

Brane to brane gravity mediation of supersymmetry breaking

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Abstract

We extend the results of Mirabelli and Peskin to supergravity. We study the compactification on S^1/\mathbb{Z}_2 of Zucker's off-shell formulation of 5D supergravity and its coupling to matter at the fixed points. We clarify some issues related to the off-shell description of supersymmetry breaking à la Scherk–Schwarz (here employed only as a tool), discussing how to deal with singular gravitino wave functions.

We then consider ‘visible’ and ‘hidden’ chiral superfields localized at the two different fixed points and communicating only through 5D supergravity. We compute the one-loop corrections that mix the two sectors and the radion superfield. Locality in 5D ensures the calculability of these effects, which transmit supersymmetry breaking from the hidden to the visible sector. In the minimal set-up visible-sector scalars get a universal squared mass $m_0^2 < 0$. In general (e.g. in the presence of a sizeable gravitational kinetic term localized on the hidden brane) the radion-mediated contribution to m_0^2 can be positive and dominant. Although we did not build a complete satisfactory model, brane-to-brane effects can cure the tachyonic sleptons predicted by anomaly mediation by adding a positive m_0^2 , which is universal up to subleading flavour-breaking corrections.

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1 Introduction

In spite of the competition from other ingenious proposals, low-energy supersymmetry remains the simplest and most realistic possibility for new physics at the electroweak scale. Among the reasons for that are its spectacular agreement with the expectations of Grand Unified Theories and its almost effortless satisfaction of the constraints posed by electroweak precision data. Nonetheless, at the theoretical level, there are still several unsatisfactory aspects, all directly related to the problem of supersymmetry breaking. Maybe the acutest problem is that supersymmetry should help with the cosmological constant problem, but it does not. Supersymmetry controls quantum corrections to the vacuum energy. However, supersymmetry must be broken at or above the electroweak scale and the generic value of the cosmological constant is then $\gtrsim (100 \text{ GeV})^4$, an excess of at least fifty orders of magnitude. In phenomenological applications of supersymmetry, the cosmological constant is tuned to be small (at least it can be done!), with the hope that some other mechanism will explain that tuning. Another problem concerns the flavour structure of the squark and slepton mass matrices. This structure should be very specific in order to satisfy the experimental constraints on Flavour-Changing Neutral Currents (FCNC). This requires theoretical control on the mechanism that generates the soft terms. Finally, the Higgs sector and electroweak symmetry breaking are crucially controlled by the μ -parameter, which does not itself break supersymmetry. The special status of μ compared to the other mass terms, which do break supersymmetry, is often a serious obstacle to the construction of simple and realistic theories for the soft terms. Indeed, after the completion of the LEP/SLC program, lacking the discovery of any superparticle, there is yet another source of embarrassment for supersymmetry: why is supersymmetry hiding in experiments at the weak scale if its role is to explain the weak scale itself? Quantitatively: with the present lower bounds on the sparticle masses, the reproduction of the measured Z -mass requires a fine-tuning of at least $1/20$ among the parameters of all popular models. Basically, more than 95% of their parameter space is already ruled out. If we want to stick to supersymmetry, is there a message in the need for this tuning? Is it possible that this tuning is not accidental, and that the underlying model naturally selects sparticle masses somewhat heavier than expected?

All in all, the above problems are probably telling us that we have not yet a fully realistic model for the soft terms. The hope and the assumption in the quest for such a model is usually that, because of its hugely different nature, a separate solution will be found the the first problem, the cosmological constant problem, that will not affect physics at the weak scale. In this paper we will follow this standard path and concentrate of the flavour problem.

In the Standard Model (SM) all flavour violation arises in the fermion mass matrices themselves. FCNC are then naturally suppressed, in agreement with experimental data, by powers of the fermion masses and mixing angles. This is the Glashow–Iliopoulos–Maiani (GIM) mechanism. In the Minimal Supersymmetric Standard Model (MSSM), a generic sfermion mass matrix represents a new source of flavour mixing, not aligned with the fermion mass matrices. The GIM mechanism generically does not work in the MSSM, and FCNC bounds are not satisfied. A model for the soft terms enforcing the GIM

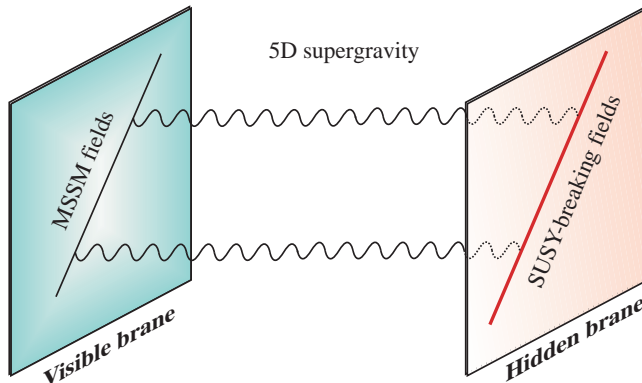


Figure 1: *One-loop supergravity diagrams inducing an effective interaction between visible and hidden sector.*

mechanism would tackle this difficulty. Gauge mediated models [1] (see [2] for a review) are such an example. In that case soft terms are mediated by gauge interactions at a scale M much below the flavour scale Λ_F . The resulting soft terms are flavour symmetric up to small effects due to the SM Yukawa matrices themselves. Extra dangerous flavour violating effects are further suppressed by powers of M/Λ_F . The resulting FCNC are then analogous to those of the SM. Gauge-mediated models are very attractive in this respect, but they require extra inelegant complications to solve the μ -problem. The so-called gravity-mediated models [3], on the other hand, fare better on the μ -problem (thanks to the possibility of the Giudice–Masiero mechanism [4]) but are in trouble with flavour. At first this seems surprising, since gravity is as flavour universal as the SM gauge interactions. However, the point is that gravity is universal, or more precisely it respects GIM, only in the IR. On the other hand, gravity-mediated models effectively represent the generation of soft masses by UV phenomena in the fundamental theory of quantum gravity. Now, this unknown fundamental theory has to explain why the top quark is so much heavier than the up quark and everything else: it should also be the theory of flavour. Then it is not obvious why it should generate soft terms respecting the GIM mechanism. The presence of extra-dimensions can however change this state of affairs. The key is a new scale associated to the radius of compactification R . The prototypical example is provided by the ‘sequestered sector’ scenario suggested by Randall and Sundrum [5], and inspired by string [6] and M -theory orbifolds [7] (although it seems difficult to realize this scenario in string models [9]). The model involves one extra dimension compactified on the orbifold S_1/\mathbb{Z}_2 . The MSSM lives at one boundary, say $x^5 = 0$, while the supersymmetry-breaking sector lives at the other boundary, a distance πR away. It is assumed that R is parametrically larger than the 5D Planck length $1/M_5$. Locality in 5D ensures the absence of direct tree-level couplings between the two sectors [8]. The direct uncalculable couplings were the origin of flavour violation in ordinary 4D models. At the quantum level the two sectors couple through virtual graviton exchange, see Fig. 1. These loops are saturated at virtuality $\sim 1/\pi R$: as

long as $R \gg 1/M_5$, they are dominated by the IR flavour-universal regime of gravity.² Indicating by $F_{\Phi_\pi} \sim M_{\text{susy}}^2$ the Vacuum Expectation Value (VEV) of the auxiliary fields in the hidden sector, at one-loop the universal scalar mass m_0 is of order

$$m_0^2 \sim \frac{1}{16\pi^2} \frac{|F_{\Phi_\pi}^2|}{M_5^6 (\pi R)^4}. \quad (1.1)$$

This effect was never computed so far. The reason is that for $RM_5 \rightarrow \infty$ the leading contribution to soft terms comes from another quantum effect, where gravity enters only at the classical level: the so-called anomaly-mediated supersymmetry breaking (AMSB) [5, 10]. The auxiliary field F_{S_0} acting as a source in AMSB is the one in the gravitational supermultiplet 4D Poincaré supergravity. This field couples to the MSSM only via the superconformal anomaly. Being an anomaly, this effect is completely saturated in the IR. Again, only the universal aspects of gravity play a rôle, and the anomaly-mediated sfermion masses beautifully enforce the GIM mechanism.

Unfortunately, the sleptons turn out to be tachyonic, as a sharp consequence of $SU(2)_L$ not being asymptotically free in the MSSM. Moreover the μ -problem affects AMSB very much as GMSB. Various proposals have been made to fix these problems. Indeed if one assumes that some unspecified flavour-universal contribution $m_0^2 > 0$ lifts the sleptons, then the low-energy phenomenology is quite peculiar [11]. The main purpose of this paper is to study whether and how the brane-to-brane-mediated term in eq. (1.1) can realize this situation.

The anomaly-mediated gaugino masses $m_{1/2}$ and scalar masses m_s depend on the auxiliary scalar F_{S_0} of supergravity and scale roughly like

$$m_{1/2} \sim m_s \sim \frac{g^2}{16\pi^2} |F_{S_0}|. \quad (1.2)$$

In the minimal situation, $F_{S_0} \sim F_{\Phi_\pi}/M_4$, where the 4D reduced Planck mass M_4 is defined as $M_4^2 = M_5^3 \pi R$. Although AMSB scalar masses squared arise at two loops, they dominate eq. (1.1) for $(M_5 \pi R)^3 \gtrsim 16\pi^2 = (M_5 \pi R_{\text{cr}})^3$ (naïve dimensional analysis [12] estimates that quantum gravity effects become important around or below the energy $\Lambda_5 \sim 4\pi M_5$). If the radius were stabilized at the critical value R_{cr} , and if the brane-to-brane contribution were positive, the tachyon problem could be overcome while preserving a certain control on flavour universality. Notice indeed that R_{cr} is still parametrically larger than the Planck length. Notice also that gaugino masses are not affected by the brane-to-brane loops. Therefore, if $R < R_{\text{cr}}$ gauginos are parametrically lighter than sfermions, which requires extra fine tuning in electroweak breaking. In ref. [13] a simple mechanism of radius stabilization which can plausibly give $R \sim R_{\text{cr}}$ was pointed out.

As we said, the purpose of the present work is to calculate the brane-to-brane term m_0^2 . In fact we do more and calculate the full one-loop correction to the Kähler potential, or better its IR-saturated part. Along the way, we also study in some detail the interactions of boundary fields with bulk supergravity. The paper is organized as follows. In the next

²For example, extra particles with mass $M \sim M_5$ propagating in the extra dimension might be present in a fundamental theory of gravity, giving extra contributions suppressed by $\exp(-MR)$.

section we outline the strategy that we use to perform our computation. In section 3 we discuss the Lagrangian for off-shell 5D supergravity and its coupling to the boundary. In section 4 we show in a sample computation that supergravity cancellations are correctly reproduced. Section 5 is a detailed discussion of Scherk–Schwarz supersymmetry breaking, which we need only as a tool to compute the one-loop correction to the Kähler potential; in our phenomenological applications, supersymmetry is not broken just by the Scherk–Schwarz mechanism. In section 6 we present our computation. In section 7 we discuss our results and their consequences. Finally, section 8 is devoted to conclusions.

2 Outline

In this section, we will describe the general context in which we will work and outline the main steps of the computation that we will perform.

2.1 The model

We consider a 5D supergravity model compactified on S_1/\mathbb{Z}_2 , following closely the study by Luty and Sundrum [13]. We parametrize S_1/\mathbb{Z}_2 by $x^5 \simeq x^5 + 2\pi$ and $x^5 \simeq -x^5$. We assume that all the fields of the MSSM live at $x^5 = 0$, while at $x^5 = \pi$ there is a field theory breaking supersymmetry in the flat limit, the hidden sector. For the purpose of our calculation it is enough to consider a toy MSSM consisting of just one chiral superfield Φ_0 (containing a sfermion ϕ_0 , a Weyl fermion χ_0 and the auxiliary field F_{Φ_0}). The result for the MSSM will just be a straightforward generalization. Similarly we assume that the hidden sector is effectively described by an O’Raifeartaigh model involving just one chiral superfield Φ_π . We will assume that all interactions in the hidden sector are characterized by just one scale Λ , corresponding to its interpretation as the low energy description of a dynamical supersymmetry-breaking model. Since the radius R is also a massless field in the lowest order description of the scenario, we will have to include it in the effective 4D description and to determine the vacuum dynamics. At low energy the effective tree level (classical) Kähler function $K_{\text{cl}} = -\frac{3}{2} \ln[-\frac{2}{3}\Omega_{\text{cl}}]$ is then specified by

$$\Omega_{\text{cl}} = -\frac{3}{2}(T + T^\dagger)M_5^3 + \Omega_0(\Phi_0, \Phi_0^\dagger) + \Omega_\pi(\Phi_\pi, \Phi_\pi^\dagger), \quad (2.1)$$

where T is the radion superfield, and Ω_0 and Ω_π are the contributions to the gravitational kinetic function, coming respectively from the 0 and π fixed points. The gravitational action is proportional to the D -term $[\Omega_{\text{cl}}S_0S_0^\dagger]_D$, where S_0 is the chiral compensator³. By the above additive form of Ω_{cl} , the VEVs of the auxiliary fields F_T and F_{Φ_π} do not generate soft terms in the visible sector. (Notice, however, that in the Einstein frame the two sectors have mixed kinetic terms.) At this stage the visible sector soft terms are generated through anomaly mediation and are proportional to $F_{S_0} \sim m_{3/2}$. In this paper we will calculate the one-loop correction $\Delta\Omega$ to Ω_{cl} , which introduces direct coupling between visible, hidden and radion sectors. The soft terms generated by $\Delta\Omega$ depend on

³We are using here the superconformal formulation of the effective 4D theory [14].

F_{Φ_π} , F_T and T . The relations among these parameters are strongly dependent on the mechanism that stabilizes T .

2.2 The computation

We now illustrate our strategy to compute the one-loop correction $\Delta\Omega$. The first remark is that, like Ω_{cl} , $\Delta\Omega$ must depend on T and T^\dagger only through the combination $T + T^\dagger$ whose lowest component is the length πR of the internal dimension. This is because the lowest component of $T - T^\dagger$ is the internal component of the graviphoton A_5 , which couples only derivatively in the tree-level Lagrangian. A dependence of $\Delta\Omega$ on $T - T^\dagger$ would lead to non-derivative terms in A_5 , which cannot arise in perturbation theory⁴. So $\Delta\Omega = \Delta\Omega(T + T^\dagger, \Phi_{0,\pi}, \Phi_{0,\pi}^\dagger)$.

We calculate $\Delta\Omega$ by a little trick: we reconstruct it by computing the one-loop effective scalar potential ΔV induced in a background with $F_T \neq 0$ and with all other auxiliary fields vanishing. This scenario is consistently realized in our model if a constant boundary superpotential $P = c$ is chosen. At tree level this is the simple supersymmetry-breaking no-scale model [15] (see [16] for a review): $F_T \sim c$ and $F_{S_0} = 0$, where the second condition ensures exact cancellation of the effective cosmological constant. In our 5D model this way of breaking supersymmetry is completely equivalent to the Scherk–Schwarz mechanism [17]. In section 5, to clarify our procedure, we will have to take a detour into explaining in detail the relation to the Scherk–Schwarz mechanism. Now, at zero momentum we have

$$\Delta V = - \left[\Delta\Omega S_0 S_0^\dagger \right]_D = -|F_T|^2 \partial_T \partial_{T^\dagger} \Delta\Omega(T + T^*, \phi_{0,\pi}, \phi_{0,\pi}^*), \quad (2.2)$$

where ΔV is the quantity we calculate, with F_T as an input. Equation (2.2) is a simple differential equation whose solution gives $\Delta\Omega$ up to two integration ‘constants’ H_0 and H_1 :

$$\Delta\Omega = \Delta\hat{\Omega}(T + T^*, \phi_{0,\pi}, \phi_{0,\pi}^*) + H_0(\phi_{0,\pi}, \phi_{0,\pi}^*) + (T + T^*)H_1(\phi_{0,\pi}, \phi_{0,\pi}^*). \quad (2.3)$$

The quantity $\Delta\hat{\Omega}$ is entirely determined and explicitly anticipated below. On the other hand, the form of the unknown $H_{0,1}$ is strongly constrained by 5D locality and the limit $R \rightarrow \infty$. Since Φ_0 and Φ_π are located at the two different boundaries and cannot talk to each other in the limit $R \rightarrow \infty$, H_0 must have the form:

$$H_0 = \Delta\Omega_0(\Phi_0, \Phi_0^\dagger) + \Delta\Omega_\pi(\Phi_\pi, \Phi_\pi^\dagger). \quad (2.4)$$

Then it is clear that H_0 is just associated to the local, UV-divergent, renormalization of each boundary kinetic function, and does not contribute to brane-to-brane mediation of supersymmetry breaking. H_1 is an ‘extensive’ contribution, growing with the volume, and must be associated to renormalization of local bulk operators. Therefore H_1 cannot depend on the boundary fields: it is a constant associated to the uncalculable renormalization of

⁴At the non-perturbative level, these terms can be generated, via for instance instanton effects, as in eq. (7.8).

the 5D Planck mass. So the only relevant quantity is the calculable one, $\Delta\hat{\Omega}$.⁵

The computation of ΔV requires in principle the knowledge of all the interactions between the boundary matter fields and the bulk supergravity fields. These can be obtained from the ordinary 4D supergravity tensor calculus, once the boundary values of bulk fields have been appropriately combined into 4D supermultiplets. We will do this in some detail in section 3, by using the off-shell description of 5D supergravity developed in [18], thereby extending the results of [19] from global to local supersymmetry. Our results do not fully agree with previous attempts (for instance [20]), and we therefore verify them in section 4, by checking that the basic cancellations demanded by supersymmetry are reproduced. Computing ΔV turns out to be an easy task. Since it vanishes in the supersymmetric limit $F_T = 0$, and since only the mass spectrum of gravitinos is affected by an $F_T \neq 0$, ΔV is simply given by the gravitino loop contribution, minus its value for $F_T = 0$ (the same remark was used in [20]). Furthermore, as a consequence of being in 5D and working at zero momentum, the whole contribution comes from diagrams involving only the scalar–scalar–gravitino–gravitino coupling. Such couplings are the same as those occurring in 4D supergravity.

For our phenomenological applications it is enough to consider the following form of the boundary kinetic functions

$$\Omega_0 = -3L_0M_5^3 + \Phi_0\Phi_0^\dagger, \quad (2.5)$$

$$\Omega_\pi = -3L_\pi M_5^3 + \Phi_\pi\Phi_\pi^\dagger. \quad (2.6)$$

The constants $L_{0,\pi}$ represent localized kinetic terms for the bulk supergravity fields, such as those considered for pure gravity in ref. [21]. Negative values of $\Omega_{0,\pi}$ correspond to positive kinetic terms. For Ω_0 , the above form is motivated by the fact that for phenomenological applications we can work close to the origin in field space. We do not consider a linear term in Φ_0 since there are no gauge singlets in the MSSM. In the hidden sector, we can always choose Φ_π such that the VEV of ϕ_π vanishes. Then a simple analysis shows that terms of cubic and higher order do not contribute to soft terms in the one-loop approximation. In general, there will however be a linear term in Φ_π , which corresponds to $\Phi_\pi \rightarrow \Phi_\pi + \text{const}$ in eq. (2.6).

Let us conclude this section by anticipating our main result. We find that the calculable one-loop correction $\Delta\hat{\Omega}$ to the Kähler potential is given by

$$\Delta\hat{\Omega} = -\frac{9}{\pi^2}M_5^2 \int_0^\infty dx x \ln \left[1 - \frac{1+x\Omega_0M_5^{-2}}{1-x\Omega_0M_5^{-2}} \frac{1+x\Omega_\pi M_5^{-2}}{1-x\Omega_\pi M_5^{-2}} e^{-6x(T+T^\dagger)M_5} \right]. \quad (2.7)$$

⁵This discussion, although correct, needs an extra remark to be made fully rigorous. This is because the one-dimensional Green function grows linearly with the separation, and contributions that are linear in T and mix the fields at the two boundaries are in principle possible. Indeed such an effect arises at tree level from the exchange of one graviton. However, it corresponds to a 4-derivative interaction in the effective theory [8], and so it does not concern us. Now the basic point is that at the quantum level we are considering 1-PI diagrams, where at least two gravitons are exchanged between each boundary: these diagrams have at least one further suppression $1/(M_5T)^3$, so that their contribution vanishes at least as $1/T^2$ for $T \rightarrow \infty$. In fact for two derivative operators (Kähler) there is an extra $1/T^2$ suppression by simple dimensional analysis, see eq. (2.8).

