Fifth Seminar on

Stochastic Analysis, Random Fields and Applications

May 30 - June 3, 2005

Centro Stefano Franscini, Ascona, Switzerland

and

Minisymposium on Stochastic Methods in Financial Models

June 2-3, 2005

Organizers

Robert Dalang
EPF-Lausanne

Marco Dozzi
Université de Nancy II

Francesco Russo
Université de Paris 13

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9:30 - 9:55 M. RÖCKNER, Universität Bielefeld
The stochastic porous media equation: a survey of recent results

9:55 - 10:20 G. DA PRATO, Scuola Normale Superiore di Pisa
Some results on the Kolmogorov equation related to the Burgers equation

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BSDEs with super-linear growth coefficient. Application to systems of degenerate semi-linear PDEs

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An approximation result for a non linear Neumann boundary value problem via BSDEs

17:50 - 18:15 F. CASTELL, Université de Provence, Marseille
Large deviations for the Brownian motion in a random scenery

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9:55 - 10:20	B. ZEGARLINSKI, Imperial College London Nonlinear Markov semigroups for large interacting systems
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Elliptic diffusions with reflecting boundary condition and an application to continuous N-particle system

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17:20 - 17:45 C. TUDOR, Université de Paris 1
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17:50 - 18:15 Z. HABA, University of Wroclaw
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Complex networks and random graphs: From structure to functions

9:55 - 10:20 A. VILLA, Université J. Fourier - Grenoble 1
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Some results on the stochastic 2*d*-Euler equation

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Approximation of small jumps of multivariate Lévy processes with applications to operator stable laws

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15:25 - 15:50 M. ZAKAI, Technion - Haifa
On mutual information, likelihood-ratios and estimation error for the additive Gaussian channel

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16:50 - 17:15 J.M. CORCUERA, Universitat de Barcelona
Power variation of some integral long-memory processes

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18:40 ***Bus departs to Locarno***

19:00 *Visit of Castello Visconteo*

19:30 *Reception offered by the Town of Locarno*

Thursday, June 2, 2005

Minisymposium on Stochastic Methods in Financial Models

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9:30 - 9:55	E. EBERLEIN, Universität Freiburg Symmetries and pricing of exotic options in Lévy models
9:55 - 10:20	F. LEGLAND, IRISA Rennes Filtering a diffusion process observed in singular noise
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10:50 - 11:35	N. BOULEAU, ENPC Paris Dirichlet forms methods in finance
11:40 - 12:05	M. PRATELLI, Università di Pisa Generalizations of "Merton's Mutual Fund Theorem" in infinite dimensional financial models
12:05 - 12:30	S. BIAGINI, Università di Perugia Utility maximization in a general framework and properties of the optimal solution
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Dynamic risk preferences and optimal behavior

15:05 - 15:30 J. WOLF, BaFin, Bonn
Valuation of participating life insurance

15:35 - 16:00 F. MORICONI, Università di Perugia
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16:05 - 16:30 H. GEMAN, ESSEC + Dauphine
Understanding the Fine Structure of Electricity Prices

16:30 – 17:00 *Break*

Public lectures

17.00 - 17.20 *Opening and welcome by a representative of the Ticino government*

17:20 - 17.45 Presentation by P. ROSSI, Director of Azienda Elettrica Ticinese (AET)

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Energy trading: new challenges in financial mathematics.

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10:50 - 11:35	B. OKSENDAL, University of Oslo The value of information in stochastic control and finance
11:40 - 12:05	A. SULEM, INRIA - Rocquencourt Utility maximization in an insider-influenced market
12:05 - 12:30	E. VALKEILA, Helsinki University of Technology Asymmetric information in pricing models - a Bayesian approach
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15:00 - 15:25	P. GUASONI, Boston University No arbitrage with transaction costs
15:25 - 15:50	Ch. STRICKER, Université de Franche-Comté Minimal entropy-Hellinger martingale measure
15:50 - 16:20	*Break*
16:20 - 17:05	W. J. RUNGGALDIER, Università di Padova On portfolio optimization in discontinuous markets and under incomplete information
17:10 - 17:35	T. VARGIOLU, Università di Padova Robustness of the Hobson-Rogers model
17:35 - 18:00	J.-P. AUBIN, Université de Paris Dauphine A tychastic approach to financial problems
18:00	*End of Meeting*

ABSTRACTS

of the Fifth Seminar on

Stochastic Analysis, Random Fields and Applications

Khaled Bahlali (**Université du Sud-Toulon-Var**)

FBSDEs with continuous continuous generators. Application to degenerate semilinear PDEs

We establish the existence of solutions for multidimensional weakly coupled FBSDEs with continuous and almost quadratic growth generators. We cover, for instance, the generators of the form $f(t,x,y,z) = -y|z|$. This is done without imposing the L^2-domination condition on the diffusion matrix. As a consequence, we prove the existence of weak solutions (in Sobolev sense) for degenerate semilinear partial differential equations with continuous and almost quadratic nonlinearities. Our main tool is a Bouleau-Hirsch theorem on the absolute continuity of the marginal laws of the solution of SDE with Lipschitz coefficient.

