . \end{aligned} \tag{9}$$

In the low energy effective supergravity field theories, the possible contributions to the scattering amplitude in the eikonal approximation come from vector exchange in the RR sector and scalar and graviton exchange in the NSNS sector. The respective contributions have a peculiar dependence on the rapidity reflecting the tensorial nature and are:

$$\mathcal{A}^{NS}_{\phi} \sim -a^2 \ , \ \mathcal{A}^{R}_{V_\mu} \sim e^2 \,\mathrm{ch}\,v \ , \ \mathcal{A}^{NS}_{g_{\mu\nu}} \sim -M^2 \,\mathrm{ch}\,2v \ . \tag{10}$$

Thus, the interpretation of the behaviors found in the various sectors and for the various brane configurations we have considered, is the following:

$$\begin{aligned} 4\,\mathrm{ch}\,v - \mathrm{ch}\,2v - 3 \quad &\Leftrightarrow \quad N = 8 \text{ Grav. multiplet} \ , \\ \mathrm{ch}\,v - \mathrm{ch}\,2v \quad &\Leftrightarrow \quad N = 2 \text{ Grav. multiplet} \ , \\ \mathrm{ch}\,v - 1 \quad &\Leftrightarrow \quad \text{Vec. multiplet} \ . \end{aligned}$$

The patterns of cancellation suggest that all the D-brane configurations that we have considered correspond to extremal p-brane solutions of the low energy supergravity, possibly coupling to the additional twisted vector multiplets; the 3-brane configuration on the Z_3 orbifold seems to be an exception since it does not couple to the scalars, and should thus correspond to a Reissner-Nordström extremal black hole.

Finally, notice that V^2 terms in the effective action are forbidden for $N = 8$ SUSY but appear in general for $N < 8$ SUSY.

3 Emission of massless NSNS bosons

Consider two moving D-branes in interaction emitting a massless NSNS boson. The amplitude is computed inserting the usual vertex operator between the two boundary states

$$\mathcal{A} = \int_0^\infty dl \int_0^l d\tau \sum_s < B, V_1, \vec{Y}_1 | e^{-lH} V(z,\bar{z}) | B, V_2, \vec{Y}_2 >_s$$

As usual, the zero mode part ensures momentum conservation $p^\mu = k_B^\mu - q_B^\mu$; the energies and longitudinal momenta

$$k_B^0 = V_1 k_B^1 \ , \ \ k_B^1 = \frac{p}{V_1 - V_2}(1 - V_2 \cos\theta) \ ,$$
$$q_B^0 = V_2 q_B^1 \ , \ \ q_B^1 = \frac{p}{V_1 - V_2}(1 - V_1 \cos\theta) \ .$$

are complitely fixed ($\cos\theta = p^1/p$, $p = p^0$).

The amplitude can be rewritten ($q^\mu \equiv q_B^\mu$, $k^\mu \equiv k_B^\mu$ and $v = v_1 - v_2$) as an integral over the proper times τ and l' of the states emitted by the branes:

$$\mathcal{A} = \frac{1}{\operatorname{sh} v} \int_0^\infty d\tau \int_0^\infty dl' \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} < e^{ip\cdot X} > \sum_s Z_B Z_F^s \mathcal{M}_s \ ,$$

3.1 Correlators

The boundary state formalism provides a systematic way of computing correlators with non trivial b.c., such as those needed here, through the definitions

$$< X^\mu X^\nu >= \frac{< B_1, V_1 | e^{-lH} X^\mu X^\nu | B_2, V_2 >_B}{< B_1, V_1 | e^{-lH} | B_2, V_2 >_B} \ , \tag{11}$$

$$< \psi^\mu \psi^\nu >_s= \frac{< B_1, V_1, \eta | e^{-lH} \psi^\mu \psi^\nu | B_2, V_2, \eta' >_F^s}{< B_1, V_1, \eta | e^{-lH} | B_2, V_2, \eta' >_F^s} \ . \tag{12}$$