We believe that this result is valid for general $\Omega_{0,\pi}$, and not just those in eqs. (2.5) and (2.6), but to prove this rigorously would require some more precise discussion into which we will not enter. In the standard situation $L_{0,\pi} = 0$ (or negligibly small), expanding at the lowest order in Φ_0 and Φ_π we find

$$\Delta\hat{\Omega} = \frac{\zeta(3)}{4\pi^2(T+T^\dagger)^2} + \frac{\zeta(3)}{6\pi^2} \frac{\Phi_0\Phi_0^\dagger + \Phi_\pi\Phi_\pi^\dagger}{(T+T^\dagger)^3 M_5^3} + \frac{\zeta(3)}{6\pi^2} \frac{\Phi_0\Phi_0^\dagger\Phi_\pi\Phi_\pi^\dagger}{(T+T^\dagger)^4 M_5^6} + \dots \quad (2.8)$$

The first term in (2.8) is the well known Casimir energy correction. The third term gives *brane-to-brane mediation* of supersymmetry breaking. The second term induces *radion-mediated* supersymmetry breaking, if the radion field T also gets a non-zero F -term (F_T has dimension zero). It was previously computed in [20], and we find the same result. The order of magnitude of the coefficients agrees with a naïve estimate performed in the effective 4D theory, where these terms are UV-divergent, with a cut-off $\Lambda_{UV} \sim 1/\pi R$.

3 Full five-dimensional theory

In this section we consider 5D supergravity compactified on S^1/\mathbb{Z}_2 , with 4D chiral and vector multiplets localized at the two fixed points $x^5 = 0$ and $x^5 = \pi$, which we will refer to as respectively the visible and the hidden branes. Our aim is to write the couplings between bulk and brane fields. This can be done by working with an off-shell formulation of supergravity, as was done in [19] for the simpler case of rigid supersymmetry. Our discussion is based on the work of Zucker [18], in which both the 5D off-shell Lagrangian and the projected multiplets at the boundary were derived.

A few words on notation are in order. We set $M_5 = 1$. We use Latin capitals $A, B, \dots = \dot{1}, \dots, \dot{5}$ for the flat 5D space-time indices and Latin capitals from the middle of the alphabet $M, N, \dots = 1, \dots, 5$ for the curved 5D indices. Similarly we use $\alpha, \beta, \dots = \dot{1}, \dots, \dot{4}$ for the flat 4D indices and $\mu, \nu, \dots = 1, \dots, 4$ for the 4D curved ones. The 5D fermions are symplectic Majorana spinors, and carry $SU(2)_R$ indices denoted by i, j, \dots ; they satisfy the condition $\bar{\Psi}_i = \varepsilon_{ij} \Psi^{jT} C$, where C is the charge conjugation matrix, and can thus be decomposed in terms of two Weyl spinors χ^i as follows: $\Psi^i = (\chi^i, \varepsilon^{ij} \bar{\chi}_j)^T$. As usual, the Weyl spinors χ^i can be equivalently described in terms of Majorana spinors $\psi^i = (\chi^i, \bar{\chi}^i)^T$. Occasionally we shall also use the $SU(2)_R$ doublet of Weyl spinors $\chi = (\chi^1, \chi^2)^T$. Our conventions are such that $\gamma^{\dot{5}} = \text{diag}(-i, -i, i, i)$ and $\varepsilon^{12} = 1$.

Consider first the bulk theory on S^1 . The on-shell version contains the fünfbein e_M^A , the gravitino Ψ_M^i and the graviphoton A_M , and has a global $SU(2)_R$ symmetry under which the gravitino is a doublet [22]. Its minimal off-shell extension has been described in [18]. It involves a minimal supergravity multiplet $(e_M^A, \Psi_M, A_M; \vec{t}, v_{AB}, \vec{V}_M, \lambda, C)$ containing the physical degrees of freedom and a set of auxiliary fields, where we indicate by an upper arrow the $SU(2)_R$ triplets. In particular \vec{V}_M gauges the $SU(2)_R$ symmetry. In addition, there is a compensator multiplet containing only auxiliary fields. The most convenient choice is a tensor multiplet $(\vec{Y}, B_{MNP}, \rho, N)$, which is related to a linear multiplet in which the constraint is solved by Poincaré duality with a vector component defined as

$$W^M = \frac{1}{12} \varepsilon^{MNPQR} \partial_N B_{PQR} + \frac{1}{4} \bar{\Psi}_P \vec{\tau} \gamma^{PMQ} \Psi_Q \vec{Y} - \frac{i}{2} \bar{\rho} \gamma^{MN} \Psi_N. \quad (3.1)$$

The theory on S^1/\mathbb{Z}_2 is defined by assigning each field a \mathbb{Z}_2 parity such that the Lagrangian is an even density. The orbifold projection then globally breaks $\mathcal{N} = 2$ down to $\mathcal{N} = 1$ and $SU(2)_R$ down to $U(1)$. There is a two-parameter family of possible choices, determined by which $U(1)$ is preserved. A standard choice is to preserve the T_3 generator, which corresponds to the following \mathbb{Z}_2 transformation properties for the gravitinos: $\Psi_M(-x^5) = i\tau_3\gamma^{\dot{5}}\Psi_M(x^5)$. The full parity assignments are then listed in Table 1.

Field	e_M^A	Ψ_M	A_M	\vec{t}	v_{AB}	\vec{V}_M	λ	C	\vec{Y}	B_{MNP}	ρ	N
+	$e_\mu^a, e_5^{\dot{5}}$	ψ_μ^1, ψ_5^2	A_5	$t^{1,2}$	$v_{\alpha\dot{5}}$	$V_\mu^3, V_5^{1,2}$	λ^1	C	$Y^{1,2}$	$B_{\mu\nu\rho}$	ρ^1	N
-	$e_\mu^{\dot{5}}, e_5^a$	ψ_μ^2, ψ_5^1	A_μ	t^3	$v_{\alpha\beta}$	$V_\mu^{1,2}, V_5^3$	λ^2		Y^3	$B_{\mu\nu 5}$	ρ^2	

Table 1: *Parity assignments for the bulk multiplets.*

At the fixed points, the even components of the 5D multiplets decompose into multiplets of the supersymmetry preserved by the orbifold projection. The even components associated to the 4D vielbein e_μ^α fill up a so-called intermediate multiplet [23] given by $I = (e_\mu^\alpha, \psi_\mu^1; a_\mu, b_\alpha, t^1, t^2, \lambda^1, S)$ with the identifications

$$S = C - \frac{1}{2}e_5^{\dot{5}}(\partial_5 t^3 - \bar{\lambda}^1 \psi_5^2 + V_5^1 t^2 - V_5^2 t^1), \quad (3.2)$$

$$a_\mu = -\frac{1}{2}\left(V_\mu^3 - \frac{2}{\sqrt{3}}F_{\mu 5} e_5^{\dot{5}} + 4e_\mu^a v_{a\dot{5}}\right), \quad (3.3)$$

$$b_a = v_{a\dot{5}}. \quad (3.4)$$

The vector a_μ gauges the R-symmetry [23], and chiral multiplets are characterized by their chiral charge (or weight). The set of remaining even components forms a chiral multiplet $E_5^{\dot{5}} = (e_5^{\dot{5}}, \frac{2}{\sqrt{3}}A_5, \psi_5, V_5^1 - 4t^2 e_5^{\dot{5}}, V_5^2 + 4t^1 e_5^{\dot{5}})$ of weight $w = 0$. However $E_5^{\dot{5}}$ also transforms under 5D local translations and under the projected supersymmetry ϵ_2 . For instance, $\delta\psi_5^2 = \partial_5\epsilon_2 + \dots$, which is not zero, even at the boundary. Because of this, $E_5^{\dot{5}}$ cannot be used to write boundary Lagrangians. However, the zero mode of $E_5^{\dot{5}}$, which is the only object that cannot be eliminated by choosing a suitable gauge for 5D local supersymmetry and diffeomorphisms, remains as a chiral multiplet, the radion, of 4D supersymmetry. Finally, all the even components of the compensator multiplet arrange into a chiral multiplet $S_0 = (Y^2, Y^1, \rho; \text{Re } F_{S_0}, \text{Im } F_{S_0})$ of weight $w_0 = 2$ with:

$$\text{Re } F_{S_0} = -2N + \hat{D}_5 Y^3, \quad (3.5)$$

$$\text{Im } F_{S_0} = 2W^{\dot{5}} + 12(Y^2 t^1 - Y^1 t^2). \quad (3.6)$$

The fields localized at the fixed points can be either chiral multiplets, made of a complex scalar ϕ , a chiral fermion χ , and auxiliary fields: $\Phi = (\text{Re } \phi, \text{Im } \phi, \chi; \text{Re } F_\Phi, \text{Im } F_\Phi)$, or vector multiplets, consisting of a vector boson B_μ , a Majorana fermion ψ , and an auxiliary field (in the Wess–Zumino gauge): $V = (B_\mu, \psi, D)$.

The Lagrangian of the complete theory describing interactions between bulk and brane multiplets has the general form

$$\mathcal{L} = \mathcal{L}_5 + \delta(x^5)\mathcal{L}_{4,0} + \delta(x^5 - \pi)\mathcal{L}_{4,\pi}, \quad (3.7)$$

where \mathcal{L}_5 describes the dynamics of the minimal and compensator multiplets, whereas $\mathcal{L}_{4,0}$ and $\mathcal{L}_{4,\pi}$ describe the dynamics of the chiral and vector multiplets of the visible and hidden sectors and their interactions with the minimal and compensator multiplets.

3.1 Bulk Lagrangian

The bulk Lagrangian has been derived in [18]. It is given by the sum $\mathcal{L}_5 = \mathcal{L}_{\text{min}} + \mathcal{L}_{\text{tens}}$ of the Lagrangians for the gravity and compensator multiplets:

$$\begin{aligned} \mathcal{L}_{\text{min}} = & \left[-32t^2 - \frac{1}{\sqrt{3}}F_{AB}v^{AB} + \bar{\Psi}_M\vec{\tau}\gamma^{MN}\Psi_N\vec{t} - \frac{1}{6\sqrt{3}}\varepsilon^{MNPQR}A_MF_{NP}F_{QR} \right. \\ & \left. + \frac{i}{8\sqrt{3}}\varepsilon^{MNPQR}\bar{\Psi}_M\gamma_N\Psi_P F_{QR} \right] + \left[-4C - 2i\bar{\lambda}\gamma^M\Psi_M \right], \quad (3.8) \\ \mathcal{L}_{\text{tens}} = & \left[-\frac{1}{4}Y\mathcal{R}(\hat{\omega}) - \frac{i}{2}Y\bar{\Psi}_P\gamma^{PMN}\mathcal{D}_M\Psi_N - \frac{1}{6}Y\hat{F}_{MN}\hat{F}^{MN} - \frac{1}{4}Y^{-1}\mathcal{D}_M\vec{Y}\mathcal{D}^M\vec{Y} \right. \\ & + Yv_{AB}v^{AB} + 20Yt^2 + Y^{-1}W_AW^A - Y^{-1}(N + 6t\vec{Y})^2 - Y\bar{\Psi}_M\vec{\tau}\gamma^{MN}\Psi_N\vec{t} \\ & - \frac{i}{2}Y\bar{\Psi}_A\Psi_Bv^{AB} - \frac{i}{4\sqrt{3}}Y\bar{\Psi}_M\gamma^{MNPQ}\Psi_N\hat{F}_{PQ} - \frac{1}{24}Y^{-1}\varepsilon^{MNPQR}\vec{Y}\vec{G}_{MN}B_{PQR} \\ & \left. + \frac{1}{24}Y^{-3}\varepsilon^{MNPQR}\vec{Y}(\mathcal{D}_M\vec{Y} \times \mathcal{D}_N\vec{Y})B_{PQR} - \frac{1}{4}Y^{-1}\bar{\Psi}_A\vec{\tau}\gamma^{ABC}\Psi_B(\vec{Y} \times \mathcal{D}_C\vec{Y}) \right] \\ & + \left[\text{terms involving } \rho \text{ but not } C \text{ or } \lambda \right] + \left[4YC + 2iY\bar{\lambda}\gamma^A\Psi_A - 4\bar{\lambda}\vec{\tau}\rho\vec{Y} \right]. \quad (3.9) \end{aligned}$$

The quantities F_{MN} and \vec{G}_{MN} are the field strengths of A_M and \vec{V}_M respectively, and $\hat{F}_{MN} = F_{MN} + i\frac{\sqrt{3}}{2}\bar{\Psi}_M\Psi_N$. The covariant derivative \mathcal{D}_M involves the $SU(2)_R$ and super-Lorentz connections \vec{V}_M and $\hat{\omega}_{MAB} = \omega_{MAB} - \frac{i}{2}(\bar{\Psi}_A\gamma_M\Psi_B + \bar{\Psi}_M\gamma_A\Psi_B - \bar{\Psi}_M\gamma_B\Psi_A)$, so that, for instance, $\mathcal{D}_M\vec{Y} = \partial_M\vec{Y} + \vec{V}_M \times \vec{Y}$ and $\mathcal{D}_M\Psi_N = D(\hat{\omega})_M\Psi_N - \frac{i}{2}\vec{V}_M\vec{\tau}\Psi_N$.

In the situation that we shall consider in the following, matter does not couple to the Lagrange multipliers C and λ . Their Lagrangian is thus given by the sum of the last brackets in \mathcal{L}_{min} and $\mathcal{L}_{\text{tens}}$, and their equations of motion imply $Y = 1$ and $\rho = 0$. All the terms in the second bracket in $\mathcal{L}_{\text{tens}}$ are thus irrelevant, and the Lagrangian simplifies to:

$$\begin{aligned} \mathcal{L}_5 = & -\frac{1}{4}\mathcal{R}(\hat{\omega}) - \frac{i}{2}\bar{\Psi}_P\gamma^{PMN}\mathcal{D}_M\Psi_N - \frac{1}{6}\hat{F}_{MN}\hat{F}^{MN} - \frac{1}{\sqrt{3}}\hat{F}_{AB}v^{AB} + v_{AB}v^{AB} \\ & - 12t^2 + W_AW^A - (N + 6t\vec{Y})^2 - \frac{1}{4}\mathcal{D}_M\vec{Y}\mathcal{D}^M\vec{Y} - \frac{1}{24}\varepsilon^{MNPQR}\vec{Y}\vec{G}_{MN}B_{PQR} \\ & - \frac{i}{4\sqrt{3}}\bar{\Psi}_M\gamma^{MNPQ}\Psi_N\hat{F}_{PQ} - \frac{1}{6\sqrt{3}}\varepsilon^{MNPQR}(A_MF_{NP} - \frac{3i}{4}\bar{\Psi}_M\gamma_N\Psi_P)F_{QR} \\ & + \frac{1}{24}\varepsilon^{MNPQR}\vec{Y}(\mathcal{D}_M\vec{Y} \times \mathcal{D}_N\vec{Y})B_{PQR} - \frac{1}{4}\bar{\Psi}_M\vec{\tau}\gamma^{MNP}\Psi_N(\vec{Y} \times \mathcal{D}_P\vec{Y}). \quad (3.10) \end{aligned}$$

The auxiliary scalar \vec{Y} is forced to acquire a non-zero VEV, since it is constrained to satisfy $Y = 1$. $SU(2)_R$ is thus spontaneously broken, and a suitable gauge fixing is given

by $\vec{Y} = (0, 1, 0)$. Note that the VEV of Y preserves the symmetry generated by T_2 , while the orbifold preserves the one associated to T_3 , so that no residual gauge symmetry survives the compactification. The bulk Lagrangian then is

$$\begin{aligned} \mathcal{L}_5 = & -\frac{1}{4}\mathcal{R}(\widehat{\omega}) - \frac{i}{2}\bar{\Psi}_P\gamma^{PMN}\mathcal{D}'_M\Psi_N - \frac{1}{6}\widehat{F}_{MN}\widehat{F}^{MN} - \frac{1}{\sqrt{3}}\widehat{F}_{AB}v^{AB} + v_{AB}v^{AB} \\ & - 12\bar{t}^2 + W_A W^A - (N + 6t^2)^2 - \frac{1}{4}(V_{1A}V_1^A + V_{3A}V_3^A) - \frac{1}{12}\varepsilon^{MNPQR}\partial_M V_N^2 B_{PQR} \\ & - \frac{i}{4\sqrt{3}}\bar{\Psi}_M\gamma^{MNPQ}\Psi_N\widehat{F}_{PQ} - \frac{1}{6\sqrt{3}}\varepsilon^{MNPQR}(A_M F_{NP} - \frac{3i}{4}\bar{\Psi}_M\gamma_N\Psi_P)F_{QR}. \end{aligned} \quad (3.11)$$

Only V_2^A , corresponding to the unbroken T_2 , now appears in the covariant derivative $\mathcal{D}'_M = D_M - \frac{i}{2}V_M^2\tau_2$. The terms involving the other vector auxiliary fields V_1^A and V_3^A have canceled against analogous interactions coming from the last term in (3.10). Note also that V_2^A enters only linearly the Lagrangian, and after integrating by parts and using eq. (3.1) all terms sum up to $V_2^A W_A$.

3.2 Boundary Lagrangians

The Lagrangians $\mathcal{L}_{4,0}$ and $\mathcal{L}_{4,\pi}$ are constructed by using the tensor calculus of 4D supergravity in the formalism of [23]; they consist of generic interactions involving the matter multiplets Φ and V , the gravitational intermediate multiplet I and the compensator S_0 . It is useful to briefly recall how 4D Lagrangians are constructed in the intermediate multiplet formalism [23]. The presence of an extra set of auxiliary fields leads to constraints on the chiral matter multiplets: with $n + 1$ chiral multiplets in the off-shell formulation, the constraints eliminate one combination of them, leading to an n -dimensional Kähler manifold. This is why there is, for a given physical on-shell Lagrangian, a family of off-shell Lagrangians that reduce to it. To make computations simpler it is useful to write the off-shell Lagrangian in such a way that the constraint involves just one multiplet with non-zero chiral weight, the compensator Ξ . Without losing generality, but making contact with our 5D model (see below), we can take the compensator to have weight $w_\Xi = 2$. The construction of a generic Lagrangian for n chiral multiplets Φ_i is then straightforward. Again, without loss of generality, we can choose all the Φ_i to have zero chiral weight. (If Φ_i had weight w_{Φ_i} , we could make it zero by a field redefinition $\Phi_i \rightarrow \Phi_i \Xi^{-w_i/2}$.) Then any function $\Omega(\Phi_i, \Phi_i^\dagger)$ will be a vector superfield according to the tensor calculus of ref. [23], where vector superfields have zero chiral weight. Moreover the expression $\Xi P(\Phi_i)$, for arbitrary P , is a chiral superfield of weight 2, whose F component has zero chiral weight and can be used to write a Lagrangian density. Then the 4D Lagrangian can be written as

$$\mathcal{L}_4^{\text{chi}} = \left[\Omega(\Phi, \Phi^\dagger) \left((\Xi \Xi^\dagger)^r - (1 - 3r) \right) \right]_D + \left[P(\Phi) \Xi \right]_F + \left[P(\Phi) \Xi \right]_F^\dagger. \quad (3.12)$$

Note that the dependence of the D -term on Ξ is to a large extent arbitrary, as long as it comes just through the vector multiplet $\Xi \Xi^\dagger$. Here we explicitly emphasized this fact by choosing an arbitrary exponent⁶ r . The equation of motion of the auxiliary scalar S in

⁶In the superconformal approach [14], Weyl invariance constrains the D -term to be just $[\Omega(\Xi \Xi^\dagger)^{\frac{1}{3}}]_D$.

the gravitational multiplet leads to a simple constraint for the scalar component ξ of Ξ :

$$\left(\frac{1}{3} - r\right) \left(|\xi|^{2r} - 1\right) = 0. \quad (3.13)$$

Note, however, that for $r = \frac{1}{3}$ the constraint disappears. For instance, after compactification on S_1/\mathbb{Z}_2 , in the absence of boundary terms, the effective off-shell Lagrangian for the light modes is

$$\mathcal{L}^{\text{eff}} = \left[(T + T^\dagger) \left(\sqrt{\Xi \Xi^\dagger} + \frac{1}{2} \right) \right]_D, \quad (3.14)$$

where the radion T and the compensator Ξ are just the zero modes of the E_5^\ddagger and S_0 supermultiplets, respectively, defined previously.

