Numerical simulation of the retreat of Alpine glaciers

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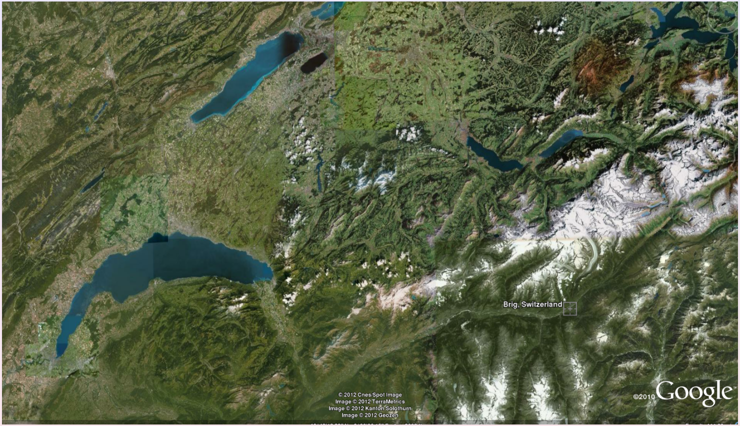
Europe

Source: google earth

Geneva lake area

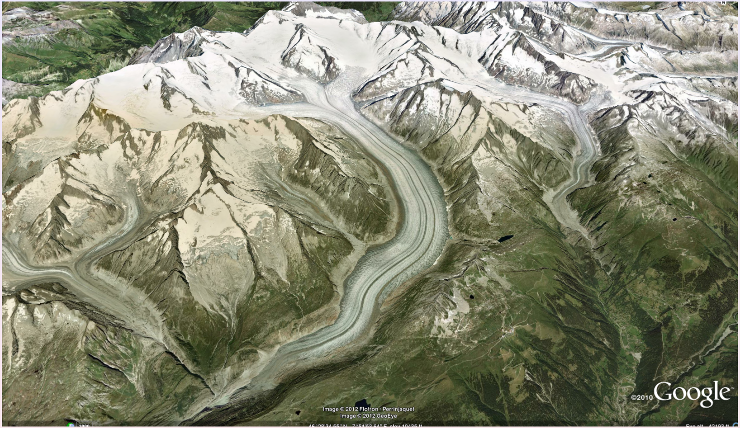


Rhone valley

OberWallis

Aletsch and Rhone glaciers

Aletsch



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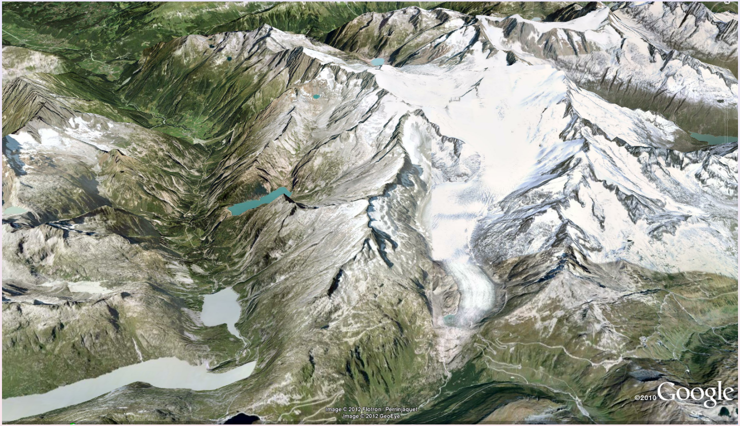
Rhone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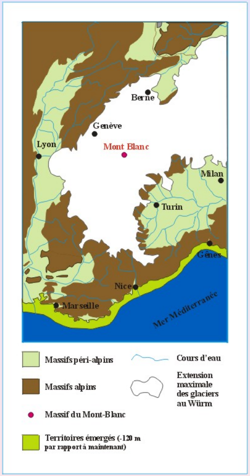
Rhône's glacier 20 000 years ago (Würm ice age)