For the bosons, one obtains an infinite series of logarithms corresponding to the propagation of all the string states with growing mass ($z = \sigma + i\tau$):

$$< X^0(z)\bar{X}^0(\bar{z}) >=< X^1(z)\bar{X}^1(\bar{z}) >=$$
$$= \frac{1}{4\pi} \sum_{n=0}^\infty \{ \operatorname{ch} 2[(v_1 - v_2)n - v_2] f_n(\tau) - \operatorname{ch} 2[(v_2 - v_1)n - v_1] f_n(l') \} \ ,$$
$$< X^0(z)\bar{X}^1(\bar{z}) >=< X^1(z)\bar{X}^0(\bar{z}) >=$$
$$= -\frac{1}{4\pi} \sum_{n=0}^\infty \{ \operatorname{sh} 2[(v_1 - v_2)n - v_2] f_n(\tau) + \operatorname{sh} 2[(v_2 - v_1)n - v_1] f_n(l') \} \ .$$

with

$$f_n(x) = \ln(1 - q^{2n} e^{-4\pi x})$$

For the fermions one finds a similar structure

$$< \psi^0(z)\bar{\psi}^0(\bar{z}) >^s = < \psi^1(z)\bar{\psi}^1(\bar{z}) >^s =$$
$$= F_o^s - i \sum_{n=0}^{\infty} (\mp)^n \{ \operatorname{ch} 2[(v_1 - v_2)n - v_2] g_n^s(\tau) \pm \operatorname{ch} 2[(v_2 - v_1)n - v_1] g_n^s(l') \} \ ,$$
$$< \psi^0(z)\bar{\psi}^1(\bar{z}) >^s = < \psi^1(z)\bar{\psi}^0(\bar{z}) >^s =$$
$$= G_o^s + i \sum_{n=0}^{\infty} (\mp)^n \{ \operatorname{sh} 2[(v_1 - v_2)n - v_2] g_n^s(\tau) \pm \operatorname{sh} 2[(v_2 - v_1)n - v_1] g_n^s(l') \} \ ,$$

with

$$g_n^{NS\pm}(x) = \frac{q^n e^{-2\pi x}}{1 - q^{2n} e^{-4\pi x}} \ , \ \ g_n^{R\pm}(x) = \frac{q^{2n} e^{-4\pi x}}{1 - q^{2n} e^{-4\pi x}}$$

and the zero mode contributions

$$F_o^{NS\pm} = G_o^{NS\pm} = 0 \ ,$$
$$F_o^{R+} = -\frac{i}{2} \frac{\operatorname{ch}(v_1 + v_2)}{\operatorname{ch}(v_1 - v_2)} \ , \ \ F_o^{R-} = -\frac{i}{2} \frac{\operatorname{sh}(v_1 + v_2)}{\operatorname{sh}(v_1 - v_2)} \ ,$$
$$G_o^{R+} = -\frac{i}{2} \frac{\operatorname{sh}(v_1 + v_2)}{\operatorname{ch}(v_1 - v_2)} \ , \ \ G_o^{R-} = -\frac{i}{2} \frac{\operatorname{ch}(v_1 + v_2)}{\operatorname{sh}(v_1 - v_2)} \ .$$

As usual, world-sheet supersymmetry means (here for osc.) a relation between the odd fermions and the derivative of the bosons

$$< \partial X^\mu(z) \bar{X}^\nu(\bar{z}) > = \frac{1}{2} < \psi^\mu(z) \bar{\psi}^\nu(\bar{z}) >_{R-} \ . \tag{13}$$

There are also non vanishing equal-point correlators, and all the correlators can be actually expressed in terms of twisted ϑ-functions [5].

3.2 Axion

For the axion, $G_{ij} = 1/2\epsilon_{ijk}p^k/p$, and only the odd spin structure in the twisted sector of the Z_3 orbifold can contribute (only two fermionic zero modes in the 2-3 pair). However, after integrating by parts the two-derivative bosonic term appearing in the contraction $\mathcal{M}_s$, and using world-sheet supersymmetry (13), the final amplitude reduces to a total derivative and vanishes

$$\mathcal{A}_{ax} = 0 \ . \tag{14}$$

3.3 Dilaton

For the dilaton, $G_{ij} = \delta_{ij} - p^i p^j / p^2$ and only the even spin structures can contribute. Again, the two-derivative bosonic term in $\mathcal{M}_s$ is conveniently integrated by parts, and proceeding as in the next subsection for the graviton, one finds a total derivative in the large distance limit $l \to \infty$, yielding