In writing the boundary action, we should apply the above rules, with S_0 playing the role of the compensator Ξ . The freedom we have in the off-shell formulation can be exploited to make the calculations simpler. In particular, the dependence on the bulk auxiliary fields can be kept to a minimum by writing the boundary Lagrangian as

$$\mathcal{L}_4^{\text{chi}} = \left[\Omega(\Phi, \Phi^\dagger) \left(S_0 S_0^\dagger \right)^{\frac{1}{3}} \right]_D + \left[P(\Phi) S_0 \right]_F + \left[P(\Phi) S_0 \right]_F^\dagger. \quad (3.15)$$

This will become clear in the examples below. Actually, the basic situation that we will be mostly interested in is $\Omega(\Phi, \Phi^\dagger) = \Phi \Phi^\dagger$ and $P(\Phi) = 0$. In this special case, it is convenient to choose $w_\Phi = \frac{2}{3}$ and write the boundary Lagrangian as

$$\mathcal{L}_4^{\text{chi}} = \left[\Phi \Phi^\dagger \right]_D. \quad (3.16)$$

We will see that with this specific off-shell formulation several auxiliary fields do not couple to matter and can be integrated out at the classical level, to yield a formulation that is still sufficiently off-shell to correctly describe interactions and reproduce supersymmetric cancellations at the quantum level.

Let us now work out the component expressions of the boundary actions describing the interaction of chiral and vector multiplets Φ and V with the intermediate multiplet I . To simplify the formulae, we will only write the relevant pieces of the Lagrangians, neglecting all interaction terms involving fermions. Consider first the Lagrangian (3.16) for a chiral multiplet Φ with generic chiral weight w . Defining the complex auxiliary field $t = t^2 + it^1$, its explicit component expression reads:

$$\begin{aligned} \mathcal{L}_4^{\text{chi}} = & |\mathcal{D}_\mu \phi|^2 + i \bar{\chi} \gamma^\mu \mathcal{D}_\mu \chi + |F_\Phi - 4\phi t^*|^2 + \frac{w}{4} |\phi|^2 \left(\mathcal{R} + 2i \bar{\psi}_\mu^1 \gamma^{\mu\nu\rho} D_\nu \psi_\rho^1 \right) \\ & + 6 \left(w - \frac{2}{3} \right) |\phi|^2 \left[b_\mu^2 - 2S - 8|t|^2 \right] + \dots, \end{aligned} \quad (3.17)$$

where the chiral covariant derivatives are given by

$$\mathcal{D}_\mu \phi = \partial_\mu \phi + iw \left(a_\mu + \frac{2}{w} b_\mu \right) \phi, \quad (3.18)$$

$$\mathcal{D}_\mu \chi = D_\mu \chi - i(1-w) \gamma^\ddagger \left(a_\mu + \frac{1}{1-w} b_\mu \right) \chi. \quad (3.19)$$

The Lagrangian $\mathcal{L}_4^{\text{vec}}$ for a vector multiplet V has already been worked out in [18], and we therefore quote only the result:

$$\mathcal{L}_4^{\text{vec}} = -\frac{1}{4}G_{\mu\nu}^2 + i\bar{\psi}\gamma^\mu\mathcal{D}_\mu\psi + \frac{1}{2}D^2 + \dots, \quad (3.20)$$

where

$$\mathcal{D}_\mu\psi = D_\mu\psi - \gamma^{\dot{5}}(a_\mu + 3b_\mu)\psi. \quad (3.21)$$

As anticipated, a substantial simplification occurs when the chiral multiplet has weight $w = \frac{2}{3}$. In this case, the second line in (3.17) drops out and there is therefore no tadpole for S , as already assumed in the previous subsection. Moreover, the same combination of auxiliary fields $a_\mu + 3b_\mu = -\frac{1}{2}(V_\mu^3 - \frac{2}{\sqrt{3}}F_{\mu\dot{5}}^{\dot{5}} - 2e_\mu^\alpha v_{\alpha\dot{5}})$ is left in all the covariant derivatives (3.18), (3.19) and (3.21).

There is actually a simple generalization of the basic situation $\Omega(\Phi, \Phi^\dagger) = \Phi\Phi^\dagger$ that we would like to consider. It consists in adding a real constant kinetic function $\Omega = -3L$. The simplest way to construct the additional terms in the off-shell boundary Lagrangians is now to use eq. (3.15). In this case, a non-trivial dependence on the compensator auxiliary fields N and $W_{\dot{5}}$ will appear. The possibility of having $\Omega = -3L$ corresponds to adding localized kinetic terms for the bulk supergravity fields, and is required to construct kinetic functions of the form (2.5) and (2.6). The component expansion of the corresponding action is easily found to be:

$$\mathcal{L}_4^{\text{loc}} = -\frac{L}{2}\left[\mathcal{R} + 2i\bar{\psi}_\mu^1\gamma^{\mu\nu\rho}D_\nu\psi_\rho^1 + \frac{8}{3}(a_\mu + 3b_\mu)^2 + \frac{8}{3}\left(N + 6t^2 - \frac{1}{2}V_{\dot{5}}^1\right)^2 + \frac{8}{3}W_{\dot{5}}^2 + \dots\right]. \quad (3.22)$$

As in the minimal situation, the auxiliary fields a_μ and b_μ appear only in the universal combination $a_\mu + 3b_\mu$. Moreover, the additional dependence on the auxiliary fields N , t^2 , $V_{\dot{5}}^1$ and $W_{\dot{5}}^2$ occurs only in the two combinations $N + 6t^2 - \frac{1}{2}V_{\dot{5}}^1$ and $W_{\dot{5}}^2$. This will be important in the next section, in which most of these fields will be integrated out.

3.3 Partially off-shell formulation

The only auxiliary fields that are influenced by the boundary are V_μ^3 , $v_{\alpha\dot{5}}$, t_1 and t_2 , as well as N , $V_{\dot{5}}^1$ and B_{MNP} if constant kinetic functions are included. All the other auxiliary fields can then be integrated out just by using (3.11), to give a partially off-shell formulation, which is still powerful enough to correctly describe all bulk-to-boundary interactions. The equations of motion of the fields t_3 , $v_{\alpha\beta}$, V_1^α , V_2^A and $V_3^{\dot{5}}$ are trivial and imply $t_3 = 0$, $v_{\alpha\beta} = \frac{1}{2\sqrt{3}}\hat{F}_{\alpha\beta}$, $V_1^\alpha = 0$, $W^A = 0$ and $V_3^{\dot{5}} = 0$. Since $W^{\dot{5}} = 0$, the dependence on B_{MNP} coming from the boundary Lagrangian (3.22) trivializes, and its equation of motion can be derived from the bulk Lagrangian (3.11) as well. It leads to the condition that the field strength of V_2^M vanish: $\partial_M V_{2N} - \partial_N V_{2M} = 0$. This implies that the connection V_2 is closed. Since space-time is in this case not simply connected, V_2 is not necessarily exact and can have a physical effect, parametrized by the gauge-invariant quantity $\epsilon = \int dx^5 V_2^{\dot{5}}(x^5)$. This is a Wilson line for the unbroken $U(1)_{T_2}$, and it is equivalent to Scherk–Schwarz supersymmetry breaking with twist ϵ [24]. In section 6 we will explain this in more detail.

The auxiliary fields N and V_5^1 appear both in the bulk Lagrangian (3.11) and in the boundary Lagrangian (3.22), but their effect is nevertheless trivial. This is most easily seen by first substituting them with the two new combinations $N_\pm = N + 6t^2 \pm \frac{1}{2}V_5^1$. These appear in the bulk Lagrangian (3.11) only through a term proportional to N_+N_- , whereas in the boundary Lagrangian (3.22) only a term proportional to N_-^2 appears. The equation of motion of N_- therefore fixes the value of N_+ , but that of N_+ implies $N_- = 0$, so that all the dependence on N_\pm finally has no effect. This is perfectly analogous to what happens in 4D no-scale models, where the equation of motion of F_T enforces the condition $F_{S_0} = 0$.

To proceed further, it is convenient to redefine the remaining auxiliary fields in such a way as to disentangle those combinations of them which do not couple to matter, and to integrate them out. This is most conveniently done by defining the following new vector and scalar auxiliary fields:

$$V_\alpha = e_\alpha^M V_M^3 - \frac{2}{\sqrt{3}} e_\alpha^M F_{M5} e_5^5 - 2v_{\alpha\dot{5}}. \quad (3.23)$$

Note that since we have $e_\mu^{\dot{5}} = e_5^\alpha = 0$ at the boundary, the vector that couples to the boundary is $V_\mu \equiv e_\mu^\alpha V_\alpha = -2(a_\mu + 3b_\mu)$. Thanks to the above redefinitions, $v_{\alpha\dot{5}}$ no longer couples to matter and can be integrated out through its equation of motion $v_{\alpha\dot{5}} = \frac{1}{2\sqrt{3}} \hat{F}_{\alpha\dot{5}}$. Similarly, the equation of motion of V_5^3 now trivially implies $V_5^3 = 0$. After a straightforward computation, splitting the covariant derivatives and factoring out the volume element $e = \det(e_M^A)$ explicitly⁷, we finally find

$$\begin{aligned} e^{-1}\mathcal{L} = & \frac{1}{6}\Omega(x^5) \left[\mathcal{R} + 2i\bar{\Psi}_M \gamma^{MNP} D_N \Psi_P + \frac{2}{3}V_\alpha V^\alpha \right] - 12|t|^2 \\ & + \Omega_{\phi\phi^*}(x^5) \left[|\partial_\mu \phi|^2 + i\bar{\chi} \not{D} \chi + |F_\Phi - 4\phi t^*|^2 \right] + e_5^5 \delta(x^5) \left[-\frac{1}{4}G_{\mu\nu}^2 + i\bar{\psi} \not{D} \psi + \frac{1}{4}D^2 \right] \\ & - \frac{1}{4}F_{\alpha\beta}^2 + \frac{1}{3} \left(J_\alpha^{\text{mat}}(x^5) - \sqrt{3}F_{\alpha\dot{5}} \right) V^\alpha + \dots. \end{aligned} \quad (3.24)$$

In this expression, $\Omega(x^5)$ is a generalized kinetic function defined as

$$\Omega(x^5) = -\frac{3}{2} + \left(-3L + |\phi|^2 \right) e_5^5 \delta(x^5). \quad (3.25)$$

It is understood that the localized part of $\Omega(x^5)$ multiplies only the restrictions of the kinetic terms to the boundary. Similarly, $J_\mu^{\text{mat}}(x^5) = J_\mu^{\text{chi}}(x^5) + J_\mu^{\text{vec}}(x^5)$ is a generalized matter R -symmetry current, defined by⁸:

$$J_\mu^{\text{chi}}(x^5) = i(\Omega_\phi(x^5)\partial_\mu \phi - \text{c.c.}) - \frac{i}{2}\Omega_{\phi\phi^*}(x^5)\bar{\chi}\gamma_\mu\gamma^{\dot{5}}\chi + \dots, \quad (3.26)$$

$$J_\mu^{\text{vec}}(x^5) = \frac{3i}{2}e_5^5 \delta(x^5)\bar{\psi}\gamma_\mu\gamma^{\dot{5}}\psi. \quad (3.27)$$

Finally, the dots denote boundary terms describing the standard 4D supergravity interactions of the gravitino with matter, the only truly novel interaction between bulk and brane being those with V_μ .

⁷ $e_5^5 \delta(x^5)$ is the scalar δ -function density.

⁸The R -charges of ϕ , χ and ψ are equal respectively to $\frac{2}{3}$, $-\frac{1}{3}$ and -1 , but for convenience we take out an overall factor of $\frac{2}{3}$ in the definition of the current.

The field V_μ is the analogue of the vector auxiliary field b_μ of Poincaré supergravity [14, 25], but it mixes with the graviphoton A_M , and is therefore no longer an ordinary auxiliary field. The graviphoton has also changed its dynamics: the Kaluza–Klein (KK) mass term $\frac{1}{2}F_{\mu 5}^2$ has disappeared. 5D covariance is not manifest because of the non-covariant field redefinition of eq. (3.23); by integrating out V_α , however, we would recover the fully covariant graviphoton kinetic term.⁹

The Lagrangian (3.24) that we find is perfectly analogous to the one found by Mirabelli and Peskin [19] in the case of a 4D chiral multiplet interacting with a 5D vector multiplet. There the role of V_μ^3 and A_μ is played respectively by X^3 , the T_3 -singlet component of the auxiliary field \vec{X} , and by Σ , the extra physical scalar of the 5D vector multiplet. The boundary couples only to the combination $X = X^3 - \partial_5 \Sigma$, which plays the role of V_μ . The propagation of X , Σ and their interaction with the boundary is described by

$$\mathcal{L}_{X,\Sigma} = \frac{1}{2} \partial_\mu \Sigma \partial^\mu \Sigma + X \partial_5 \Sigma + \frac{1}{2} X^2 + \delta(x^5) X |\phi|^2. \quad (3.28)$$

Note that, as in our case, the auxiliary field Σ propagates in the 5th dimension only via its mixing to X .

From eq. (3.24) one would normally go ahead and eliminate the remaining auxiliary fields to write the physical Lagrangian. For F_Φ and t this can be trivially done. On the other hand, V_μ has sources proportional to $\delta(x^5)$ so that, after solving its equation of motion, the physical Lagrangian contains seemingly ambiguous expressions involving powers of $\delta(x^5)$. Indeed, since the kinetic term of V_μ has a coefficient given by eq. (3.25), the effective Lagrangian, proportional to $1/\Omega(x^5)$, will formally involve infinite powers of $\delta(x^5)$. This should be compared with the global case of ref. [19], see eq. (3.28), where one has ‘just’ to deal with $\delta^2(x^5)$. Now, the presence of tree-level UV divergences is a normal fact in theories with fixed points: the momentum in the orbifolded directions is not conserved, so that the momentum on the external lines does not fix the virtual momenta even at tree level. For propagating fields in n extra dimensions the sum over the transverse momentum p_T gives rise to an amplitude

$$\int \frac{d^n p_T}{p^2 + p_T^2}, \quad (3.29)$$

which leads to UV divergences when $n \geq 2$. For an auxiliary field, the propagator is just 1, so the UV divergences appear already when $n = 1$. However, in the case at hand, these UV divergences are a spurious effect of integrating out an incomplete supermultiplet. In physical quantities they will never appear. Physically we should also account for the propagation of the graviphoton A_μ (or of Σ in the global case). Notice that A_5 plays no rôle, as we can choose the gauge $\partial_5 A_5 = 0$, where it has no local 5D degrees of freedom. The mixed A_μ, V_μ kinetic matrix then has the form

$$K_{A,V} = \begin{pmatrix} p^2 \eta_{\mu\nu} - p_\mu p_\nu & \frac{1}{2\sqrt{3}} p_5 \eta_{\mu\nu} \\ \frac{1}{2\sqrt{3}} p_5 \eta_{\mu\nu} & \frac{1}{3} \eta_{\mu\nu} \end{pmatrix}. \quad (3.30)$$

⁹The field t is similar to the auxiliary field M of Poincaré supergravity [14, 25], but it does not coincide with it.

The propagator of A_μ and V_μ is obtained by inverting this matrix. Since the AA entry does not involve any p_5^2 , the $\langle V_\mu V_\nu \rangle$ propagator scales like p^2/p_5^2 , and the exchange of V_μ between boundary localized sources does not lead to any UV divergences.

One example of a physical object that is calculated by integrating out the auxiliary KK modes is the low-energy two-derivative effective Lagrangian after compactification. In order to compute it, we will pick the zero modes of the physical fields $e_\mu^\alpha(x, x^5) \equiv e_\mu^\alpha(x)$ and similarly for ψ_μ^1, ψ_5^2 and A_5 without changing notation. On the other hand, we set $e_5^\alpha = e_\mu^{\dot{5}} \equiv 0$, so that indices are raised and lowered according to 4D rules. Finally we define the radion field by $e_5^{\dot{5}}(x, x^5) \equiv R(x)$ and normalize the radion supermultiplet¹⁰ as $T/\pi = (R + i\frac{2}{\sqrt{3}}A_5, \psi_5^2)$. The graviphoton A_μ does not have zero modes and it is conveniently integrated out by working in the gauge $\partial_5 A_5 = 0$, where only the physical zero mode of A_5 is turned on. The $\partial_\mu A_5/R$ piece in $F_{\mu\dot{5}}$ corresponds to the radion contribution to the generalized R -symmetry current:

$$J_\mu^{\text{rad}}(x^5) = -\frac{3i}{2(T + T^\dagger)}(\partial_\mu T - \text{c.c.}) . \quad (3.31)$$

This reconstructs the total R -current $J_\mu(x^5) = J_\mu^{\text{mat}}(x^5) + J_\mu^{\text{rad}}(x^5)$ in the last term of (3.24). The graviphoton A_μ can now be integrated out at the classical level. Neglecting the $F_{\mu\nu}^2$ term, which only affects higher-derivative terms in the low-energy action, the A_μ equation of motion amounts to the constraint

$$\partial_5 V_\mu = 0 , \quad (3.32)$$

saying that only the zero mode of V_μ survives. As V_μ is constant, to obtain the low-energy effective action we just need to integrate eq. (3.24) over x^5 ; the result is

$$\begin{aligned} \mathcal{L}^{\text{eff}} = & \frac{1}{6}\Omega \left[\mathcal{R} + 2i\bar{\psi}_\mu^1 \gamma^{\mu\nu\rho} D_\nu \psi_\rho^1 + \frac{2}{3}V_\mu^2 \right] + \frac{1}{3}J_\mu V^\mu \\ & + \Omega_{\phi\phi^*} \left[|\partial_\mu \phi|^2 + i\bar{\chi} \not{D} \chi \right] + \left[-\frac{1}{4}G_{\mu\nu}^2 + i\bar{\psi} \not{D} \psi \right] + \dots . \end{aligned} \quad (3.33)$$

In this expression, the 4D quantities Ω and J are obtained by integrating the corresponding generalized 5D quantities $\Omega(x^5)$ and $J(x^5)$, defined by eq. (3.25) and the sum of (3.26), (3.27), (3.31), over the internal space. Denoting the former by X and the latter by $X(x^5)$, the precise relation is $X = \int_{-\pi}^{\pi} dx^5 e_5^{\dot{5}} X(x^5)$. The kinetic function is found to be

$$\Omega = -\frac{3}{2}(T + T^\dagger) - 3L + |\phi|^2 , \quad (3.34)$$

and the total R -symmetry current of the light fields $J_\mu = J_\mu^{\text{chi}} + J_\mu^{\text{vec}} + J_\mu^{\text{rad}}$ is correctly reproduced with

$$J_\mu^{\text{chi}} = i(\Omega_\phi \partial_\mu \phi - \text{c.c.}) - \frac{i}{2}\Omega_{\phi\phi^*} \bar{\chi} \gamma_\mu \gamma^{\dot{5}} \chi + \dots , \quad (3.35)$$

$$J_\mu^{\text{vec}} = \frac{3i}{2}\bar{\psi} \gamma_\mu \gamma^{\dot{5}} \psi , \quad (3.36)$$

$$J_\mu^{\text{rad}} = i(\Omega_T \partial_\mu T - \text{c.c.}) . \quad (3.37)$$

¹⁰The relative coefficients of the real and imaginary parts of T agree with Luty and Sundrum (LS) [13], after noticing that our A_5 equals $\frac{1}{\sqrt{2}}A_5^{LS}$, owing to our different normalization of the supergravity kinetic terms. Notice also our different overall normalizations: $T_{LS} = 3T$.

In the Lagrangian (3.33) V_μ is identified with the standard vector auxiliary field of 4D supergravity. It is easy to check, using for instance the formulae in [25], that all coefficients in the above equations are correct.