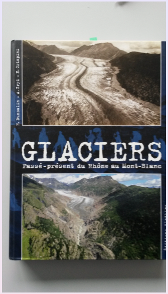


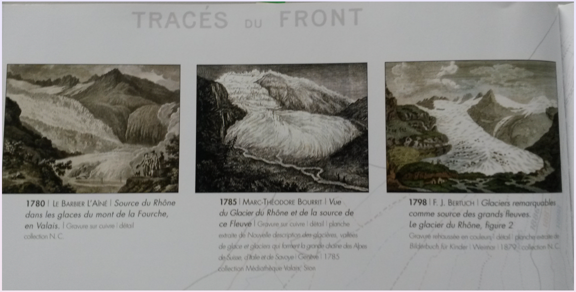
Source: geologie-montblanc.fr

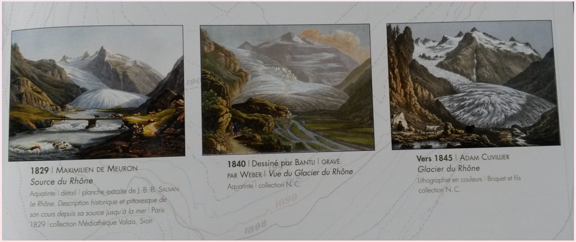
Rhône's glacier in 1850

Source: unifr.ch/geosciences/geographie/glaciers

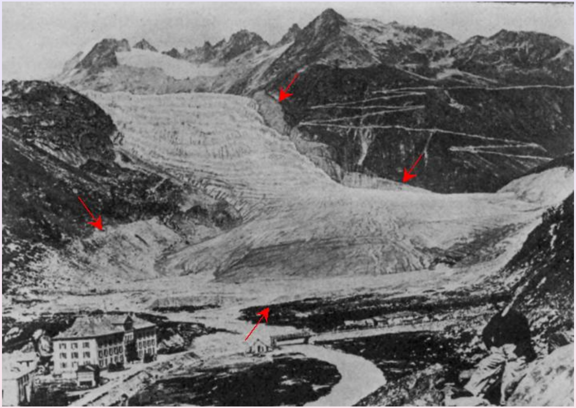
Rhône's glacier before 1850







Rhône's glacier in 1870



200 km / 20 000 years = 10 m / year = 200 m / 20 years

Rhône's glacier in 1900

Source: unifr.ch/geosciences/geographie/glaciers

Rhône's glacier in 1914

Source: unifr.ch/geosciences/geographie/glaciers

Rhône's glacier in 1925

Source: unifr.ch/geosciences/geographie/glaciers

Rhône's glacier in 1985

Source: unifr.ch/geosciences/geographie/glaciers

Rhône's glacier: comparison at 2000 m

Rhône's glacier in 1860 (M. Funk's reconstruction)

Rhône's glacier in 1970 (M. Funk's reconstruction)

Rhône's glacier in 2050 (M. Funk's prediction)

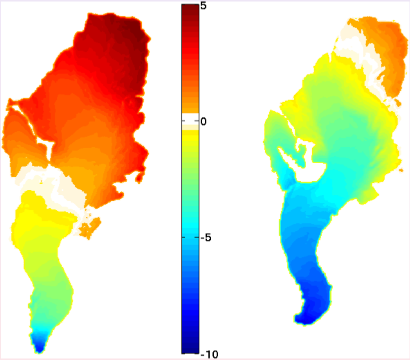
Mathematical model: ice flows in glaciers

- For long time scales, ice behaves as a fluid: Trift glacier, one picture a day in 2003, Animation. Free surface flow.
- Climatic input: meters of ice per year, model based on 150 years of measurements.

Accumulation

Eq. line Acc.

Ablation



Eq. line Acc.

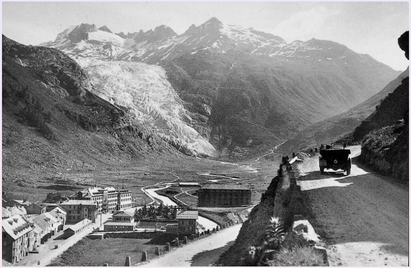
Cold year 1913

Warm year 2003

Rhône's glacier: 1900

Only one parameter to tune: sliding coefficient along the bedrock.

Rhône's glacier: 1932



Only one parameter to tune: sliding coefficient along the bedrock.

Rhône's glacier: 1960

Only one parameter to tune: sliding coefficient along the bedrock.

Rhône's glacier: 1985

Only one parameter to tune: sliding coefficient along the bedrock.

Rhône's glacier: numerical prediction from 2008 to 2100

- Median climatic scenario (occh.ch, Organe Consultatif pour les Changements Climatiques), temperature trend $+3.8^o C$, precipitation trend -6%: Animation.