$$\mathcal{A}_{dil} = 0 \ . \tag{15}$$

3.4 Graviton

For the graviton, $G_{ij} = h_{ij} = h_{ji}$, $p^i h_{ij} = h^i_i = 0$, with two independent components. In the large distance limit $l \to \infty$, the contraction are

$$\begin{aligned}
\mathcal{M}^{R+}_{grav} = &-\frac{1}{4}\left[h_{ij}k^i k^j - p\,\mathrm{tgh}\,v h_{i1}k^i\right] \\
&-V_2\gamma_2\left[p^{(2)}\left(h_{i1}k^i - \frac{p}{2}\,\mathrm{tgh}\,v h_{11}\right) + \frac{1}{4}(k^2-q^2)V_2\gamma_2 h_{11}\right]\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} \\
&+V_1\gamma_1\left[p^{(1)}\left(h_{i1}k^i - \frac{p}{2}\,\mathrm{tgh}\,v h_{11}\right) + \frac{1}{4}(k^2-q^2)V_1\gamma_1 h_{11}\right]\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}} \ , \\
\mathcal{M}^{NS\pm}_{grav} = &-\frac{1}{4}\left[h_{ij}k^i k^j \mp 4e^{-2\pi l}\left(p\,\mathrm{sh}\,2v h_{i1}k^i - p^2\,\mathrm{sh}^2 v h_{11}\right)\right] \\
&-V_2\gamma_2\left[p^{(2)}\left(h_{i1}k^i \mp 2e^{-2\pi l} p\,\mathrm{sh}\,v h_{11}\right) + \frac{1}{4}(k^2-q^2)V_2\gamma_2 h_{11}\right]\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} \\
&+V_1\gamma_1\left[p^{(1)}\left(h_{i1}k^i \mp 2e^{-2\pi l} p\,\mathrm{sh}\,v h_{11}\right) + \frac{1}{4}(k^2-q^2)V_1\gamma_1 h_{11}\right]\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}} \ .
\end{aligned}$$

Also, the bosonic exponential reduces to

$$< e^{ip\cdot X} >= \left(1-e^{-4\pi\tau}\right)^{-\frac{p^{(2)2}}{2\pi}} \left(1-e^{-4\pi l'}\right)^{-\frac{p^{(1)2}}{2\pi}} \ , \tag{16}$$

where $p^{(1,2)} = p\gamma_{1,2}(1 - V_{1,2}\cos\theta) = p(\,\mathrm{ch}\,v_{1,2} - \,\mathrm{sh}\,v_{1,2}\cos\theta)$.

Integrating by parts in the final amplitude, one finds the following rules for the τ and l' poles in the contraction:

$$\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} \doteq -\frac{1}{4}\frac{q^2}{p^{(2)2}} \ , \quad \frac{e^{-4\pi l'}}{1-e^{-4\pi l'}} \doteq -\frac{1}{4}\frac{k^2}{p^{(1)2}} \ . \tag{17}$$

One can then use these equivalence relations to write $\mathcal{M}^s_{grav}$ in a τ, l'-independent form, parametrizing it with three independent functions of the momenta

$$\mathcal{M}^s_{grav} = B^s(p,k,q) + q^2 C^s_1(p,k,q) + k^2 C^s_2(p,k,q) \ .$$

The kinematical integrals over the two proper times τ, l' can then be easily evaluated, finding the usual dual structure with a double serie of poles

$$I_1 = \int_0^\infty d\tau e^{-\frac{q^2}{2}\tau}\left(1-e^{-4\pi\tau}\right)^{-\frac{p^{(2)2}}{2\pi}} = -\frac{1}{4\pi}\frac{\Gamma[\frac{q^2}{8\pi}]\Gamma[-\frac{p^{(2)2}}{2\pi}+1]}{\Gamma[\frac{q^2}{8\pi}-\frac{p^{(2)2}}{2\pi}+1]} \underset{p\to 0}{\longrightarrow} -\frac{2}{q^2} ,$$
$$I_2 = \int_0^\infty dl' e^{-\frac{k^2}{2}l'}\left(1-e^{-4\pi l'}\right)^{-\frac{p^{(1)2}}{2\pi}} = -\frac{1}{4\pi}\frac{\Gamma[\frac{k^2}{8\pi}]\Gamma[-\frac{p^{(1)2}}{2\pi}+1]}{\Gamma[\frac{k^2}{8\pi}-\frac{p^{(1)2}}{2\pi}+1]} \underset{p\to 0}{\longrightarrow} -\frac{2}{k^2} .$$

The last limit is required by the eikonal approximation ($p \ll M = 1$) and selects the massless part of the states emitted by the branes.