3.4 On-shell formulation

In this section we will compute the on-shell Lagrangian. We do that mainly to make contact with the standard approach followed by Mirabelli and Peskin [19]. We believe that our discussion completes or even corrects previous treatments of this issue in the supergravity case [20, 26].

Let us start from eq. (3.24). The most natural way to proceed is to complete the quadratic form depending on the auxiliary field V_α through a shift. This is achieved by defining the new auxiliary field

$$\tilde{V}_\alpha = V_\alpha + \frac{3}{2\Omega(x^5)} \left[J_\alpha^{\text{mat}}(x^5) - \sqrt{3} F_{\alpha\dot{5}} \right], \quad (3.38)$$

where $\Omega(x^5)$ has been defined in eq. (3.25) and $J_\mu^{\text{mat}}(x^5) = J_\mu^{\text{chi}}(x^5) + J_\mu^{\text{vec}}(x^5)$ in (3.26) and (3.27). Note that we are working with the ill-defined distribution $1/\Omega(x^5)$. In what follows, one could think of $\delta(x^5)$ as being regulated. In the end, as is evident from the discussion in the previous section, the regulation will not matter in the computation of physical quantities. After some straightforward algebra, and integrating out the trivial auxiliary fields Q , F_Φ and D , the Lagrangian can be rewritten as

$$\begin{aligned} e^{-1} \mathcal{L} = & \frac{1}{6} \Omega(x^5) \left[\mathcal{R} + 2i \bar{\Psi}_M \gamma^{MNP} D_N \Psi_P + \frac{2}{3} \tilde{V}_\alpha^2 \right] \\ & + \Omega_{\phi\phi^*}(x^5) \left[|\partial_\mu \phi|^2 + i \bar{\chi} \not{D} \chi \right] + e_5^5 \delta(x^5) \left[-\frac{1}{4} G_{\mu\nu}^2 + i \bar{\psi} \not{D} \psi \right] \\ & - \frac{1}{4} F_{\alpha\beta}^2 - \frac{3}{4\Omega(x^5)} \left[F_{\alpha\dot{5}} - \frac{1}{\sqrt{3}} J_\alpha^{\text{mat}}(x^5) \right]^2 + \dots \end{aligned} \quad (3.39)$$

Note that we have not truly integrated out V_α , but just rewritten the Lagrangian in terms of the classically irrelevant field \tilde{V}_α . The reason for keeping \tilde{V}_α is that its kinetic term is field-dependent and gives rise to a Jacobian at the quantum level. The above Lagrangian differs from the one advocated in [20]; in particular, the interaction of the chiral multiplet with the graviphoton involves a non-trivial denominator with δ -functions, which is crucial to correctly reproduce the quartic coupling of the effective 4D theory (and of course to obtain the supersymmetric cancellations at the quantum level). More insight in these couplings can be obtained by expanding the perfect square to isolate the complete bulk kinetic term of the graviphoton. At leading order in a power series expansion in the scalar fields, one finds that the exceeding $F_{\alpha\dot{5}}^2 |\phi|^2 e_5^5 \delta(x^5)$ term just provides the correct scalar seagull correction to the coupling $J^\alpha(x^5) F_{\alpha\dot{5}}$ to turn it into a minimal coupling through a covariant derivative, so that the R -symmetry appears to be gauged by $F_{\alpha\dot{5}}$.

We now show once more that the correct low-energy effective 4D theory is obtained when integrating out the heavy KK modes. Again, since we take $e_5^\alpha = e_\mu^5 \equiv 0$ we can restore the curved indices to integrate out the massive modes of the graviphoton. As before

we work in the gauge $\partial_5 A_5 = 0$, and the $\partial_\mu A_5/R$ piece in $F_{\mu\dot{5}}$ again corresponds to the radion contribution to the generalized R -symmetry current. Neglecting as before terms with 4D space-time derivatives with respect to x^5 -derivatives in the low energy limit, and defining the total generalized R -symmetry current $J_\mu(x^5) = J_\mu^{\text{mat}}(x^5) + J_\mu^{\text{rad}}(x^5)$, with $J_\mu^{\text{rad}}(x^5)$ given by (3.31), the Lagrangian for the heavy field A_μ can be written as

$$\mathcal{L}_A \simeq -\frac{3}{4\Omega(x^5)} \left[\partial_5 A_\mu - \frac{1}{\sqrt{3}} J_\mu(x^5) \right]^2. \quad (3.40)$$

The corresponding equation of motion yields

$$\partial_5 A_\mu = \frac{1}{\sqrt{3}} \left[J_\mu(x^5) - \frac{\Omega(x^5)}{\Omega} J_\mu \right], \quad (3.41)$$

where the 4D kinetic function Ω and R -current J_μ arise again as integrals of their 5D generalizations $\Omega(x^5)$ and $J_\mu(x^5)$. Plugging this expression back into the Lagrangian, discarding the auxiliary field and integrating over x^5 , one finally finds the standard on-shell expression for a 4D chiral no-scale supergravity model with kinetic function Ω and vanishing superpotential:

$$\begin{aligned} \mathcal{L}^{\text{eff}} = & \frac{1}{6} \Omega \left[\mathcal{R} + 2i\bar{\psi}_\mu^1 \gamma^{\mu\nu\rho} D_\nu \psi_\rho^1 \right] - \frac{1}{4\Omega} J_\mu^2 \\ & + \Omega_{\phi\phi^*} \left[|\partial_\mu \phi|^2 + \bar{\chi} \mathcal{D} \chi \right] + \left[-\frac{1}{4} G_{\mu\nu}^2 + i\bar{\psi} \mathcal{D} \psi \right] + \dots. \end{aligned} \quad (3.42)$$

4 Loop corrections to matter operators

Before starting the computation outlined in the introduction, we shall verify in this section that the one-loop corrections to operators involving scalar fields and no derivatives correctly cancel as a consequence of the supersymmetry surviving the orbifold projection. In order to do that, we need to discuss the structure of the propagators of 5D fields. For the gauge field A_M and the graviton h_{MN} defined by expanding the metric around the flat background as $g_{MN} = \eta_{MN} + 2\sqrt{2}h_{MN}$, one can proceed along the lines of [27]. For the gravitino, that we can now describe with an ordinary Dirac spinor¹¹ $\Psi_M = (\chi_M^1, \bar{\chi}_M^2)^T$, we refer instead to [28, 29]. The mode expansions are standard and lead to towers of KK states with masses $m_n = n/R$. As usual it is convenient to use the doubling trick and run n from $-\infty$ to $+\infty$, including $n = 0$, with the same weight. For the gravitino, we use Dirac modes $\Psi_n^M = (\chi_n^{1M}, \bar{\chi}_n^{2M})^T$. For simplicity we restrict our analysis to the basic case of a simple quadratic kinetic function and set $L = 0$.

4.1 On-shell formulation

We consider first the completely on-shell formulation (3.39), and focus on the simplest example of the class of operators we want to study: the scalar two-point function at zero momentum, i.e. the correction to the scalar mass. The relevant interactions on the

¹¹The kinetic term then has an additional factor of 2.

brane are easily obtained by expanding all interactions in (3.39) to quadratic order and recalling the usual supersymmetric interaction between the gravitino and the improved supersymmetric current of the chiral multiplet. To switch to the new Dirac notation for the gravitino, we use the projectors $P_{L,R} = \frac{1}{2}(1 \pm i\gamma^5)$. The terms that are relevant at zero momentum are given by:

$$\begin{aligned} \mathcal{L}^{\text{int}} \simeq \delta(x^5) e_4 \left[\frac{1}{3} |\phi|^2 \left(\frac{1}{2} \mathcal{R}_4 + 2i \bar{\Psi}_\mu \gamma^{\mu\nu\rho} P_L \partial_\nu \Psi_\rho + \frac{1}{3} \tilde{V}_\mu^2 + F_{\mu\dot{5}}^2 \right) \right. \\ \left. + \frac{1}{3} \left(\sqrt{2} \phi^* \bar{\chi} \gamma^{\mu\nu} P_L \partial_\mu \Psi_\nu - i\sqrt{3} F_{\mu\dot{5}} \phi^* \partial^\mu \phi - \text{c.c.} \right) \right. \\ \left. + \frac{1}{6} |\phi^* \partial^\mu \phi - \text{c.c.}|^2 e_5^{\dot{5}} \delta(0) \right], \end{aligned} \quad (4.1)$$

where:

$$e_4 \mathcal{R}_4 = 2\sqrt{2} \left[\partial_\mu \partial_\nu h^{\mu\nu} - \partial^2 h \right] + 2 \left[h \partial^2 h - h_{\mu\nu} \partial^2 h^{\mu\nu} - 2h^{\mu\nu} \partial_\mu \partial_\nu h + 2h^{\mu\nu} \partial_\nu \partial_\rho h^\rho_\mu \right]. \quad (4.2)$$

As advertised in the last section, the couplings between scalars and graviphotons reconstruct a minimal coupling with a covariant derivative given by $\mathcal{D}_\mu = \partial_\mu + \frac{i}{\sqrt{3}} F_{\mu\dot{5}}$.

To derive the propagators of the bulk fields, we have to chose a gauge. Unitary gauges [27] have the advantage of explicitly disentangling physical and unphysical modes for massive KK modes, which will therefore have the propagators of standard massive particles. However, in general they do not fully fix the gauge for the zero modes, which must be separately specified. Moreover, the latter remain entangled in any gauge. For these reasons, it is more convenient to use covariant gauges, which treat massless and massive modes on an equal footing. For the graviphoton, the above problem does not exist, because A_μ does not have zero modes, and for later convenience we will thus choose the unitary gauge $\partial_5 A_5 = 0$. The propagators of the various modes are then given by

$$\langle A_\mu A_\nu \rangle_n = - \left[\eta_{\mu\nu} - \frac{p_\mu p_\nu}{m_n^2} \right] \frac{i}{p^2 - m_n^2}, \quad \langle A_5 A_5 \rangle_0 = \frac{i}{p^2}. \quad (4.3)$$

For the graviton and the gravitino, we shall instead choose the harmonic gauges (called de Donder in the case of the graviton) and add to the 5D Lagrangian the gauge-fixing terms

$$\mathcal{L}_h^{\text{GF}} = - \left[\partial_M (h^{MN} - \frac{1}{2} \eta^{MN} h) \right]^2, \quad (4.4)$$

$$\mathcal{L}_\Psi^{\text{GF}} = \frac{i}{2} \bar{\Psi}_M \gamma^M \not{\partial} \gamma^N \Psi_N. \quad (4.5)$$

In these gauges, the propagators have a structure that is reminiscent of the 5D origin of the fields, and can be deduced by repeating the analysis of [28] on the orbifold after decomposing the fields in KK modes. For the 4D components, relevant to our computation, one finds:

$$\langle h_{\mu\nu} h_{\alpha\beta} \rangle_n = \frac{1}{2} \left[\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha} - \frac{2}{3} \eta_{\mu\nu} \eta_{\alpha\beta} \right] \frac{i}{p^2 - m_n^2}, \quad (4.6)$$

$$\langle \Psi_\mu \bar{\Psi}_\nu \rangle_n = \frac{1}{6} \left[-\gamma_\nu (\not{p} - m_n) \gamma_\mu + \left(\eta_{\mu\nu} - 2 \frac{p_\mu p_\nu}{p^2 - m_n^2} \right) (\not{p} + m_n) \right] \frac{i}{p^2 - m_n^2}. \quad (4.7)$$

Finally, the propagator of the auxiliary field \tilde{V}_μ is given in the same notation by

$$\langle \tilde{V}_\mu \tilde{V}_\nu \rangle_n = -3i \eta_{\mu\nu} . \quad (4.8)$$

Note that in our computation at vanishing external momentum, the longitudinal pieces of the propagators are actually irrelevant, because the couplings in (4.1) feel only the transverse polarizations and each diagram is gauge-independent on its own.

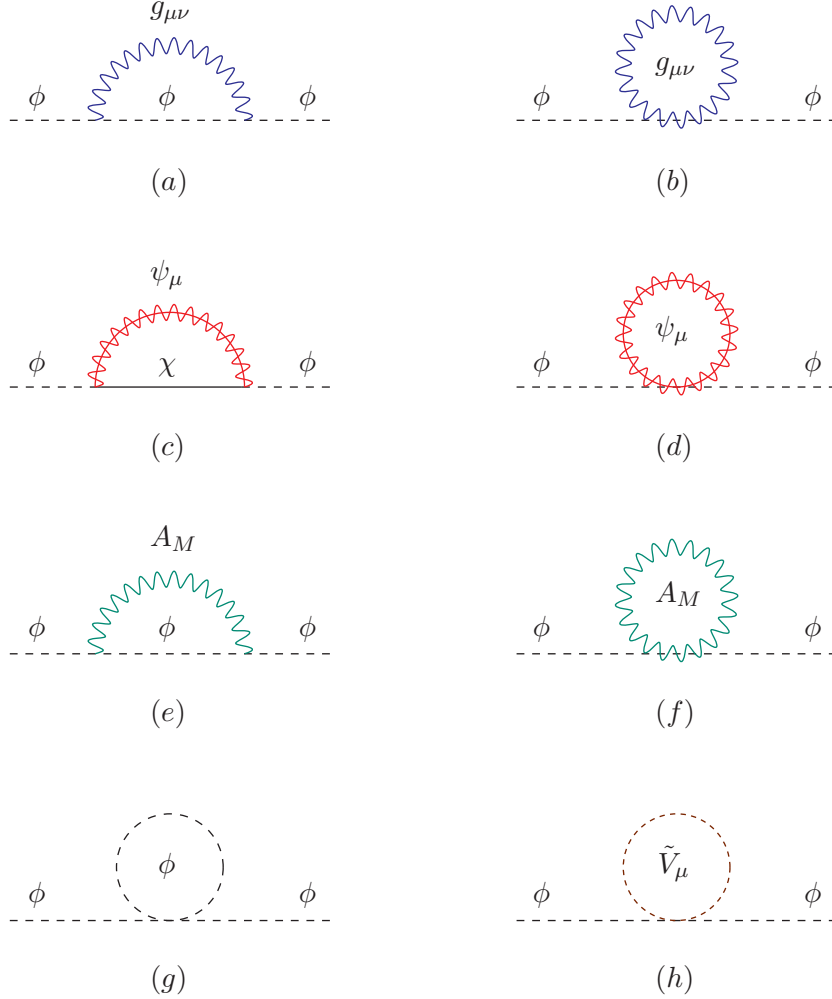


Figure 2: *The diagrams contributing to the mass of the scalar ϕ .*

The 8 diagrams contributing to the one-loop mass correction are depicted in Fig. 2. As in the rigid case [19], the singular couplings proportional to $\delta(0)$ play a crucial rôle in the supersymmetric cancellation. Note, however, that the auxiliary field \tilde{V}_μ gives a non-vanishing contribution as well, which is in fact the only contribution left over in the effective action when integrating it out. Using the representation [19]

$$e^{\frac{5}{3}} \delta(0) = \frac{1}{2\pi R} \sum_{n=-\infty}^{\infty} \frac{p^2 - m_n^2}{p^2 - m_n^2} , \quad (4.9)$$

all the diagrams can be brought into the form

$$\Delta m_\alpha^2 = \frac{i}{2\pi R} \sum_{n=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4} \frac{N_\alpha}{p^2 - m_n^2}. \quad (4.10)$$

After a straightforward computation, it can be verified that the diagrams indeed cancel each other level by level, the contributions of the single diagrams being¹²:

$$\begin{aligned} N_a &= 0, & N_b &= \frac{5}{3} p^2, \\ N_c &= 0, & N_d &= -\frac{8}{3} p^2, \\ N_e &= \frac{1}{3} (p^2 - m_n^2), & N_f &= -\frac{1}{3} (p^2 - 4 m_n^2), \\ N_g &= -\frac{1}{3} (p^2 - m_n^2), & N_h &= \frac{4}{3} (p^2 - m_n^2). \end{aligned} \quad (4.11)$$

The diagrams (a) and (c) involving cubic vertices vanish, since the graviton or gravitino going out of a cubic vertex turns out to be longitudinal, so that it cannot couple to another cubic vertex. Indeed, it can be easily verified that $(p^2 \eta^{\mu\nu} - p^\mu p^\nu) \langle h_{\mu\nu} h_{\alpha\beta} \rangle \propto p_\alpha p_\beta$ and $\gamma^{\mu\nu} p_\mu \langle \Psi_\nu \bar{\Psi}_\alpha \rangle \propto p_\alpha$. The singular diagram (g) arising from the quartic scalar coupling proportional to $\delta(0)$ cancels the divergent part of diagrams (e), (f) and (h), similarly to what happens in the rigid case [19]. Actually, the diagrams in the left column ((a), (c), (e) and (g)), which involve virtual matter particles, cancel separately. This is because the theory with frozen matter fields, where there are only the diagrams in the right column, is a consistent construction on its own (see section 7), for which the cancellation must hold true as well.

It is expected that this pattern of cancellation will continue for operators with higher powers of the scalar fields. Unlike what happens in the rigid case [19], expanding the Lagrangian (3.39) to higher powers in ϕ generates higher powers of $\delta(0)$. The associated singular scalar diagrams are expected to contribute to cancel the divergences coming from the graviphoton, but we will not proceed further.

4.2 Partially off-shell formulation

In the partially off-shell formulation defined by (3.24), things are easier, and one can verify the supersymmetric cancellation of the full effective scalar potential. The graviton and gravitino propagators are exactly the same as before. In this case, the graviphoton does not couple to matter, and correspondingly singular self-couplings for matter fields are absent. The propagator of the auxiliary vector field V_μ is in this case non-trivial, as a consequence of its mixing with the graviphoton, and inverting (3.30) one easily finds:

$$\langle V_\mu V_\nu \rangle_n = -3 \left[p^2 \eta_{\mu\nu} - p_\mu p_\nu \right] \frac{i}{p^2 - m_n^2}. \quad (4.12)$$

As before, cubic vertices involving gravitons and gravitinos are irrelevant, and cubic vertices involving the vector field vanish trivially at zero-momentum, since its propagator

¹²We believe that this corrects the computation performed in ref. [20], where the diagrams (f) and (h) were not properly taken into account, as well as that of [26].

is transverse. The relevant diagrams are then loops of gravitons, gravitinos or vector fields, with an arbitrary number of insertions of the appropriate quartic vertex with scalar fields. In order to perform an exact resummation of all these one-loop diagrams, it is extremely convenient to introduce the following projection operators:

$$P_{1/2}^{\mu\nu} = \frac{1}{3} \left(\gamma^\mu - \frac{p^\mu}{\not{p}} \right) \left(\gamma^\nu - \frac{p^\nu}{\not{p}} \right), \quad (4.13)$$

$$P_1^{\mu\nu} = \eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}, \quad (4.14)$$

$$P_{3/2}^{\mu\nu} = \left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) - \frac{1}{3} \left(\gamma^\mu - \frac{p^\mu}{\not{p}} \right) \left(\gamma^\nu - \frac{p^\nu}{\not{p}} \right), \quad (4.15)$$

$$P_2^{\mu\nu\alpha\beta} = \frac{1}{2} \left(\eta^{\mu\alpha} - \frac{p^\mu p^\alpha}{p^2} \right) \left(\eta^{\nu\beta} - \frac{p^\nu p^\beta}{p^2} \right) - \frac{1}{6} \left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \left(\eta^{\alpha\beta} - \frac{p^\alpha p^\beta}{p^2} \right) + (\alpha \leftrightarrow \beta). \quad (4.16)$$

These are all idempotent, $P_i^2 = P_i$, and transverse, $p \cdot P_i = 0$. The spin-3/2 projector also satisfies $\gamma \cdot P_{3/2} = 0$. Defining for notational convenience $\rho = \frac{1}{3} |\phi|^2$, the quartic interaction vertices in mixed momentum/configuration space can then be written as

$$\begin{aligned} \mathcal{L}^{\text{int}} = \rho \delta(x^5) & \left[p^2 h_{\mu\nu} \left(P_2^{\mu\nu\alpha\beta} - \frac{2}{3} P_1^{\mu\nu} P_1^{\alpha\beta} \right) h_{\alpha\beta} \right. \\ & \left. + 2 \bar{\Psi}_\mu \not{p} \left(P_{3/2}^{\mu\nu} - 2 P_{1/2}^{\mu\nu} \right) P_L \Psi_\nu + \frac{1}{3} V_\mu V^\mu \right]. \end{aligned} \quad (4.17)$$

The longitudinal parts of the graviton and gravitino propagators are irrelevant. It is thus convenient using this fact to choose the longitudinal part in such a way as to reconstruct for each propagator the appropriate projection operator, respectively $P_2^{\mu\nu\alpha\beta}$ and $P_{3/2}^{\mu\nu}$. The vector propagator, happily, is already proportional to the projection operator $P_1^{\mu\nu}$. Furthermore, the mass insertion in the gravitino propagator drops in the diagrams because of the P_L projectors at the vertices. In practice, one can therefore use the following propagators:

$$\Delta_{(h)}^{\mu\nu\alpha\beta} = P_2^{\mu\nu\alpha\beta} \Delta, \quad \Delta_{(\Psi)}^{\mu\nu} = \frac{1}{2} \not{p} P_{3/2}^{\mu\nu} \Delta, \quad \Delta_{(V)}^{\mu\nu} = 3 p^2 P_1^{\mu\nu} \Delta. \quad (4.18)$$

where

$$\Delta = \frac{1}{2\pi R} \sum_{n=-\infty}^{\infty} \frac{i}{p^2 - m_n^2}. \quad (4.19)$$

Since $P_1 \perp P_2$ and $P_{1/2} \perp P_{3/2}$, the quartic vertex acting on the graviton and gravitino propagators is just proportional to respectively P_2 and $P_{3/2}$. The effective potential is then easily computed by resumming insertions in the graviton, gravitino and graviphoton vacuum diagrams. One finds

$$\begin{aligned} W_{h+\psi+A}(\rho) &= -\frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \sum_{k=1}^{\infty} \frac{(-i\rho p^2)^k}{k} \text{Tr} \left[(P_2 \Delta)^k - 2(P_{3/2} P_R \Delta)^k + (P_1 \Delta)^k \right] \\ &= \left(\text{Tr} P_2 - \text{Tr} P_{3/2} + \text{Tr} P_1 \right) \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \ln \left[1 + i\rho p^2 \Delta \right]. \end{aligned} \quad (4.20)$$

The vanishing of the one-loop effective potential is thus a direct consequence of the standard balancing of degrees of freedom in supergravity: $\text{Tr} P_2 - \text{Tr} P_{3/2} + \text{Tr} P_1 = 5 - 8 + 3 = 0$.