Michel Benaïm (**Université de Neuchâtel**)

A Bakry-Emery criterion for self-interacting diffusions

Let M be a smooth compact connected Riemannian manifold without boundary and $V : M \times M \to \mathbb{R}$ a smooth function. For every Borel probability measure μ on M let $V\mu : M \to \mathbb{R}$ denote the function defined by $V\mu(x) = \int_M V(x,u)\,\mu(du)$, and let $\nabla(V\mu)$ denote its gradient.

A *self-interacting diffusion process* associated to V is a continuous time stochastic process (X_t) living on M solution to the stochastic differential equation

$$\begin{cases} dX_t &= dW_t(X_t) - \frac{1}{2}\nabla(V\mu_t)(X_t)\,dt \\ X_0 &= x \in M \end{cases}$$

where (W_t) is a Brownian vector field on M and $\mu_t = \frac{1}{t}\int_0^t \delta_{X_s}\,ds$ is the *empirical occupation measure* of $\{X_t\}$.

This type of process with reinforcement was introduced in [2] and further studied in [3], [4], with the ultimate goal to:

(a) provide tools allowing to analyze the long term behavior of $\{\mu_t\}$;

(b) understand the relations connecting this behavior to the nature of V and,

(c) the geometry of M.

Let $\mathcal{P}(M)$ denote the space of Borel probability measures over M, λ the Riemannian probability on M and $\mathcal{P}_{cd}(M) \subset \mathcal{P}(M)$ the set of measures having a continuous density with respect to λ. Let X_V be the vector field defined on $\mathcal{P}_{cd}(M)$ by

$$X_V(\mu) = -\mu + \Pi_V(\mu)$$

where

$$\frac{d\Pi_V(\mu)}{d\lambda} = \frac{\exp -V\mu}{\int_M \exp -V\mu(y)\,\lambda(dy)}.$$

Point (a) was mainly addressed in [2] where it was shown that the asymptotic behavior of $\{\mu_t\}$ can be precisely described in terms of the deterministic dynamical system induced by X_V.

Depending on the nature of V, the dynamics of X_V can either be convergent, globally convergent or non-convergent, leading to a similar behavior for $\{\mu_t\}$. A key step toward (b) is the next result recently proved in [4].

Theorem 1 *Suppose V is a symmetric function. Then the limit set of $\{\mu_t\}$ (for the topology of weak* convergence) is almost surely a connected subset of $X_V^{-1}(0) = \mathsf{Fix}(\Pi_V)$.*

In (the generic) case where the equilibrium set $X_V^{-1}(0)$ is finite, Theorem 1 implies that $\{\mu_t\}$ converges almost surely. If furthermore, $X_V^{-1}(0)$ reduces to a singleton $\{\mu^*\}$, then $\{\mu_t\}$ converges almost surely to μ^* and we say that $\{\mu_t\}$ is *globally convergent*.

A function $K : M \times M \to \mathbb{R}$ is a *Mercer kernel* provided K is continuous symmetric and defines a positive operator in the sense that

$$\int_{M\times M} K(x,y) f(x) f(y)\, \lambda(dx)\, \lambda(dy) \geqslant 0$$

for all $f \in L^2(\lambda)$. The following result is proved in [4].

Theorem 2 *Assume that (up to an additive constant) V is a Mercer Kernel. Then $\{\mu_t\}$ is* globally convergent.

Example 1 Suppose $M \subset \mathbb{R}^n$ and $V(x,y) = f(-\|x-y\|^2)$ where $\|\cdot\|$ is the Euclidean norm of $\mathbb{R}^n$ and $f : \mathbb{R}^+ \mapsto \mathbb{R}^+$ is a smooth function whose derivatives of all order f', f'', ... are nonnegative. Then it was proved by Schoenberg [6] that V is a Mercer Kernel.