- Cold scenario: Oct 1977-Sep 1978.
- Current scenario: pick randomly years between 2000 and 2008.
- Hot scenario: Oct 2002-Sep 2003.

Cold Current Hot

Numerical simulation of Aletsch's glacier

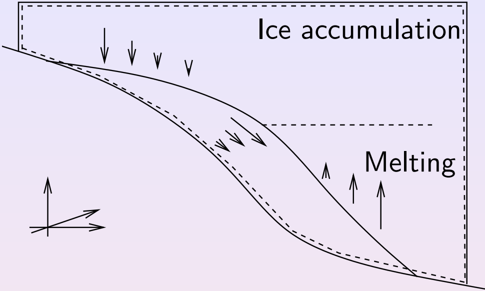
- Largest alpine glacier, 25% of Swiss ice, 23 km long, max. depth 900 m:
- Three scenario: Animation.

The mathematical model: 3D fluid flows with complex free surfaces

- Several formulations/numerical methods
  - Volume of Fluid: Tryggvason Scardovelli Zaleski 2011
  - Level Set: Osher Fedkiw 2003 Fedkiw 1 Fedkiw 2
  - Smooth Particle Hydrodynamics: Monaghan 2012
  - Lattice Boltzmann...
- Our experience (CFD free surface codes are versatile):
  - Newtonian flows: Maronnier Picasso Rappaz 2003, Caboussat Picasso Rappaz 2005, Mould filling Dams
  - Viscoelastic flows: Bonito Picasso Laso 2006
    - Jet buckling
    - Fingering instabilities Experiment (G. McKinley MIT) Simulation
  - Elastic flows: Picasso 2014
    - Bouncing ball
    - Beam bending
  - Dynamics of glaciers: Jouvet Huss Blatter Picasso Rappaz 2009, Jouvet Huss Blatter Funk 2011

Climatic input : b.

Unknowns :
velocity u and pressure p,
volume fraction of ice φ.



$$\rho\frac{\partial u}{\partial t} + \rho(u\cdot\nabla)u - \text{div}\,\Big(2\mu\epsilon(u)\Big) + \nabla p = \rho g,$$

$$\text{div}\; u = 0,$$

$$\frac{\partial\varphi}{\partial t} + u\cdot\nabla\varphi = b\delta_\Gamma,$$

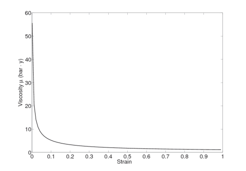
on the ice/air interface Γ : $(2\mu\epsilon(u) - pI)n = 0$,

along the bedrock : slip or no-slip.

Mathematics of free surface flows

- Theory is far behind numerical simulations...
  - Existence: a Newtonian fluid with free surface (Solonnikov 1997), two immiscible Newtonian fluids (Lions 1996).
  - Numerical analysis: two immiscible Newtonian fluids (Liu Walkington 2007).
  - Numerical simulations:
    - type "level set" in MathSciNet $>$ 1300 hits,
    - Sethian's book in google scholar $>$ 7700,
    - "A level set approach for computing solutions to incompressible two-phase flow" Sussman Smereka Osher JCP 94 in google scholar $>$ 3200.
  - Numerical analysis on simplified problems: transport, Stokes, Navier-Stokes.
- Same remark for viscous compressible flows...
  - Existence (without turbulence model): Lions 96 Bresch Desjardins 2007.
  - Numerical analysis: Gallouet Herbin Latché 2007.
  - Numerical simulations: design of airplanes. adaptation

Ice rheology

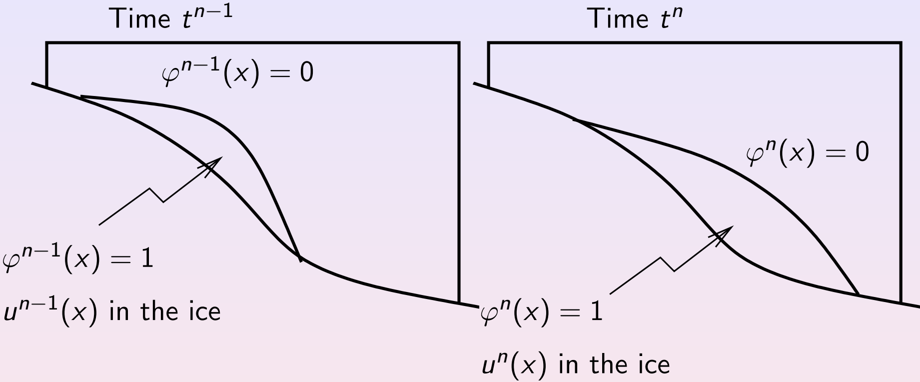


- Glen's law: viscosity $\mu(|\epsilon(u)|) = O\left(\frac{1}{(1+|\epsilon(u)|)^{1-\frac{1}{m}}}\right)$.
- Sliding law (Hutter 83) $u \cdot n = 0$ and $(2\mu\epsilon(u)n) \cdot t_i = -\alpha u \cdot t_i$, $i = 1, 2$ with