The quantity multiplied by this coefficient is easily recognized to be the effective potential induced by a real scalar field φ , corresponding to a single degree of freedom, with the following Lagrangian:

$$\mathcal{L}_\varphi = \partial_M \varphi \partial^M \varphi + \rho \delta(x^5) \partial_\mu \varphi \partial^\mu \varphi. \quad (4.21)$$

Indeed, defining $f_n = i/(p^2 - m_n^2)$, in terms of which $\Delta = (2\pi R)^{-1} \sum_n f_n$, one computes

$$\begin{aligned} W_\varphi(\rho) &= \frac{1}{2} \ln \det \left[1 - \rho \delta(x^5) \frac{\partial_\mu \partial^\mu}{\partial_M \partial^M} \right] = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \ln \det_{\text{KK}} \left[\delta_{n,n'} + \frac{i\rho p^2}{2\pi R} f_n \right] \\ &= \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \ln \left[1 + \frac{i\rho p^2}{2\pi R} \sum_n f_n \right] = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \ln \left[1 + i\rho p^2 \Delta \right]. \end{aligned} \quad (4.22)$$

The determinant over the infinite KK modes (needed in the third equality) is most easily computed by considering recursively finite truncations of increasing dimensionality.

5 Scherk–Schwarz supersymmetry breaking

We want to consider a situation where supersymmetry is broken by the VEV of the radion auxiliary field. As argued in [30], this case corresponds to Scherk–Schwarz supersymmetry breaking. This correspondence has been further elucidated in [24] by considering the off-shell formulation of 5D supergravity. Furthermore the same supersymmetry-breaking spectrum has been obtained in [31] by considering constant superpotentials localized at the fixed points. The latter realization can be simply understood in the effective field theory. The boundary term leads to a constant 4D superpotential, so that eq. (3.14) becomes

$$\mathcal{L}^{\text{eff}} = \left[(T + T^\dagger) \left(\sqrt{S_0 S_0^\dagger} + \frac{1}{2} \right) \right]_D + P [S_0]_{\mathcal{F}} + P^* [S_0]_{\mathcal{F}}^\dagger, \quad (5.1)$$

corresponding to the following structure as far as the auxiliary fields are concerned:

$$\mathcal{L}_{F_{S_0, T}}^{\text{eff}} = (T + T^*) |F_{S_0}|^2 + (F_T + P^*) F_{S_0}^* + (F_T^* + P) F_{S_0}. \quad (5.2)$$

Solving the auxiliary equations of motion we find the standard no-scale result: $F_{S_0} = 0$, $F_T = -P^*$, with the scalar potential exactly zero for any T .

For the purpose of our calculation, as will become clear below, it is important to understand in some detail the way F_T is generated in the full 5D theory. From the discussion in section 3 we have

$$F_T = \frac{1}{2} \int_{-\pi}^{\pi} dx^5 \left[E_5^{\dot{5}} \right]_F = \frac{1}{2} \int_{-\pi}^{\pi} dx^5 \left[V_5^1 + iV_5^2 + 4e_5^{\dot{5}}(it^1 - t^2) \right]. \quad (5.3)$$

Note that all components of $E_5^{\dot{5}}$ can be locally gauged away, so that when $F_T \neq 0$ supersymmetry is broken by global effects at the compactification scale. This is very similar to what happens for a U(1) gauge symmetry in the presence of localized Fayet–Iliopoulos terms [19]. We are interested in the situation in which F_T is the only auxiliary with non-zero VEV. Therefore, t^1 and t^2 , which are part of the gravitational multiplet, should vanish and we have just $F_T \propto V_5^1 + iV_5^2$. To generate F_T we add boundary superpotentials

[31] in our off-shell formulation. The superpotential being a complex object, there are two independent real covariant densities that we can write at each boundary [23, 18]:

$$\text{Re}[S_0]_F = \frac{1}{2}\bar{\Psi}_a\gamma^{ab}(Y^1\tau_1 + Y^2\tau_2)\Psi_b - 2N - 12(Y^2t^2 + Y^1t^1) + D_5^3Y^3 + \dots, \quad (5.4)$$

$$\text{Im}[S_0]_F = -\frac{1}{2}\bar{\Psi}_a\gamma^{ab}(Y^1\tau_2 - Y^2\tau_1)\Psi_b + 2W^5 + \dots. \quad (5.5)$$

In both equations the dots indicate ρ -dependent terms, which trivially vanish on shell and can thus be discarded. $\text{Re}F_{S_0}$ and $\text{Im}F_{S_0}$ are fairly different objects when written in terms of 5D fields, see eqs. (3.5) and (3.6). There are therefore important technical differences in working out the implications of adding the $\text{Re}F$ and the $\text{Im}F$ terms. In the next two subsections we will study the two cases separately.

5.1 Generating V_5^2

Let us consider adding to the action a superpotential term

$$\mathcal{L}_\epsilon = -P_\epsilon(x^5)\text{Im}[S_0]_F, \quad (5.6)$$

where

$$P_\epsilon(x^5) = 2\pi\epsilon_0\delta(x^5) + 2\pi\epsilon_\pi\delta(x^5 - \pi). \quad (5.7)$$

Using eq. (3.1) and writing eq. (5.5) in terms of B_{MNR} and Ψ_M , we immediately encounter a problem. The gravitino bilinear cancels out and what remains is just a total derivative:

$$\text{Im}[S_0]_F = \frac{1}{6}\epsilon^{\dot{5}\mu\nu\rho\sigma}\partial_\mu B_{\nu\rho\sigma}. \quad (5.8)$$

Naively this term is trivial, although a more correct statement is that it is topological, as it can be formally associated to an integral at the boundary of our 4D space (not the boundaries of the orbifold!). This result indicates that, as it stands, the off-shell Lagrangian with a tensor multiplet compensator of ref. [18] is not fully adequate to describe this particular superpotential. In deriving the Lagrangian no attention was paid to total derivative terms. Now, it is known that, for certain auxiliary formulations of supergravity, some ways of breaking supersymmetry are triggered by global, instead of local, charges¹³. The basic point is that the set of auxiliary fields we are using is perfectly fine locally, but there can be physical situations where a global definition of our fields, in particular B_{MNR} , is impossible and our set of fields inadequate. This is the analogue of what happens for monopole configurations of a gauge vector field. These are the situations where there is a non-zero 4D-flux for dB . This may not come as a big surprise. The tensor B was originally introduced to locally solve the constraint on the vector of a linear multiplet. After gauge-fixing, this constraint reads:

$$\partial_M W^M + \partial_M J_\Psi^M = 0, \quad (5.9)$$

in terms of the $U(1)_{T_2}$ gravitino current

$$J_\Psi^M = -\frac{1}{4}\bar{\Psi}_A\gamma^{AMB}\tau_2\Psi_B. \quad (5.10)$$

¹³We thank C. Kounnas for pointing this out to us.

Using the language of differential forms, eq. (5.9) reads $d^*(W + J_\Psi) = 0$, and this is solved by eq. (3.1) with $\rho = 0$: $W = -J_\Psi + \frac{1}{12} *dB$. When the space has non-trivial 4-cycles this parametrization is missing the closed 4-forms ω , which are not exact $\omega \neq dB$, but which are perfectly acceptable solutions of the constraint. Fortunately, for the purpose of our computations, this lack of completeness does not pose any serious limitations. This will become clear in the following discussion. It would nevertheless be very interesting to address this issue within the potentially more powerful formalism developed in [32, 33].

From inspection of the low energy effective theory, eq. (5.2), the superpotential $\text{Im } F$ -term we are considering would correspond to imaginary P and would induce a VEV for $\text{Im } F_T \propto V_5^2$. The terms in the bulk Lagrangian (eq. (3.11)) that are relevant to a discussion of the VEV of V_5^2 and its consequences are

$$\mathcal{L} = -\frac{i}{2} \bar{\Psi}_M \gamma^{MNP} D_N \Psi_P + V_M^2 J_\Psi^M - \frac{1}{12} \epsilon^{ABMNP} \partial_A V_B^2 B_{MNP} + W_A W^A. \quad (5.11)$$

Integrating by parts and using the definition (3.1) of W^M (with $\rho = 0$), this equation becomes

$$\mathcal{L} = -\frac{i}{2} \bar{\Psi}_M \gamma^{MNP} D_N \Psi_P + W^A V_A^2 + W^A W_A. \quad (5.12)$$

Note that the coupling between V_M^2 and the gravitino no longer shows up explicitly. Eq. (5.12) has precisely the structure of the no-scale Lagrangian (5.2); in particular there is no $V_M^2 V^{M2}$ term. If we treat W_M as an independent field, the sum of eqs. (5.12) and (5.6), integrated over x^5 and reduced to the zero modes $\text{Im } F_T = \pi V_5^2$ and $\text{Im } F_{S_0} = 2W^5$, agrees perfectly with eq. (5.2). However, from the 5D point of view W^M cannot be independent, otherwise the gauge symmetry $U(1)_{T_2}$ would be explicitly broken; the constraint (5.9) represents precisely the condition for (5.12) to be gauge-invariant. These considerations will matter in moment. Before then, let us consider the equations of motion that follow from the unconstrained fields in eq. (5.11). The equation for V_A^2 gives $W^A = 0$, so that the equation for B_{MNR} implies $\partial_M V_N^2 - \partial_N V_M^2 = 0$. Up to gauge transformations, the resulting class of solutions is conveniently parametrized by a constant $V_5^2 = 2\epsilon$ [18, 24]. Indeed the corresponding gauge-invariant Wilson line operator is $e^{i\oint dx^5 V_5^2 \tau_2/2}$ so that physics should be unchanged by the shift $\epsilon \rightarrow \epsilon + 1$. Going back to the explicit Lagrangian, eq. (5.11), we find a gravitino mass $\propto \epsilon$, in correct relation with F_T , see eq. (5.17)¹⁴. The existence of this family of supersymmetry-breaking solutions is in direct correspondence with eq. (5.6) being a total derivative. The compensator auxiliary field W^5 is not the most general scalar: it is basically the field strength of a 3-form (modulo the gravitino term). The variation of W^5 imposes a slightly weaker constraint than usual. The actual value of ϵ cannot be decided with the sole use of our local description: the source of ϵ is a global

¹⁴Actually there is a subtlety in deriving this mass term, which is related to the fact that our formulation is not completely satisfactory at the global level. The equation of motion of V_2 sets $W = 0$, i.e. $*dB = 12J_\Psi$. This does not just fix the value of the auxiliary field B : when integrated over a 4D surface, it also gives the condition $\int_4 J_\Psi^5 = 0$. Since this is a constraint on the physical fields, the Lagrangian obtained by substituting the solution for the auxiliary fields would miss the terms associated to the constraint: substituting $W = 0$ into eq. (5.12), one finds no gravitino mass term at all. The correct procedure is to first derive the equation of motion of the gravitino, and then solve the constraint from the auxiliary fields.

flux. However, for the purpose of our computation all we need is a *locally* consistent way to generate $F_T \neq 0$. We have just shown that the *local* Lagrangian [18] we use admits automatically these solutions, although it formally lacks the *global* degrees of freedom needed to associate ϵ to a Lagrangian parameter (a charge)¹⁵.

The bottom line of the above discussion is that, in terms of W^M , the 5D Lagrangian looks precisely like what one would have liked, and reproduces nicely the 4D structure. But in terms of B_{MNR} there are differences. In fact if one could do without B_{MNR} and just work with a constrained W^M , these issues would not arise: the most general $*W$ includes closed forms with non-zero flux. Unfortunately the fully off-shell Lagrangian (3.10) cannot be written just in terms of W^M , not even after integrating by parts. Indeed all the obstruction is coming from the second to last term in eq. (3.10). This problem is fully analogous to the case of $\mathcal{N} = 2$ supergravity in 4D, which was discussed in ref. [35]. The basic remark is that the Lagrangian can be written in terms of W , but at the price of losing manifest $SU(2)_R$ invariance. Now, after gauge-fixing $Y_i \propto \delta_i^2$ the obstructive term in eq. (3.10) vanishes, and we can write the Lagrangian just in terms of W^M , i.e. eq. (5.12). As in ref. [35] we can enforce the constraint on W^M by adding a Lagrange multiplier X

$$\mathcal{L}_X = \partial_M X (W^M + J_{\Psi}^M) . \quad (5.13)$$

Note that X shifts under $U(1)_{T_2}$ gauge rotations, restoring invariance of the unconstrained Lagrangian (5.12). Now, the addition of eqs. (5.6), (5.12) and (5.13) leads to the equations of motion $W^M = 0$ and $V_M^2 = 2P_\epsilon(x^5)\delta_M^5 + \partial_M X$. The latter equation is manifestly gauge-invariant, and fixes just the VEV of the Wilson line. Defining $\epsilon = \epsilon_1 + \epsilon_2$ we find:

$$\oint dx^5 V_5^2 = 2 \oint dx^5 P_\epsilon(x^5) = 4\pi\epsilon . \quad (5.14)$$

Notice also that on shell the gravitino mass is still determined by V_5^2 . So a convenient gauge to study the gravitino spectrum is the one in which $V_5^2 = 2\epsilon$ is constant, and there are no δ -function terms. We will better explain below the advantages of working in a gauge with no δ -function terms. We see that in this approach with W instead of B we end up with the same conclusions.

Let us now study the spectrum and the wave function of the gravitino in the presence of the Wilson line. With a constant $V_5^2 \neq 0$, the zero mode of ψ_5^2 plays the role of the Goldstino, so it can be gauged away. The gravitinos can be described through a doublet of Weyl spinors $\chi_\mu = (\chi_\mu^1, \chi_\mu^2)^T$. They can then be decomposed as $\chi^\mu(x^5) = \sum_n \xi_n(x^5)\chi_n^\mu$ in terms of Weyl KK modes χ_n^μ and the standard wave functions

$$\xi_n(x^5) = \begin{pmatrix} \cos nx^5 \\ -\sin nx^5 \end{pmatrix} . \quad (5.15)$$

¹⁵In ref. [34] the point of view was taken that the one-loop effective action should be minimized with respect to ϵ . However, as V_M^2 satisfies the equation $\partial_N V_M^2 - \partial_M V_N^2 = 0$, there is no local propagating degree of freedom associated to the VEV $V_5^2 = 2\epsilon$. There is no dynamics that can make ϵ evolve locally, and we do not fully understand the meaning of that minimization. Our viewpoint is that, in the correct treatment, V_5^2 should be fixed at tree level by a global charge, so that the issue of minimizing in ϵ should not arise.

The mass eigenstates are Majorana KK modes defined as $\psi_\mu^n = (\chi_\mu^n, \bar{\chi}_\mu^n)^T$, with masses given by

$$m_n(\epsilon) = \frac{n - \epsilon}{R} . \quad (5.16)$$

Notice that the periodicity $\epsilon \rightarrow \epsilon + 1$ is respected. Moreover, for the $n = 0$ mode, which for $\epsilon \ll 1$ represents the 4D gravitino, we reproduce the well-known relation of the no-scale model

$$m_{3/2} = \left| \frac{F_T}{T + T^*} \right| = \left| \frac{\pi V_5^2}{2\pi R} \right| = \left| \frac{\epsilon}{R} \right| . \quad (5.17)$$

Also note that the wave functions of the modes are unaffected by supersymmetry breaking. In particular they are smooth at the boundaries. It will become clear below why this matters.

The above scenario is shown to be equivalent to Scherk–Schwarz supersymmetry breaking by performing a multi-valued $U(1)_{T_2}$ gauge transformation $e^{-2i\alpha_2 T_2}$ to eliminate V_5^2 , in the spirit of ref. [36]. This is achieved with $\alpha_2(x^5) = \epsilon x^5$. In this new basis, $V_5'^2 = V_5^2 - 2\epsilon = 0$, but the new charged fields $\phi' = e^{-2i\epsilon x^5 T_2} \phi$ get twisted boundary conditions. Defining the matrices $U(\epsilon) = e^{-4i\epsilon T_2}$ and $Z = \eta_\phi(-1)^{T-T_3}$ for any given multiplet ϕ with isospin T and overall parity η_ϕ , the new boundary conditions are

$$\phi'(x^5 + 2\pi) = U(\epsilon)\phi'(x^5) , \quad \phi'(-x^5) = Z\phi'(x^5) . \quad (5.18)$$

The $SU(2)_R$ group algebra ensures that the consistency condition $ZU(\epsilon)Z = U(\epsilon)^{-1}$ is automatically satisfied.

In the primed basis it is manifest that supersymmetry is broken non-locally. It amounts to the fact that the two different fixed points locally preserve different combinations of the supercharges. Indeed, the reflection condition of the new gravitino around each fixed point $x^5 = k\pi$ involves a different matrix $Z(\epsilon, k) = e^{-ik\pi\epsilon\tau_2}\tau_3 e^{ik\pi\epsilon\tau_2}$ and reads

$$\chi'_\mu(k\pi + y) = Z(\epsilon, k)\chi'_\mu(k\pi - y) . \quad (5.19)$$

The supersymmetry that is locally preserved at $x^5 = k\pi$ is aligned with the $+1$ eigenvalue of $Z(\epsilon, k)$. The fields diagonalizing the latter are nothing but the gravitinos in the unprimed basis, $e^{ik\pi\epsilon\tau_2}\chi'_\mu(k\pi) = \chi_\mu(k\pi)$, and the combination of gravitinos associated with the supersymmetry preserved at $x^5 = k\pi$ is thus $\cos k\pi\epsilon\chi_\mu^1(k\pi) + \sin k\pi\epsilon\chi_\mu^2(k\pi) = \chi_\mu^1(k\pi)$. Therefore, working in the Scherk–Schwarz picture, i.e. in the primed basis, one has to be careful to use the right combination of the two gravitinos when writing boundary actions. The couplings are straightforward in the unprimed basis where the fields are single valued. In this basis the wave functions of the appropriate gravitino components at each fixed point are therefore given by:

$$\chi^\mu(0) = \sum_{n=-\infty}^{\infty} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \chi_n^\mu , \quad \chi^\mu(\pi) = \sum_{n=-\infty}^{\infty} \begin{pmatrix} 1 \\ 0 \end{pmatrix} (-1)^n \chi_n^\mu . \quad (5.20)$$

Notice that no dependence on ϵ appears in these couplings. In ref. [20] this dependence was not eliminated, leading, in general, to incorrect results. We will have more to say on this issue in the next section.