Finally, the amplitude assumes a simple field theory form

$$\mathcal{A}_{grav} = \frac{4}{\operatorname{sh} v}\int \frac{d^2\vec{k}_T}{(2\pi)^2}e^{i\vec{k}\cdot\vec{b}}\left\{B^s\frac{1}{q^2k^2}+C_1^s\frac{1}{k^2}+C_2^s\frac{1}{q^2}\right\} . \tag{18}$$

The B^s factor corresponds to an annihilation process occurring far away from both branes, with a double pole, whereas the C_1^s and C_2^s factors correspond to absorption-emission bremsstrahlung-like processes occurring on the first and the second brane respectively, with a simple pole.

3.5 Field theory interpretation

It is interesting to compare the string theory results to a field theory computation in the limit of large impact parameters $\vec{b}$.

For the axion and the dilaton, there is no coupling in supergravity allowing the emission process, and therefore the vanishing of the string amplitude is understood. For the annihilation term of the graviton, there are three possible diagrams in supergravity, involving the exchange of vectors, scalarss and gravitons. Their contributions in the eikonal approximation are

$$\begin{aligned} &B_\phi^{NS} \sim -a^2 h_{ij}k^ik^j , \\ &B_{V_\mu}^R \sim e^2\left[\operatorname{ch} v h_{ij}k^ik^j - p \operatorname{sh} v h_{i1}k^i\right] , \\ &B_{g_{\mu\nu}}^{NS} \sim -M^2\left[\operatorname{ch} 2v h_{ij}k^ik^j - 2p\operatorname{sh} 2v h_{i1}k^i + 2p^2\operatorname{sh}^2 v h_{11}\right] . \end{aligned} \tag{19}$$

The annihilation part of the string amplitude in the various compactification schemes is instead the following:

0-brane: untwisted sector & 3-brane on $T_2 \otimes T_4/Z_2, T_6$

One finds an $e^{2\pi l}$ enhancement from $Z^{NS+} + Z^{NS-}$ (with the "wrong sign"), and in the final result we recognize a leading order cancellation between the

RR vector and both the NSNS scalar and graviton exchange:

$$B_{grav} = B^R_{V_\mu} + B^{NS}_\phi + B^{NS}_{g_{\mu\nu}} \sim V^4 h_{ij} k^i k^j + V^3 p h_{i1} k^i + V^2 p^2 h_{11} \ . \qquad (20)$$

0-brane: twisted sector
In this case, there is no enhancement, and the leading order cancellation occurs between the RR vector and NSNS scalar exchange:

$$B_{grav} = B^R_{V_\mu} + B^{NS}_\phi \sim V^2 h_{ij} k^i k^j + V p h_{i1} k^i + V^2 p^2 h_{11} \ . \qquad (21)$$

3-brane on T_6/Z_3
In this case there is again an $e^{2\pi l}$ enhancement from $Z^{NS+} + Z^{NS-}$, and the cancellation occurs between the RR vector and the NSNS graviton exchange:

$$B_{grav} = B^R_{V_\mu} + B^{NS}_{g_{\mu\nu}} \sim V^4 h_{ij} k^i k^j + V^3 p h_{i1} k^i + V^2 p^2 h_{11} \ . \qquad (22)$$

The patterns of cancellation in the various cases confirm the interpretation in terms of supermultiplets coming from the computation of the potential.

3.6 Radiated energy

The average energy radiated when two D-branes pass each other at impact parameter $\vec{b}$ is, in the colinear case $\theta = 0$ ($n = 2, 4$ depending on SUSY),

$$< p > \sim g_s^2 l_s^2 \frac{V^{1+2n}}{b^3} \ . \qquad (23)$$

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