$$\alpha(|u|) = O\left(\frac{1}{(1+|u|)^{1-\frac{1}{m}}}\right)$$

- Following Barrett Liu 94, the nonlinear Stokes problem with sliding in a given domain has a solution in $W^{1,1+1/m}$ (minimum of a strictly convex functional), Jouvet Rappaz 2011.

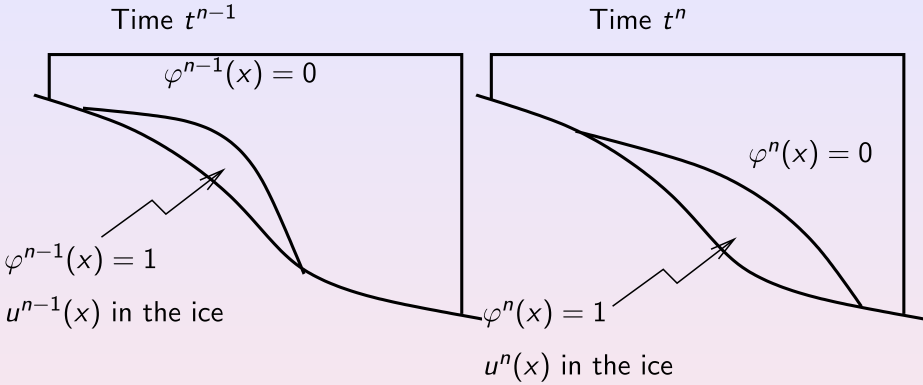
Time discretization : a splitting scheme



- Shape computation : solve between $t = t^{n-1}$ and $t = t^n$

$$\frac{\partial u}{\partial t} + (u \cdot \nabla) u = 0,$$
$$\frac{\partial \varphi}{\partial t} + u \cdot \nabla \varphi = b\delta_\Gamma.$$

Time discretization : a splitting scheme



- Velocity computation : solve

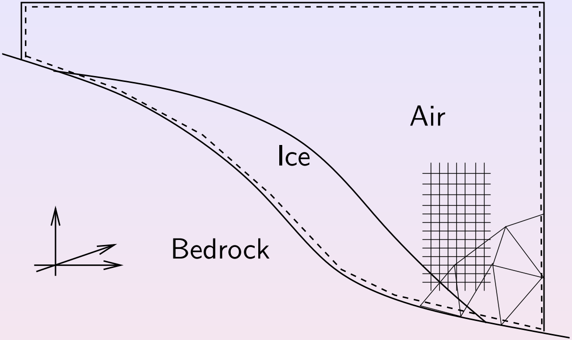
$$\rho\frac{\partial u}{\partial t} - \text{div}\,\Big(2\mu\epsilon(u)\Big) + \nabla p = \rho g,$$

$$\text{div}\ u = 0,$$

on the ice/air interface Γ : $(2\mu\epsilon(u) - pI)n = 0$,

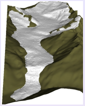
on the bedrock : no-slip or sliding.

Space discretization : structured cells and finite elements

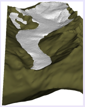


- Shape computation (ice advection + accumulation/melting) : small structured cells
- Velocity computation (nonlinear Stokes) : unstructured coarse finite elements
- To avoid numerical diffusion : $\frac{\text{FE spacing}}{\text{cells spacing}} \simeq 5$.
- CFL numbers from 1 to 10.