Summarizing: *the net effect of a Scherk–Schwarz twist in the five-dimensional theory amounts to a shift in the masses of the gravitino KK modes.* The gravitino wave functions that determine the couplings to the fixed points are instead insensitive to the twist and coincide with those of the supersymmetric case; the n -th mode has therefore wave function 1 at $x^5 = 0$ and $(-1)^n$ at $x^5 = \pi$.

5.2 Generating V_5^1

Consider now the addition of a superpotential $\mathcal{L}_\eta = -P_\eta(x^5)\text{Re}[S_0]_F$, with $\text{Re}[S_0]_F$ given by eq. (5.4) and

$$P_\eta(x^5) = 2\pi\eta_0 \delta(x^5) + 2\pi\eta_\pi \delta(x^5 - \pi) . \quad (5.21)$$

In the gauge $Y^{1,3} = 0$, $Y^2 = 1$, eq. (5.4) reproduces the structure of eq. (5.1) for real P . The auxiliary field N is an ordinary scalar, so that the problem of the previous section does not arise. Let us then consider the equations of motion in the presence of this superpotential. As we already discussed in section 3.3, by introducing $N_\pm = N + 6t^2 \pm \frac{1}{2}V_5^1$, the auxiliary fields do not contribute any term to the on-shell action, because their equation of motion implies $N_- = 0$. In terms of the original fields t^2 , V_5^1 and N , one finds $t^2 = 0$, $V_5^1 = 2P_\eta(x^5)$ and $N = P_\eta(x^5)/R$. In particular, a non-vanishing VEV for the zero mode of V_5^1 is generated; defining $\eta = \eta_0 + \eta_\pi$, we have

$$\oint dx^5 V_5^1 = 2 \oint dx^5 P_\eta(x^5) = 4\pi\eta . \quad (5.22)$$

Let us now study the gravitino spectrum and wave functions. The mass operator for the doublet of Weyl spinors $\chi_\mu = (\chi_\mu^1, \chi_\mu^2)^T$ describing the gravitino is given by the matrix

$$M = \begin{pmatrix} -iP_\eta(x^5) & -\partial_5 \\ \partial_5 & 0 \end{pmatrix} . \quad (5.23)$$

This leads to singularly behaved wave functions at the boundary [31]: χ_μ^1 has a cusp and χ_μ^2 is discontinuous. As was the case in the Scherk–Schwarz example, this situation leads to ambiguities when trying to decide which combination couples to matter at the boundary. Note, however, that this singular behaviour comes along with a singular profile $V_5^1(x^5) = 2P_\eta(x^5)$. Therefore, a natural guess is that by going to a gauge in which V_5^1 is smooth, the gravitinos will also be smooth, and their interactions straightforward. This is indeed what happens. What we need is a $U(1)_{T_1}$ rotation $e^{-2i\alpha_1 T_1}$ with a parameter α_1 such that V_5^1 is made constant. One finds

$$\alpha_1(x^5) = -\epsilon(x^5) [\eta_0(|x^5| - \pi) + \eta_\pi|x^5|] , \quad (5.24)$$

where $\epsilon(x^5)$ is the completely odd step function, which jumps from -1 to 1 at $x^5 = 2k\pi$ and from 1 to -1 at $x = (2k+1)\pi$. Notice that α_1 is defined to be single-valued on the circle, although it is discontinuous at the fixed points. This should be contrasted to the improper gauge transformation of the previous section. In the new gauge, the gravitino is transformed to a new field Ψ'_μ , the gauge field is shifted to

$$V_5'^1 = V_5^1 - 2\partial_5\alpha_1 = 2\eta , \quad (5.25)$$

and Y_2 and Y_3 are rotated to

$$Y'^2 = \cos 2\alpha_1 Y^2 - \sin 2\alpha_1 Y^3 = \cos 2\alpha_1 , \quad (5.26)$$

$$Y'^3 = \cos 2\alpha_1 Y^3 + \sin 2\alpha_1 Y^2 = \sin 2\alpha_1 . \quad (5.27)$$

In order to compute the gravitino mass term in this new gauge, one has to go back to eq. (3.10), where invariance under the full $SU(2)_R$ is still manifest. In particular, one has to consider the last term in eq. (3.10), which, because of the non-constant profile for Y^i , gives rise to an extra contribution to the gravitino mass operator¹⁶

$$\mathcal{L}_m^{\text{bulk}} = -\frac{1}{4}\bar{\Psi}'_a\gamma^{ab5}\tau_1\Psi'_b\left(Y'^2\partial_5 Y'^3 - Y'^3\partial_5 Y'^2\right) = -\frac{1}{2}\partial_5\alpha_1\bar{\Psi}'_a\gamma^{ab5}\tau_1\Psi'_b . \quad (5.28)$$

The value of Y'^i at the fixed points is more subtle. At first sight $Y'^2 = \cos 2\alpha_1$ would seem continuous, even though α_1 flips sign at the boundaries. Then one would conclude that, in eq. (5.4), one should use $Y'^2(0) = \cos \pi\eta_0$ and $Y'^2(\pi) = \cos \pi\eta_\pi$. However, this simple reasoning is incorrect. The point is that Y^2 , strictly at the fixed points, is invariant under $U(1)_{T_1}$. This is because the orbifold projection breaks $SU(2)_R$ down to $U(1)_{T_3}$ and $U(1)_{T_1}$ is not active at the boundaries. Therefore, based on gauge invariance, we must impose $Y'^2(0) = Y'^2(\pi) \equiv 1$. This is equivalent to taking $\alpha_1(0) = \alpha_1(\pi) \equiv 0$, which makes qualitative sense since α_1 is on average zero at the fixed points. Y'^2 is discontinuous at the fixed points, even though, being even, it has the same limit when approaching the fixed points from opposite sides. In the end we must take $Y'^2 = 1$ and $Y'^3 = 0$ at the fixed points, even though they rotate in the bulk. The mass term induced by the boundary superpotential is then given by

$$\mathcal{L}_m^{\text{bound}} = \frac{1}{2}P_\eta(x^5)\bar{\Psi}'_a\gamma^{ab}\tau_2\Psi'_b . \quad (5.29)$$

The total mass term is found by adding (5.28) and (5.29). Using the fact that $i\tau_3\gamma^5 = 1$ on the gravitino at the boundary, one can verify that the contributions in (5.28) that are localized at the boundaries exactly cancel (5.29), and only a constant bulk mass term is left:

$$\mathcal{L}_m = \frac{\eta}{2}\bar{\Psi}'_a\gamma^{ab5}\tau_1\Psi'_b . \quad (5.30)$$

The most appropriate basis of KK wave functions is in this case obtained from the standard one through a $U(1)_{T_1}$ rotation that diagonalizes the constant bulk mass terms (5.30):

$$\xi'_n(x^5) = \begin{pmatrix} e^{i\pi/4} \cos nx^5 \\ -e^{-i\pi/4} \sin nx^5 \end{pmatrix} . \quad (5.31)$$

This leads to the following mass eigenvalues:

$$m_n = \frac{n + \eta}{R} . \quad (5.32)$$

Again, the mass of the lightest mode agrees with eqs. (5.25) and (5.17). In the case where both auxiliaries V_5^1 and V_5^2 are turned on, the parameter describing the twisted gravitino spectrum becomes $|\eta + i\epsilon| \propto |V_5^1 + iV_5^2|$.

¹⁶Note that at the same time the covariant derivative \mathcal{D}'_M in eq. (3.11) will involve the rotated vector $V_M'^2 = \cos 2\alpha_1 V_M^2 - \sin 2\alpha_1 V_M^3$. This does not influence the gravitino mass since both V_M^2 and V_M^3 are zero.

If we rotate the eigenmodes back to the original gauge, their wave functions become $\xi_n(x^5) = e^{i\alpha_1(x^5)\tau_1}\xi'_n(x^5)$. From this expression, and from the rule $\alpha_1(0) = \alpha_1(\pi) \equiv 0$, we deduce that χ^1 not only has a cusp but is truly discontinuous at the fixed points. This suggests that a derivation of the spectrum based on the operator (5.23) assuming continuity of χ^1 is flawed. Indeed under these assumptions we would get different eigenvalues that do not satisfy the periodicity under $\eta \rightarrow \eta + 1$:

$$m_n = \frac{n + \arctan \eta_0 + \arctan \eta_\pi}{R}. \quad (5.33)$$

Also note that in the singular basis it is the continuous combination of χ^1 and χ^2 that couples to the boundary. In ref. [20] this point was missed: only χ^1 was coupled in the computations. As a consequence, the coupling of the gravitinos to matter at $x^5 = 0, \pi$ was weighted by the wave-function factor $\cos \eta_{0,\pi}\pi$. This way if one were to repeat the computation of section 4, one would find that the supersymmetric cancellation is spoiled and that the scalar masses are UV-divergent. Note that this disaster would also survive the case $\eta_0 = -\eta_\pi \neq 0$ in which half of the supercharges are preserved (there is a killing spinor [18]) and not even a finite scalar mass is tolerated. We believe that our approach makes it clear how to avoid these errors.

6 One-loop effective action

We can now compute the one-loop correction to the Kähler potential. We consider a ‘visible sector’ consisting of a chiral multiplet Φ_0 with kinetic function $\Omega_0(\Phi_0, \Phi_0^\dagger)$ localized at $x^5 = 0$ and a ‘hidden sector’ consisting of a chiral multiplet Φ_π with kinetic function $\Omega_\pi(\Phi_\pi, \Phi_\pi^\dagger)$ at $x^5 = \pi$. In the notation of section 3, the 5D microscopic theory is described at tree level by the generalized kinetic function

$$\Omega(x^5) = -\frac{3}{2} + \Omega_0(\Phi_0, \Phi_0^\dagger)e_5^5 \delta(x^5) + \Omega_\pi(\Phi_\pi, \Phi_\pi^\dagger)e_5^5 \delta(x^5 - \pi). \quad (6.1)$$

The 4D low-energy effective supergravity theory obtained by integrating out all the massive KK modes is then specified at leading order by $\Omega = \int_{-\pi}^{\pi} dx^5 e_5^5 \Omega(x^5)$, which leads to eq. (2.1). As explained in section 2, one-loop diagrams involving the massive supergravity KK modes will induce a correction $\Delta\hat{\Omega}$ mixing the two sectors. This $\Delta\hat{\Omega}$ can be fully reconstructed by calculating the scalar potential ΔV in a background with $F_T \neq 0$. For definiteness we can consider the case discussed in subsection 5.1, where $F_T = \pi V_5^2 = 2\pi\epsilon$ and the gravitino KK modes are already well behaved in the original basis.

In the presence of a non-zero $F_T = 2\pi\epsilon$, the component expansion for the D -term of the correction

$$\Delta\hat{\Omega} = \frac{A}{(T + T^\dagger)^2} + B \frac{\Phi_0\Phi_0^\dagger + \Phi_\pi\Phi_\pi^\dagger}{(T + T^\dagger)^3} + C \frac{\Phi_0\Phi_0^\dagger\Phi_\pi\Phi_\pi^\dagger}{(T + T^\dagger)^4} + \dots \quad (6.2)$$

leads to the scalar potential

$$\begin{aligned} \Delta V &= -|F_T|^2 \frac{\partial^2 \Delta\hat{\Omega}}{\partial(T + T^\dagger)^2} \\ &= -\frac{3A\epsilon^2}{2\pi^2 R^4} - \frac{3B\epsilon^2}{2\pi^3 R^5} (|\phi_0|^2 + |\phi_\pi|^2) - \frac{5C\epsilon^2}{4\pi^4 R^6} |\phi_0|^2 |\phi_\pi|^2 + \dots \end{aligned} \quad (6.3)$$

Each of the interactions in ΔV receives contributions from many diagrams, adding up to zero in the supersymmetric limit $\epsilon \rightarrow 0$. However, only the gravitino spectrum is affected by ϵ ; the graviton and graviphoton are unaffected by supersymmetry breaking. It is thus enough to compute the contribution of gravitino diagrams $\Gamma_\Psi(\epsilon)$. The potential is then simply given by $\Delta V(\epsilon) = \Delta V_\Psi(\epsilon) - \Delta V_\Psi(0)$. As discussed in section 4, all diagrams involving cubic gravitino vertices vanish at zero momentum, so that there is a single relevant diagram for each operator in $\Delta\Omega$, involving the quartic vertex, as depicted in Fig. 3.

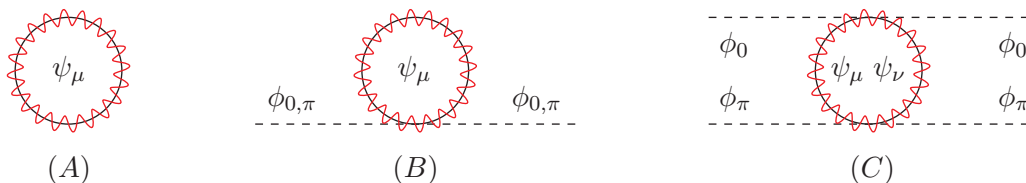


Figure 3: *Diagrams controlling (A) Casimir energy, (B) radion mediation and (C) brane to brane mediation of supersymmetry breaking.*

Since supersymmetry is broken, all the gravitino modes are massive. For this reason it is convenient to work with Majorana spinors $\psi_i^\mu = (\chi_i^\mu, \bar{\chi}_i^\mu)^T$. The relevant interaction, which is nothing but the standard field-dependent (localized) gravitino kinetic term, is then written as

$$\mathcal{L}^{\text{int}} = \frac{1}{3} \left[\Omega_0(\phi_0, \phi_0^*) \delta(x^5) + \Omega_\pi(\phi_\pi, \phi_\pi^*) \delta(x^5 - \pi) \right] i e_4 \bar{\psi}_\mu \gamma^{\mu\nu\rho} \partial_\nu \psi_\rho. \quad (6.4)$$

In this case it is convenient to work in the unitary gauge $\psi_i^5 = 0$ to decouple completely the Goldstinos. The gravitino propagator for a mode of mass m is then given by the ordinary propagator for a massive gravitino [29]:

$$\langle \psi_\mu \bar{\psi}_\nu \rangle_n = \left[\eta_{\mu\nu} - \frac{p_\mu p_\nu}{m_n^2} - \frac{1}{3} \left(\gamma_\mu - \frac{p_\mu}{m_n} \right) \left(\gamma_\nu - \frac{p_\nu}{m_n} \right) \right] \frac{i}{\not{p} + m_n}. \quad (6.5)$$

As in section 4, the longitudinal part of the propagator is irrelevant for the amplitudes with vanishing external momentum that we are interested in.

The diagrams that we have to compute consist of gravitinos propagating between interaction vertices localized at the fixed points. Therefore the sums over virtual KK modes n reconstruct in position space the propagator between the two fixed points. More precisely, each vertex occurring at the fixed point located at $x^5 = k\pi$ comes with a wavefunction factor $e^{ink\pi}$, and the sum over KK modes in a propagator connecting two fixed points separated by a distance d will therefore be weighted by a factor e^{ind} . After going to Euclidean space and performing the trace over spinor indices, all the diagrams can be reexpressed in terms of the basic quantity $G_d(p, \epsilon) = (2\pi R)^{-1} \sum_n e^{ind} / (p - i m_n(\epsilon))$. The distance d is 0 when the propagation is from a fixed point to itself, and π when the

propagation is instead from one fixed point to the other. The two relevant quantities are thus

$$G_0(p, \epsilon) = \frac{1}{2\pi R} \sum_{n=-\infty}^{\infty} \frac{1}{p - i m_n(\epsilon)} = \frac{1}{2} \coth \pi(pR + i\epsilon), \quad (6.6)$$

$$G_\pi(p, \epsilon) = \frac{1}{2\pi R} \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{p - i m_n(\epsilon)} = \frac{1}{2} \operatorname{csch} \pi(pR + i\epsilon). \quad (6.7)$$

6.1 Models without localized kinetic terms

Let us first examine the simplest situation, in which the boundary terms L_0 and L_π are set to zero in (2.5) and (2.6). In this case, we can use the standard bulk gravitino propagator to compute the diagrams of Fig. 3.

Consider first the vacuum diagram A . Its standard expression as the trace of the logarithm of the kinetic operator can be rewritten in terms of (6.6), thanks to an integration by parts which isolates the divergent ϵ -independent part:

$$\begin{aligned} \Delta V_\Psi^A(\epsilon) &= -8 \frac{1}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4} \ln[p^2 + m_n^2(\epsilon)] \\ &= \operatorname{Div.} + 2\pi R \int \frac{d^4 p}{(2\pi)^4} p \operatorname{Re} [G_0(p, \epsilon)]. \end{aligned} \quad (6.8)$$

The total vacuum amplitude is given by $\Delta V_A(\epsilon) = \Delta V_\Psi^A(\epsilon) - \Delta V_\Psi^A(0)$, so that we are left with a finite momentum integral which is easily evaluated:

$$\Delta V_A(\epsilon) = \frac{3}{16\pi^6 R^4} \left[\operatorname{Re} \operatorname{Li}_5(e^{2\pi i \epsilon}) - \zeta(5) \right] \simeq -\frac{3\zeta(3)\epsilon^2}{8\pi^4 R^4}. \quad (6.9)$$

In the last step, we have used $\operatorname{Re} \operatorname{Li}_5(e^{2\pi i \epsilon}) \simeq \zeta(5) - 2\pi^2 \zeta(3) \epsilon^2 + \mathcal{O}(\epsilon^4)$ in the limit $\epsilon \rightarrow 0$. Comparing with eq. (6.3) we get

$$A = \frac{\zeta(3)}{4\pi^2}. \quad (6.10)$$

Consider next the two-point function B . The diagram is easily evaluated, and the result in Euclidean space is given by the following expression:

$$\begin{aligned} \Delta V_\Psi^B(\epsilon) &= \frac{4}{3(2\pi R)} \sum_{n=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4} \frac{p^2}{[p^2 + m_n^2(\epsilon)]} \\ &= \frac{4}{3} \int \frac{d^4 p}{(2\pi)^4} p \operatorname{Re} [G_0(p, \epsilon)]. \end{aligned} \quad (6.11)$$

The total contribution $\Delta V_B(\epsilon) = \Delta V_\Psi^B(\epsilon) - \Delta V_\Psi^B(0)$ is finite and given by:

$$\Delta V_B(\epsilon) = \frac{1}{8\pi^7 R^5} \left[\operatorname{Re} \operatorname{Li}_5(e^{2\pi i \epsilon}) - \zeta(5) \right] \simeq -\frac{\zeta(3)\epsilon^2}{4\pi^5 R^5}. \quad (6.12)$$

Comparing with eq. (6.3), we extract:

$$B = \frac{\zeta(3)}{6\pi^2}, \quad (6.13)$$

in agreement with [20].¹⁷

Consider finally the four-point function C . After some straightforward algebra, the diagram can be simplified to:

$$\begin{aligned}\Delta V_{\Psi}^C(\epsilon) &= \frac{4}{9(2\pi R)^2} \sum_{n,n'=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4} \frac{p^2 [p^2 - m_n(\epsilon)m_{n'}(\epsilon)]}{[p^2 + m_n^2(\epsilon)][p^2 + m_{n'}^2(\epsilon)]} (-1)^{n+n'} \\ &= \frac{4}{9} \int \frac{d^4 p}{(2\pi)^4} p^2 \operatorname{Re} \left[G_{\pi}(p, \epsilon)^2 \right].\end{aligned}\quad (6.14)$$

In this case, the momentum integral is finite even before subtracting the untwisted diagram $\Delta V_{\Psi}^C(0)$, because the loop involves propagation between separated fixed points and therefore cannot shrink to a point. The final result is

$$\Delta V_C(\epsilon) = \frac{5}{48\pi^8 R^6} \left[\operatorname{Re} \operatorname{Li}_5(e^{2\pi i \epsilon}) - \zeta(5) \right] \simeq -\frac{5\zeta(3)\epsilon^2}{24\pi^6 R^6}, \quad (6.15)$$

which yields:

$$C = \frac{\zeta(3)}{6\pi^2}. \quad (6.16)$$

Since A, B, C are positive, if the operators in eq. (6.2) are dominant the radion potential is unbounded from below at $R \rightarrow 0$ and the visible-sector scalars get negative squared masses for any R .