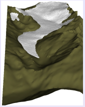
Velocity computation: the 3D finite element mesh



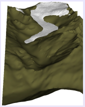
1874



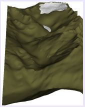
2008



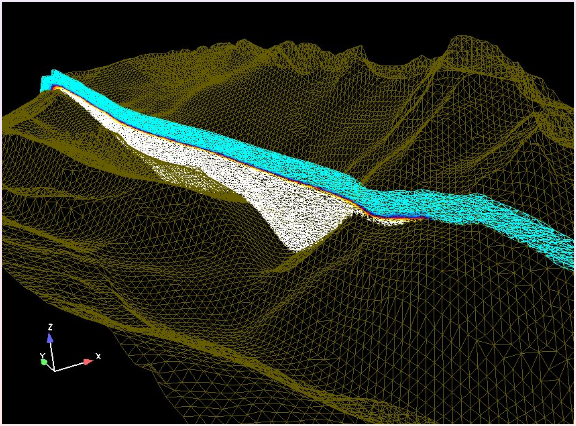
2050



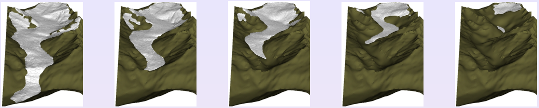
2075



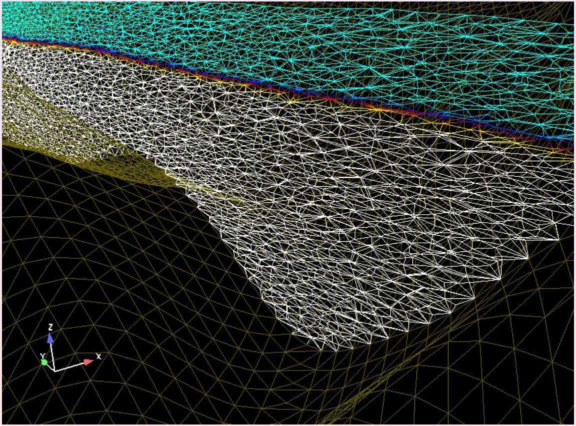
2100



Velocity computation: the 3D finite element mesh



1874 2008 2050 2075 2100

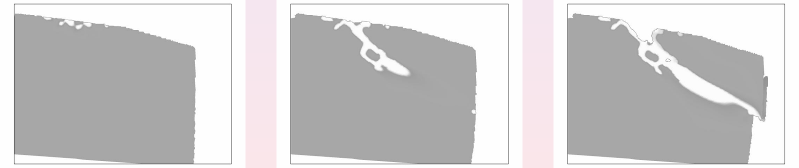


Conclusions

- Accurate model to reproduce the dynamics of alpine glaciers.
- Data: bedrock, accumulation or melting (meters of ice per year).
- Tuning: sliding coefficient along the bedrock.
- Can be used to predict the glacier shape during the next 100 years, given several climatic scenario.
- Useful for hydroelectric companies (water captation for dams).
- Modeling of crevasse formation.
- Modeling of mountaineer fatalities.

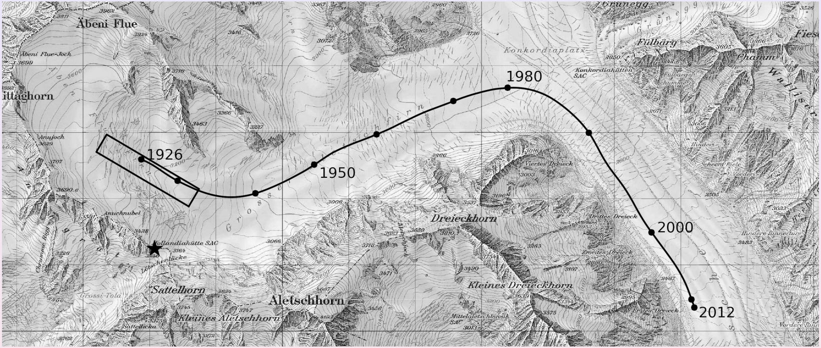
Glacier calving

Calving of a glacier



2D Simulation (Jouvet Picasso Rappaz Huss Funk 2011)

Mountaineer fatalities



Modelling the trajectory of the corpses of mountaineers who disappeared in 1926 on Aletschgletscher (Jouvet Funk 2014)

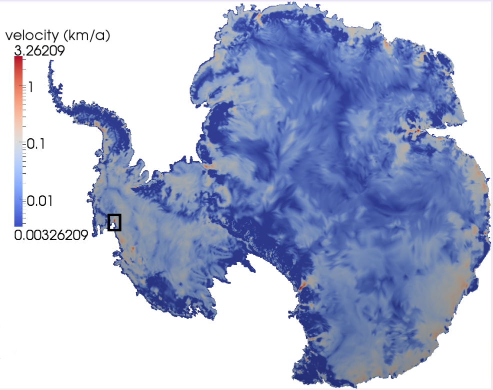
Perspectives: inverse modeling - uncertainties