6.2 Models with localized kinetic terms

In the more general situation in which non-vanishing boundary terms L_0 and L_{π} arise in (2.5) and (2.6), the computation is more involved. In this case, one has to dress the diagrams of Fig. 3 with insertions of the boundary kinetic terms for the bulk fields, and resum all of these. This is equivalent to computing the exact one-loop effective potential as a function of Ω_0 and Ω_{π} . In order to perform this computation, it is crucial to use the projection operators defined in section 4 in order to simplify the tensor structure of the interaction and the propagator. Defining for convenience $\rho_{0,\pi} = \frac{1}{3}\Omega_{0,\pi}(\phi_{0,\pi}, \phi_{0,\pi}^*)$, the scalar–scalar–gravitino–gravitino coupling can be written as:

$$\mathcal{L}^{\text{int}} = \left(\rho_0 \delta(x^5) + \rho_{\pi} \delta(x^5 - \pi) \right) \bar{\psi}_{\mu} \not{p} (P_{3/2}^{\mu\nu} - 2P_{1/2}^{\mu\nu}) \psi_{\nu}. \quad (6.17)$$

Since the longitudinal part of the gravitino propagator is irrelevant, it can be conveniently chosen in such a way as to reconstruct the projection operator $P_{3/2}$ in the polarization factor. By doing so, the gravitino propagators between two fixed points separated by a distance $d = 0, \pi$ can be written as

$$\Delta_{d(\psi)}^{\mu\nu} = \not{p} P_{3/2}^{\mu\nu} \Delta_d, \quad (6.18)$$

¹⁷In ref. [20] the result depends on whether the constant superpotential is on the visible or hidden brane. This is because of their incorrect treatment of the gravitino wave function. We are comparing here with their formula for a superpotential at the hidden brane. In this case the gravitino field is smooth at the visible brane and the result of ref. [20] correct.

with

$$\Delta_0 = \frac{1}{2\pi R} \sum_{n=-\infty}^{\infty} \frac{i}{\not{p}(\not{p} + m_n)}, \quad (6.19)$$

$$\Delta_\pi = \frac{1}{2\pi R} \sum_{n=-\infty}^{\infty} \frac{i(-1)^n}{\not{p}(\not{p} + m_n)}. \quad (6.20)$$

The Euclidean versions of these quantities reduce for $\epsilon = 0$ to $-i/p$ times (6.6) and (6.7).

The effective potential is obtained by summing up all the independent diagrams with an arbitrary number of each type of insertion. This task is complicated by the fact that the type of propagators to be used depends on the topology of the diagram, and not just on the number of each type of insertion. The easiest way to figure out the correct combinatoric is then to resum the two kinds of insertions successively. First one computes a dressed propagator that takes into account one type of insertion, say the insertion of ρ_π . Then one uses this propagator to compute diagrams with a given number of the other insertion (insertion of ρ_0) and finally resums the latter.

The effective gravitino propagator between two ρ_0 vertices corrected by insertions of ρ_π vertices is easily computed by using a geometric resummation; the result can be written as $\Delta_{0(\psi)}^{\mu\nu}(\rho_\pi) = \not{p} P_{3/2}^{\mu\nu} \Delta_0(\rho_\pi)$ with

$$\begin{aligned} \Delta_0(\rho_\pi) &= \Delta_0 + \Delta_\pi (-i\rho_\pi) p^2 \Delta_\pi + \Delta_\pi (-i\rho_\pi) p^2 \Delta_0 (-i\rho_\pi) p^2 \Delta_\pi + \dots \\ &= \Delta_0 - i\rho_\pi p^2 \Delta_\pi^2 \left(1 + i\rho_\pi p^2 \Delta_0\right)^{-1}. \end{aligned} \quad (6.21)$$

The full effective potential is then given by the sum of two pieces. The first is the sum of all the diagrams with at least one insertion of ρ_0 , but computed with the dressed propagator $\Delta_0(\rho_\pi)$. The second is the effective potential at $\rho_0 = 0$, which corresponds to all the diagrams with only ρ_π vertices and undressed propagator Δ_0 . The result is:

$$\begin{aligned} W_\Psi(\rho_0, \pi) &= \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \sum_{k=1}^{\infty} \frac{(-i p^2)^k}{k} \text{Tr} \left[\left(\rho_0 P_{3/2} \Delta_0(\rho_\pi) \right)^k + \left(\rho_\pi P_{3/2} \Delta_0 \right)^k \right] \\ &= -P_{3/2\mu}^\mu \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \ln \left[1 + i(\rho_0 + \rho_\pi) p^2 \Delta_0 - \rho_0 \rho_\pi p^4 \left(\Delta_0^2 - \Delta_\pi^2 \right) \right]. \end{aligned} \quad (6.22)$$

The trace over vector indices therefore reduces to $P_{3/2\mu}^\mu = 2$. The trace over spinor indices is less immediate, but can be easily performed as well. Introducing a matrix notation for the propagation between the two types of boundaries, and going to Euclidean space, the final result can be written in terms of the complex propagators (6.6) and (6.7) as

$$W_\Psi(\rho_0, \pi) = -\text{Tr} P_{3/2} \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \text{Re} \ln \det \begin{pmatrix} 1 - \rho_0 p G_0 & -\rho_0 p G_\pi \\ -\rho_\pi p G_\pi & 1 - \rho_\pi p G_0 \end{pmatrix}, \quad (6.23)$$

where $\text{Tr} P_{3/2} = 8$ is the total number of degrees of freedom and the determinant is now only as a 2×2 matrix. The structure of the result is therefore $V = \frac{1}{2} \text{Tr} \ln [1 - p M(\rho_0, \pi)]$, where $M(\rho_0, \pi)$ is a 2×2 matrix encoding the propagation between any pair of fixed points weighted by the appropriate coupling ρ_0 or ρ_π . In more complicated situations with N

distinct fixed points $i = 1, \dots, N$ with couplings ρ_i , M would generalize to the $N \times N$ matrix $M_{ij} = \rho_i G_{ij}$.

As in the simpler case analysed in section 4, the above result is indeed the expected induced effective potential for a theory with twisted boundary conditions and localized kinetic terms. As already explained, the twisting influences only the mass of the gravitino modes, but not the strength of their couplings to the boundaries. For this reason, the main features of the computation are already captured by the untwisted case. The contribution of a single untwisted scalar degree of freedom with Lagrangian

$$\mathcal{L}_\varphi = \partial_M \varphi \partial^M \varphi + \left(\rho_0 \delta(x^5) + \rho_\pi \delta(x^5 - \pi) \right) \partial_\mu \varphi \partial^\mu \varphi \quad (6.24)$$

has been studied in [37]. The result can be reobtained in an alternative and very simple way as a determinant. Using $f_n = i/(p^2 - m_n^2)$, in terms of which the propagators (6.19) and (6.20) for $\epsilon = 0$ read simply $\Delta_0 = (2\pi R)^{-1} \sum_n f_n$ and $\Delta_\pi = (2\pi R)^{-1} \sum_n (-1)^n f_n$, one computes:

$$\begin{aligned} W_\varphi(\rho_0, \pi) &= \frac{1}{2} \ln \det \left[1 - \left(\rho_0 \delta(x^5) + \rho_\pi \delta(x^5 - \pi) \right) \frac{\partial_\mu \partial^\mu}{\partial_M \partial^M} \right] \\ &= \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \ln \det_{\text{KK}} \left[\delta_{n, n'} + \frac{i \rho_0 p^2}{2\pi R} f_n + \frac{i \rho_\pi p^2}{2\pi R} (-1)^{n+n'} f_n \right] \\ &= \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \ln \left[1 + \frac{i(\rho_0 + \rho_\pi) p^2}{2\pi R} \sum_n f_n - \frac{4\rho_0 \rho_\pi p^4}{(2\pi R)^2} \sum_{n, n'} f_{2n} f_{2n'+1} \right] \\ &= \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \ln \left[1 + i(\rho_0 + \rho_\pi) p^2 \Delta_0 - \rho_0 \rho_\pi p^4 \left(\Delta_0^2 - \Delta_\pi^2 \right) \right]. \end{aligned} \quad (6.25)$$

The infinite-dimensional KK determinant in the third step can be computed as before by considering finite-dimensional truncations. Comparing eq. (6.22) with eq. (6.25), we see that the gravitino contributes indeed as -8 times a scalar. The result (6.23) generalizes the results of [37] to arbitrary boundary conditions. A similar computation for gauge fields can be found in [38].

The explicit expressions of the gravitino contribution to the vacuum energy (eq. (6.8)) and to the effective potential (eq. (6.23)), as functions of the supersymmetry-breaking parameter ϵ , are given by

$$E_\Psi(\epsilon) = \text{Div.} - \frac{1}{2\pi^6 R^4} \text{Re} \int_0^\infty dx x^3 \ln \left[\sinh(x + i\pi\epsilon) \right], \quad (6.26)$$

$$W_\Psi(\epsilon) = -\frac{1}{2\pi^6 R^4} \text{Re} \int_0^\infty dx x^3 \ln \left[1 - (\alpha_0 + \alpha_\pi) x \coth(x + i\pi\epsilon) + \alpha_0 \alpha_\pi x^2 \right], \quad (6.27)$$

in terms of the dimensionless parameters

$$\alpha_{0, \pi} = \frac{\rho_{0, \pi}}{2\pi R} = \frac{\Omega_{0, \pi}}{6\pi R}. \quad (6.28)$$

The full effective action $\Delta V(\epsilon)$ is then obtained by subtracting the untwisted contribution, $\Delta V(\epsilon) = [E_\Psi(\epsilon) + W_\Psi(\epsilon)] - [E_\Psi(0) + W_\Psi(0)]$, and reads:

$$\Delta V(\epsilon) = -\frac{1}{2\pi^6 R^4} \text{Re} \int_0^\infty dx x^3 \ln \left[1 - \frac{1 + \alpha_0 x}{1 - \alpha_0 x} \frac{1 + \alpha_\pi x}{1 - \alpha_\pi x} e^{-2(x+i\pi\delta)} \right]_{\delta=0}^{\delta=\epsilon}. \quad (6.29)$$

An alternative expression, which is particularly interesting in the case $L_{0,\pi} = 0$, can be obtained by first expanding the logarithm in power series and then Taylor-expanding the fractions around $\alpha_0 = \alpha_\pi = 0$ in a weak-field approximation. The first step is quite safe, but the second leads to an asymptotic series for the integrated result. Rescaling the integration variable, one finds in this way:

$$\Delta V(\epsilon) = \frac{1}{2\pi^6 R^4} \sum_{k=1}^{\infty} \frac{1}{k^5} (\cos 2\pi k \epsilon - 1) \int_0^\infty dx x^3 e^{-2x} \left[\sum_{p,q} \left(\frac{\alpha_0 x}{k} \right)^{|p|} \left(\frac{\alpha_\pi x}{k} \right)^{|q|} \right]^k. \quad (6.30)$$

Using this power expansion, one finds Li_r functions of growing order r for higher and higher order terms, which when expanded for $\epsilon \rightarrow 0$ yield $\zeta(r-2)$ functions. It is clear that for $L_{0,\pi} = 0$ all the infinite terms have the same sign, and working out the first few orders, one can easily check that $\Delta V(\epsilon) = \Delta V_A(\epsilon) + \Delta V_B(\epsilon) + \Delta V_C(\epsilon) + \dots$, therefore reproducing the diagrammatic computation.

Actually, it is possible to derive a closed integral form of the full one-loop correction (6.2) that encodes all the higher-order corrections as well. To do so, we rescale the integration variable by $1/(2\pi R)$ and switch to the R -independent quantities $\rho_{0,\pi} = \frac{1}{3}\Omega_{0,\pi}$, to push the whole R -dependence of (6.29) into the exponential. This allows derivatives with respect to ϵ and derivatives with respect to R to be related in a simple way. At leading order in ϵ , one finds;

$$\Delta V(\epsilon) \simeq \frac{\epsilon^2}{\pi^2} \frac{\partial^2}{\partial R^2} \int_0^\infty dx x \ln \left[1 - \frac{1 + \rho_0 x}{1 - \rho_0 x} \frac{1 + \rho_\pi x}{1 - \rho_\pi x} e^{-4\pi R x} \right]. \quad (6.31)$$

Comparing this expression to eq. (6.3) with $F_T = 2\pi\epsilon$ and restoring M_5 , one deduces finally the result anticipated in eq. (2.7) for the one-loop correction to the Kähler potential:

$$\Delta \hat{\Omega} = -\frac{9}{\pi^2} M_5^2 \int_0^\infty dx x \ln \left[1 - \frac{1 + x \Omega_0 M_5^{-2}}{1 - x \Omega_0 M_5^{-2}} \frac{1 + x \Omega_\pi M_5^{-2}}{1 - x \Omega_\pi M_5^{-2}} e^{-6x(T+T^\dagger)M_5} \right]. \quad (6.32)$$

7 Discussion

In this section we will study the visible soft terms arising from $\Delta \hat{\Omega}$. The result will crucially depend on the mechanism by which the radion is stabilized.

7.1 General analysis

To start with, we consider R as a free parameter and deduce from $\Delta \hat{\Omega}$, eq. (6.32), an explicit expression for the universal supersymmetry-breaking squared mass¹⁸ m_0^2 , which receives contributions from both F_{Φ_π} and F_T . Furthermore m_0^2 depends on the VEV of ϕ_π , which might be non-vanishing, and on L_0 and L_π , which parametrize the gravitational kinetic terms localized at the boundaries. We are particularly interested in studying in which cases m_0^2 is positive. Its explicit expression can be written as

$$m_0^2 = \frac{\zeta(3)}{(4\pi)^2} \left[-\frac{|F_{\Phi_\pi}|^2}{6T^4 M_5^6} f_{\Phi_\pi \Phi_\pi} - \frac{|F_T|^2}{T^5 M_5^3} f_{TT} + \frac{2\text{Re}[\phi_\pi F_T F_{\Phi_\pi}^*]}{3T^5 M_5^6} f_{\Phi_\pi T} \right]. \quad (7.1)$$

¹⁸We assume that the one-loop correction $\Delta \hat{\Omega}$ negligibly renormalizes the tree-level kinetic terms of matter fields, Ω_0 . In such a situation $\Delta \hat{\Omega}$ induces a small universal trilinear term, $|A_0| \ll |m_0|$.

The functions $f_{\Phi_\pi\Phi_\pi}, f_{TT}, f_{\Phi_\pi T}$, obtained by taking appropriate derivatives of $\Delta\hat{\Omega}$, depend on the dimensionless variables

$$\alpha_0 = \frac{\Omega_0}{6M_5^3 T} = -\frac{L_0}{2T}, \quad \alpha_\pi = \frac{\Omega_\pi}{6M_5^3 T} = -\frac{L_\pi}{2T} + \frac{|\phi_\pi|^2}{6M_5^3 T}, \quad \beta = \frac{|\phi_\pi|^2}{6M_5^3 T}. \quad (7.2)$$

The normalization has been chosen in such a way that $f_{\Phi_\pi\Phi_\pi} = f_{TT} = f_{\Phi_\pi T} = 1$ in the minimal case when their three arguments vanish. One easily finds:

$$f_{\Phi_\pi\Phi_\pi} = \frac{2}{3\zeta(3)} \int_0^\infty dx x^3 \frac{(1 + x^2\alpha_0(\alpha_\pi - 2\beta)) \sinh x - x(\alpha_0 + \alpha_\pi - 2\beta) \cosh x}{[(1 + x^2\alpha_0\alpha_\pi) \sinh x - x(\alpha_0 + \alpha_\pi) \cosh x]^3}, \quad (7.3)$$

$$f_{TT} = \frac{1}{3\zeta(3)} \int_0^\infty dx x^4 (1 - x^2\alpha_\pi^2) \frac{(1 + x^2\alpha_0\alpha_\pi) \cosh x - x(\alpha_0 + \alpha_\pi) \sinh x}{[(1 + x^2\alpha_0\alpha_\pi) \sinh x - x(\alpha_0 + \alpha_\pi) \cosh x]^3}, \quad (7.4)$$

$$f_{\Phi_\pi T} = \frac{1}{3\zeta(3)} \int_0^\infty dx x^4 \frac{(1 + x^2\alpha_0\alpha_\pi) \cosh x - x(\alpha_0 + \alpha_\pi) \sinh x}{[(1 + x^2\alpha_0\alpha_\pi) \sinh x - x(\alpha_0 + \alpha_\pi) \cosh x]^3}. \quad (7.5)$$

In the minimal case $L_0 = L_\pi = \phi_\pi = 0$, m_0^2 is negative. In the presence of $\phi_\pi \neq 0$, but still keeping $L_{0,\pi} = 0$, the third contribution to m_0^2 in eq. (7.1) can be positive, but it is competitive with the first two only if $|\phi_\pi|^2 \sim M_5^3 T$. This situation is however unphysical, as it leads to an instability: ϕ_π induces a negative localized kinetic term for the gravitational multiplet. For such a large value of ϕ_π , there is a ghost-like KK mode with a small tachyonic mass squared $m^2 \sim -1/R^2$. To avoid manifest problems, we should take such a low UV cut-off $\sim 1/R$ for our 5D supergravity that the 5D description itself is of no use. Therefore we do not consider this case.

In the presence of $L_{0,\pi} > 0$ such that the localized kinetic terms are positive (i.e. $\alpha_{0,\pi} < 0$), $f_{\Phi_\pi\Phi_\pi}$ remains positive. Therefore, pure brane-to-brane mediation gives a negative contribution to m_0^2 , corresponding to the term proportional to $|F_{\Phi_\pi}|^2$. On the contrary f_{TT} becomes negative for large enough L_π (the precise value depends on L_0). Therefore m_0^2 can be positive if the dimensionless quantity

$$y = \frac{|F_{\Phi_\pi}|^2 T}{|F_T|^2 M_5^3} \quad (7.6)$$

is small enough, i.e. in the presence of a radion-mediated contribution. Notice that generically we expect $|F_{\Phi_\pi}|^2/M_5^3 T \sim |F_T|^2/T^2 \sim m_{3/2}^2$, so that $y \sim 1$ and radion mediation competes with brane-to-brane mediation. In specific models things can, however, be different.

The situation is illustrated in Fig. 4: m_0^2 can be positive at the left of the various lines. The continuous line corresponds to $\alpha_0 = \beta = 0$. The blue long-dashed lines show how the boundary $m_0 = 0$ shifts when a non-zero $\alpha_0 = \{-1/6, -1/2\}$ is turned on, while keeping $\beta = 0$. Finally, the red short-dashed lines show how the boundary $m_0 = 0$ shifts when a non-zero $\beta = \{1/12, 1/6, 1/4, 1/3\}$ is turned on, while keeping $\alpha_0 = 0$. In the last case, as a consequence of the last term in eq. (7.1), m_0^2 can be positive even when the total localized kinetic terms vanish, $\alpha_{0,\pi} = 0$.

In conclusion m_0^2 is usually negative, but the radion-mediated contribution can make it positive in two basic circumstances: 1) if the gravitational multiplet has a sizeable kinetic

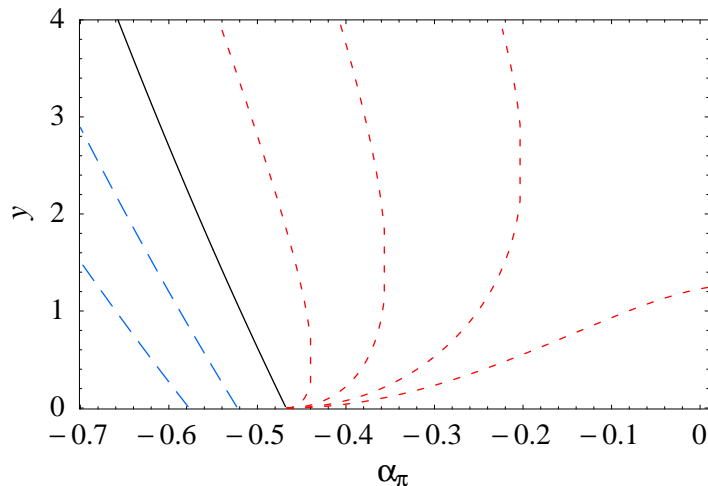


Figure 4: m_0^2 can be positive at the left of the various lines, which correspond to representative values of α_0 and β . The solid line corresponds to $\alpha_0 = \beta = 0$, whereas the blue long-dashed and red short-dashed lines describe situations with $\alpha_0 \neq 0$ and $\beta \neq 0$ respectively.

term localized on the hidden brane; 2) if the supersymmetry-breaking hidden sector field has a sizeable VEV ϕ_π .