Unknown parameters:

- Bedrock
- Accumulation or melting
- Sliding along the bedrock
- Viscosity

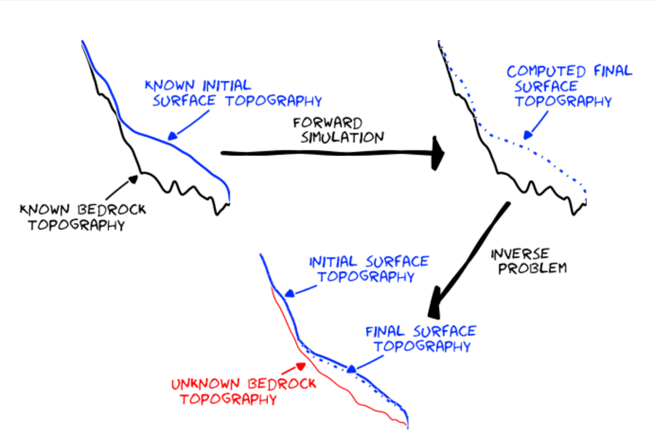
Perspectives: inverse modeling - sliding coefficient

Location dependent sliding law (Petra Zhu Stadler Hughes Ghattas 2012).



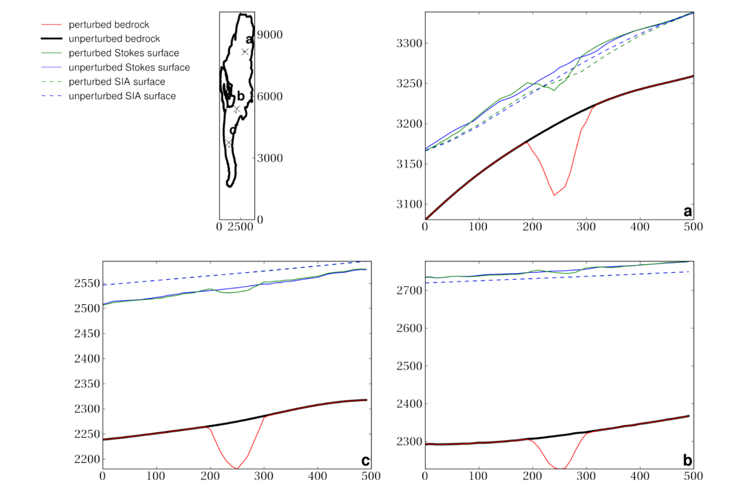
Perspectives: inverse modeling - bedrock

Bedrock optimization (PhD L. Michel): given the glacier's top surface, find the bedrock in order to minimize the discrepancy between the measured and computed top surface, under the constraint that the underlying pde's are satisfied.



Perspectives: inverse modeling - bedrock

Bedrock optimization (PhD L. Michel): sensitivity analysis



Different bedrocks may correspond to the same top surface.

Perspectives: uncertainty quantification

- The model problem: $-\text{div}\,(a\nabla u) = f$, where

$$a(x,\omega) = a_0(x) + \varepsilon \sum_{i=1}^{L} Y_i(\omega) a_i(x), \quad \mathbb{E}(Y_i) = 0, \quad \mathbb{E}(Y_i^2) = \sigma^2.$$

- Ansatz $u = u_0 + \varepsilon u_1 + O(\varepsilon^2)$.
- $O(1)$ deterministic term: $-\text{div}\,(a_0 \nabla u_0) = f$, finite element approximation $u_{0,h}$ continuous, piecewise linear

$$\int_D a_0 \nabla u_{0,h} \cdot \nabla v_h = \int_D f v_h.$$

- A posteriori error estimate (Guignard Nobile Picasso 2014):

$$\mathbb{E}\left(\int_D |\nabla (u - u_{0,h})|^2 \right) \leq \frac{2}{a_{min}^2} \left(\eta_1^2 + \eta_2^2 \right),$$

where η_1 is the usual error estimator in space and $\eta_2^2 = \varepsilon^2 \sigma^2 \int_D |\nabla u_{0,h}|^2 \left(a_1^2 + \cdots + a_L^2 \right)$ is the model error.