A phenomenologically acceptable sparticle spectrum can be obtained if some other effect generates supersymmetry-breaking masses for gauginos. We will later discuss the specific case of anomaly mediation, which is the most natural candidate within the scenario we are considering.

In general, RGE effects induced by gaugino masses can make scalar masses positive at low energy, even starting from a negative m_0^2 at some high scale $\sim 1/R$. If scalar particles will be discovered, extrapolating their masses up to high energies one could try to identify a universal brane-to-brane contribution. We remark that squared scalar masses can be negative at high energies: this instability induces vacuum decay with a negligibly slow rate (the thermal evolution of the Universe can naturally select the metastable physical vacuum).

The low energy physical sfermion masses might contain non-SM sources of flavour and CP violation. In unified theories or in the presence of large neutrino Yukawa couplings, RGE corrections imprint detectable extra sources of flavour violation in scalar masses (see e.g. [39]). Beyond these effects, we expect that brane-to-brane mediation itself does not give an exactly flavour-universal m_0^2 because gravity becomes flavour-universal only at low energy, but in general violates flavour around the Planck scale. In fact, the effective supergravity Lagrangian describing matter terms might contain dimension 6 terms, e.g. kinetic terms with extra derivatives $(\partial/\Lambda_5)^2$ and flavour-breaking coefficients. Λ_5 is the unknown energy at which new quantum gravity phenomena, not accounted by general relativity, set in. Naïve dimensional analysis suggests $\Lambda_5 \lesssim 4\pi M_5$, with approximate equality holding if quantum gravity is strongly coupled [12]. In the absence of a predictive

theory of quantum gravity and of flavour, we cannot go beyond these semi-quantitative expectations.

Since brane-to-brane mediation is dominated by loop energies $E \sim 1/\pi R$, higher-dimensional operators are expected to give small flavour-breaking corrections to the squared masses, proportional to the factor $\delta \sim 1/(\Lambda_5 \pi R)^2$. If brane-to-brane mediation is used to solve the problems of anomaly mediation, the discussion below eq. (1.2) suggests $\delta \gtrsim 1/(4\pi)^{10/3}$. On the experimental side, $\mu \rightarrow e\gamma$ and ϵ_K give the strongest bounds, $\delta \lesssim 10^{-3}$ for sfermion masses of a few hundreds of GeV [40].

We will now discuss two different scenarios of radion stabilization. We will focus on the case $\phi_\pi = 0$ (or better $\phi_\pi \ll M_5$), suggested by a strongly coupled hidden sector. In this case the 4D Planck mass is given by

$$M_{\text{P}}^2 = M_5^3 (\text{Re}T + L_0 + L_\pi) . \quad (7.7)$$

7.2 Luty–Sundrum model

In ref. [13] the superpotential of the effective low energy theory was given by

$$P_{\text{eff}} = \Lambda^2 \Phi_\pi + \frac{1}{16\pi^2} \left(\Lambda_1^3 + \Lambda_2^3 e^{-a\Lambda_2 T/\pi} \right) . \quad (7.8)$$

The first term is just the standard O’Raifeartaigh superpotential of the hidden sector. The second and third terms are generated by gaugino condensation of respectively a gauge group on the boundary and in the bulk¹⁹. Their rôle is to stabilize the radion, and also to allow a fine-tuning of the 4D cosmological constant to zero. A discussion of the minimization of the potential is given in ref. [13]. One crucial remark that simplifies the discussion is that the Φ_π sector breaks supersymmetry already in the flat limit $M_5 \rightarrow \infty$. Assuming that the superpotential for Φ_π originates from some strong 4D dynamics at the scale Λ , the Φ_π -dependent part in Ω_π has the form $\Phi_\pi \Phi_\pi^\dagger F(\Phi_\pi \Phi_\pi^\dagger/\Lambda^2)$ (see for instance ref. [41]). Then the scalar ϕ_π has a fairly large mass $\sim \Lambda \gg m_{3/2} \sim \Lambda^2/M_{\text{P}}$, so that it can be integrated out before studying the radius potential. The only light mode in the Φ_π multiplet is the fermion, which contains a component of the eaten Goldstino. For the purpose of our discussions it is useful to briefly recall the resulting relations between the various parameters and VEVs. Cancellation of the 4D cosmological constant requires to tune

$$\frac{\Lambda_1^3}{(4\pi)^2} \sim \Lambda^2 M_{\text{P}} , \quad (7.9)$$

where the effective Planck mass M_{P} is given by eq. (7.7), while the relevant VEVs are

$$F_{\Phi_\pi} \sim \Lambda^2 , \quad F_{S_0} \sim \frac{\Lambda_1^3}{(4\pi M_{\text{P}})^2} \sim \frac{\Lambda^2}{M_{\text{P}}} \sim m_{3/2} , \quad (7.10)$$

$$\frac{F_T}{T} \sim \frac{\pi F_{S_0}}{\Lambda_2 T} , \quad \frac{\Lambda_2 T}{\pi} \sim 3 \ln \frac{\Lambda_2}{\Lambda_1} . \quad (7.11)$$

From the equation for T , it follows that its natural value is small. Indeed Λ_2 represents the strongly interacting scale of a bulk gauge theory, so it is natural to expect Λ_2 not much

¹⁹ $\Lambda_{1,2}$ are the strong-interaction scales according to NDA, a is of order 1.

below the quantum gravity scale $\Lambda_5 \sim M_5\pi$. On the other hand, perturbative control of the 5D theory requires $\Lambda_2 T/\pi$ somewhat larger than 1. In order to have scalar masses that are positive and comparable to gaugino masses, two conditions must be satisfied. One is that the anomaly-mediated mass be comparable to the radion-mediated one (second term in eq. (7.1)). Using the above equations this condition reduces to

$$\left(\frac{\Lambda_2 T}{\pi}\right)^2 (M_5 T)^3 = (\Lambda_2 R)^2 (\Lambda_5 R)^3 \sim \frac{\zeta(3)}{\pi^2} \left(\frac{4\pi}{g}\right)^4. \quad (7.12)$$

There is a window for which both gravity and the bulk gauge theory are (to a good extent) perturbative at the compactification radius. The other condition is that m_0^2 itself, eq. (7.1), be positive. From the above minimum conditions we have

$$y \sim \frac{\Lambda_2^2 T^2}{\pi^2} \left(1 + \frac{L_0}{T} + \frac{L_\pi}{T}\right). \quad (7.13)$$

This equation describes a line (not shown) in the plane of Fig. 4. For small enough $\Lambda_2 T/\pi$, but parametrically still larger than 1, the line will cross the black solid curve, allowing positive masses. Considering the best case $L_0 \sim 0$, we find that the crossing point is roughly at $L_\pi/T \sim \Lambda_2 T/\pi$: to safely remain in the perturbative domain of the bulk gauge theory, the boundary contribution to M_{P}^2 should be hierarchically larger than the ordinary 5D one ($L_\pi \gg T$).

The fact that we can squeeze our parameters to get positive masses does not yet mean that we can build a definite realistic model, in which all flavour-violating contributions to the soft masses are sufficiently small. As we have seen, the parameters $1/\Lambda_2 R$ and $1/\Lambda_5 R$ controlling higher order effects are not too small. Moreover, for a large localized kinetic term (say L_π), gravity becomes strongly coupled at a fairly low scale $\sqrt{M_5/L_\pi} \ll M_5$ [42]. It would be interesting to make a thorough investigation on this point.

7.3 Radius stabilization through localized kinetic terms

We will now consider the case in which the superpotential does not depend on T . This is essentially the case we have considered in our calculation of the effective potential, and it is straightforward to include the effects of F_{Φ_π} . We are then in the genuine no-scale scenario, in which T is an exact flat direction at tree level. The effective potential we have calculated is a generalized Casimir energy lifting this flatness and one can ask if it can also stabilize the radius at some finite value. This issue has already been studied by Ponton and Poppitz [37], who have shown that appropriate localized kinetic terms for the bulk fields lead to a modified Casimir energy with a stable minimum. This result is easy to understand. The boundary kinetic operator introduces a length scale L in the theory. Therefore the Casimir energy $\sim 1/R^4$ is modified into $F(L/R)/R^4$, for which stationary points are possible at $R \sim L$. This effect of boundary terms is analogous to the one we already studied for the scalar masses.

In the presence of non-vanishing F_T and/or F_{Φ_π} at tree level, the effective radion potential from (6.32) is

$$V(T) = -\frac{\partial^2 \Delta \hat{\Omega}}{\partial(T + T^\dagger)^2} |F_T|^2 - \frac{\partial \Delta \hat{\Omega}}{\partial \Omega_\pi} |F_{\Phi_\pi}|^2. \quad (7.14)$$

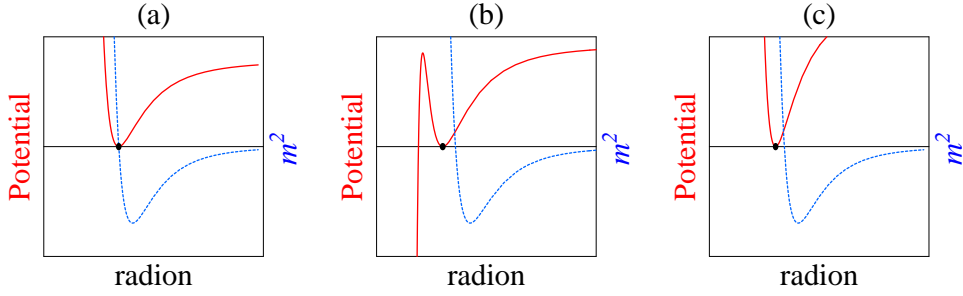


Figure 5: *The radion potential $V(R)$ (red solid line) and the soft mass $m_0^2(R)$ (blue dashed line) in cases: (a) $L_0 = 0$, $L_\pi > 0$, (b) $0 < L_0 < L_\pi$, and (c) $L_0 = 0$, $L_\pi > 0$ with extra bulk fields.*

Notice that, at tree level, we have $F_{S_0} = 0$. This implies that anomaly-mediated masses vanish at tree level in the gravitational interactions; $F_{S_0} = 0$ is also associated, through the specific no-scale form of Ω_{cl} , to a vanishing contribution to the vacuum energy from the radion sector. At tree level the vacuum energy equals $|F_{\Phi_\pi}|^2 > 0$. The inclusion of the one-loop correction $\Delta\Omega$ modifies this state of things.

The equation of motion of F_T leads to

$$F_{S_0} = \frac{2}{3M_5^3} \frac{\partial^2 \Delta\hat{\Omega}}{\partial(T + T^\dagger)^2} F_T \sim \frac{\zeta(3)}{\pi^2(M_5 T)^3} \frac{F_T}{T}. \quad (7.15)$$

In the last equality we made a simple dimensional estimate, based on the assumption that the boundary kinetic terms introduce just one length scale (say L_π) which coincides with T (see discussion below). Not surprisingly we find that F_{S_0} is suppressed with respect to its natural scale just by the gravitational loop expansion coefficient $\alpha_5 = 1/\pi^2(M_5 T)^3$. The anomaly-mediated gaugino masses are therefore similarly suppressed, as we will better discuss below.

At its minimum, $V(T)$ is negative and the parameters can be tuned so that this contribution to the cosmological constant cancels the tree-level contribution from Φ_π . This cancellation leads to the following relation (tuning) between F_T and F_{Φ_π} :

$$F_{\Phi_\pi} \sim \frac{F_T}{\pi T^2} \quad \rightarrow \quad y \sim \frac{1}{\pi^2(M_5 T)^3}. \quad (7.16)$$

Therefore in the perturbative regime we have $y \ll 1$ and radion mediation dominates the gravity-induced scalar mass in eq. (7.1).

We now consider various possibilities for the boundary kinetic terms, always assuming $\phi_\pi = 0$.

(a) $L_0 = 0$, $L_\pi > 0$.

In this case, $V(T \rightarrow \infty) \rightarrow 0^-$ and $V(T \rightarrow 0) \rightarrow +\infty$, so that there is an absolute minimum at finite $T \sim L_\pi$. The radius can thus be made strictly stable. The soft

mass $m_0^2(T)$ can be positive or negative, depending on T . But m_0^2 vanishes at the minimum of the potential, since it is given by $m_0^2(T) = -V'(T)/6\pi$. The situation is illustrated in Fig. 5a. As already mentioned above, also the leading anomaly-mediated masses vanish. In this case, to fully calculate the sparticle masses we should consider one extra gravitational loop for each quantity. We would then find that, while the MSSM gauginos have mass $m_{1/2} \sim \alpha\alpha_5|F_T|/T$, the scalars have a larger mass $m_0^2 \sim \alpha_5^2(|F_T|/T)^2$, unless extra cancellations occur in the two-loop gravitational contribution. Moreover, in this situation non-universal contributions to the scalar masses could be as important as the purely gravitational one.

(b) $L_\pi \gg L_0 > 0$.

Now there is still a local minimum allowing for a metastable situation. The induced soft mass squared can become positive, and it grows with L_0 . However, increasing L_0 lowers the barrier hiding the true minimum, and for some critical value of $L_0 \sim L_\pi$, the local minimum disappears and the potential becomes unstable. The situation is illustrated in Fig. 5b. In this case $m_{1/2} \sim \alpha\alpha_5|F_T|/T$, while $m_0^2 \sim \alpha_5(|F_T|/T)^2$ arises already at one-loop. So although the tachyons can be avoided, the resulting model is phenomenologically quite unattractive, as a huge tuning $\mathcal{O}(\alpha^2\alpha_5)$ must be made in order to keep $M_Z^2 \lesssim m_{1/2}^2$ [43] (multi-TeV universal scalars can be obtained by fine-tuning just the top Yukawa coupling [44]).

(c) $L_0 = 0$, $L_\pi > 0$, with extra bulk matter.

It is possible to add extra bulk fields that do not couple to ordinary matter and affect therefore only $V(R)$. Vector multiplets and hypermultiplets give a contribution to $V(R)$ equal respectively to $\frac{1}{2}$ and $-\frac{1}{2}$ that of the supergravity multiplet. Introducing then n_V vector multiplets and n_H hypermultiplets, with boundary kinetic terms that are independent from those of the supergravity multiplet, one can deform the effective potential to change the value of the radius at the minimum and therefore the value of the soft mass. If the localized kinetic terms of the new multiplets coincide with those of the supergravity multiplet, the minimum is not changed and m_0^2 remains zero. If they are smaller (larger), then the minimum is shifted to lower (higher) R for $n_V > n_H$ and higher (lower) R for $n_V < n_H$, leading respectively to a positive (negative) and negative (positive) m_0^2 in the stable vacuum. An example with $L_0 = 0$, $L_\pi > 0$ and extra bulk fields is illustrated in Fig. 5c. Again, this scenario leads to a nice stable vacuum and positive m_0^2 , but very light gauginos, as in case b).

8 Conclusions

We studied gravity-mediated brane-to-brane supersymmetry breaking. We considered the simplest set-up with one flat extra dimension, assumed to be a segment S_1/\mathbb{Z}_2 of length πR , with the ‘visible’ MSSM fields localized at one boundary, and supersymmetry broken at the other ‘hidden’ boundary. In this set-up, supersymmetry-breaking is transmitted to MSSM fields by two different minimal effects: anomaly mediation (which gives gaugino

masses and negative squared slepton masses) and one-loop supergravity diagrams like the one depicted in Fig. 1 (which gives an extra contribution to scalar masses).

We have computed this second contribution. Even if supergravity is plagued by UV divergences, locality implies that Fig. 1 is finite and dominated by particles with energy $E \sim 1/R$. Since gravity is flavour-universal in the IR, Fig. 1 induces a universal scalar soft mass squared m_0^2 . Unknown UV effects give extra contributions suppressed by $\delta \sim 1/(M_5 R)^2$, which presumably break flavour. While the overall coefficient of m_0^2 depends on the 5D Planck mass M_5 and on R , its sign is strongly constrained.

Knowing that many contributions must cancel, as demanded by supersymmetry, we only needed to compute one Feynman graph, plotted in Fig. 3, which involves only one non-trivial supergravity ingredient: the 4D coupling between two gravitinos and two scalars. In the first part of our paper we verified that all the rest works as expected. Starting from the Lagrangian for off-shell 5D supergravity with localized 4D fields, we derived a much simpler partially on-shell formulation which can be conveniently used in loop computations. (We also presented another, less convenient formulation in which powers of $\delta(0)$ arise in intermediate steps). We verified how supersymmetric cancellations really take place. Although not strictly necessary for our computation, we discussed these issues in great detail, correcting in various ways previous attempts. We calculated the full one-loop threshold correction to the effective Kähler potential at the compactification scale. We have done this calculation indirectly. We first computed the one-loop effective potential in a convenient and consistent supersymmetry-breaking background, where constant superpotentials are placed at the boundaries. This set-up corresponds to the well-known Scherk–Schwarz mechanism. Secondly, we have reconstructed the full Kähler potential by solving a simple differential equation. The computation is fairly simple, as it reduces to the class of gravitino loops. However, in order to fully secure our result, we had to tackle the puzzle posed by the singular gravitino wave functions. These are a well-known feature of models with boundary superpotentials and can lead, if not properly treated, to ambiguities in loop computations. We have explained a simple procedure, based on invariance under the local $SU(2)_R$ of the off-shell theory, to properly define the singular quantities and obtain consistent results.

In the most minimal case we find $m_0^2 < 0$ (the same result has been obtained by Buchbinder, Gates, Goh, Linch III, Luty, Ng and Phillips, using $N = 1$ supergraph techniques [45]). A positive m_0^2 arises in two basic circumstances: *i*) if a substantial part of the 4D graviton kinetic energy comes from terms localized on the hidden brane; *ii*) if supersymmetry-breaking fields localized on the hidden brane have a Planck-scale VEV. In both cases the radion superfield contributes to the mediation of supersymmetry breaking in an important way.

Finally, we studied how anomaly mediation and brane-to-brane effects may co-operate to give an acceptable sparticle spectrum — a goal that neither of the two mechanisms reaches by itself. The two effects are comparable when the radius R of the extra dimension is such that flavour-breaking higher order effects are suppressed by $\delta \gtrsim 1/(4\pi)^{10/3}$.

We considered two possible concrete mechanisms of radius stabilization. In the first, where the radion is stabilized by a superpotential generated by gaugino condensation of

a bulk gauge theory, we find that by stretching our parameters a little bit we can obtain positive scalar masses comparable with gaugino masses. This is a necessary requirement to construct a fully successful model. To do so we must tackle the μ -problem and carefully study all possible sources of flavour violation — a task that may be worth future work. The second scenario corresponds to a standard no-scale model in which the radion is stabilized by the quantum corrections to the Kähler potential. Although this second scenario presents peculiar phenomena, such as the scalar masses vanishing exactly at the potential minimum in the simplest realization, it has the phenomenological drawback of giving gaugino masses that are too small.

We conclude by listing a few related questions that require further work to be answered. What happens in the case of a warped extra dimension? How does our analysis generalize to more than one extra dimension? What changes if matter is localized on fluctuating branes away from orbifold fixed points?

Acknowledgements

We thank L. Andrianopoli, I. Antoniadis, S. Ferrara, F. Feruglio, L. Girardello, G. Giudice, C. Kounnas, R. Leigh, M. Luty, A. Riotto, M. Serone, L. Silvestrini, R. Sundrum, A. Zaffaroni and F. Zwirner for useful discussions. This research was partly supported by the European Commission through a Marie Curie fellowship and the RTN research network HPRN-CT-2000-00148.

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