As observed in [4], the assumption that V is a Mercer Kernel seems well suited to describe *self-repelling diffusions*. On the other hand, it is not clearly related to the geometry of M (see e.g. the preceding example).

The next theorem has a more geometrical flavor and is robust to smooth perturbations (of M and V). It can be seen as a Bakry-Emery type condition [1] for self interacting diffusions and is a first step toward (c).

Theorem 3 *Assume that V is symmetric and that for all $x \in M$, $y \in M$, $u \in T_xM$, $v \in T_yM$*

$$\mathsf{Ric}_x(u,u) + \mathsf{Ric}_y(v,v) + \mathsf{Hess}_{x,y}V((u,v),(u,v)) \geqslant K(\|u\|^2 + \|v\|^2)$$

where K is some positive constant. Then $\{\mu_t\}$ is globally convergent.

Let $\mathcal{P}_{ac}(M)$ denote the set of probability measures that are absolutely continuous with respect to λ and let J be the nonlinear free energy function defined on $\mathcal{P}_{ac}(M)$ by

$$J(\mu) = \mathsf{Ent}(\mu) + \frac{1}{2}\int_{M\times M} V(x,y)\, \mu(dx)\, \mu(dy)$$

where

$$\mathsf{Ent}(\mu) = \int_M \log(\frac{d\mu}{d\lambda})\, d\mu.$$

The key point is that $X_V^{-1}(0)$ is the critical set of J (restricted to $\mathcal{P}_{cd}(M)$) as shown in [4] (Proposition 2.9). On the other hand the condition given in the theorem makes J a *displacement K-convex* function in the sense of McCann [5]. Let us briefly explain this later statement. Let d_2^W denote the L^2-Wasserstein distance on $\mathcal{P}(M)$ (see e.g. [7] or [8]). Given ν^0, $\nu^1 \in \mathcal{P}_{ac}(M)$, McCann [5] proved that there exists a unique geodesic path $t \to \nu^t$ in $(\mathcal{P}_{ac}(M), d_2^W)$ and that ν^t is the image of ν^0 by a map of the form $F_t(x) = \exp_x(t\Phi)$, where Φ is some vector field. Moreover

$$d_2^W(\nu^0,\nu^t)^2 = \int_M d(x, F_t(x))^2\, \nu^0(dx).$$

Set $j(t) = J(\nu^t) = e(t) + \frac{v(t)}{2}$ with $e(t) = \mathsf{Ent}(\nu^t)$ and

$$v(t) = \int_{M\times M} V(x,y)\, \nu^t(dx)\, \nu^t(dy) = \int_{M\times M} V(F_t(x), F_t(y))\, \nu^0(dx)\, \nu^0(dy).$$

Sturm [7] recently proved the beautiful result that

$$\partial^2 e_t(t) = \int_M \mathsf{Ric}(\dot F_t(x), \dot F_t(x))\, \nu^0(dx)$$

where $\partial^2 e_t(t) := \liminf_{s\to 0} \frac{1}{s^2}(e(t+s) - 2e(t) + e(t-s))$. Clearly

$$\partial^2 v(t) = \int_{M\times M} \mathsf{Hess}_{F_t(x),F_t(y)} \Big((\dot F_t(x), \dot F_t(y)), (\dot F_t(x), \dot F_t(y)) \Big)\, \nu^0(dx)\, \nu^0(dy).$$

Hence, under the assumption of Theorem 3

$$\partial^2 j_t(t) \geqslant \frac{K}{2} \int_{M\times M} (\|\dot{F}_t(x)\|^2 + \|\dot{F}_t(y)\|^2)\, \nu^0(dx)\, \nu^0(dy) = K d_2^W(\nu^0, \nu^1)^2.$$

In particular, j is strictly convex. It then follows that J (respectively X_V) has a unique minimum (respectively equilibrium).

Example 2 Let $M = S^n \subset \mathbb{R}^{n+1}$ be the unit sphere of dimension n, $f : \mathbb{R} \mapsto \mathbb{R}$ a smooth convex function and

$$V(x, y) = f(-\|x - y\|^2) = g(\langle x, y\rangle)$$

with $g(t) = f(2t-2)$. By invariance of λ under the orthogonal group $O(n+1)$ it is easily seen (see e.g. Lemma 4.6 of [2]) that $V\lambda$ is a constant map. Hence $\lambda \in X_V^{-1}(0)$ and here, global convergence means convergence to λ.

For all $(x, y) \in M \times M$, $(u, v) \in T_xM \times T_yM$

$$\begin{aligned}\mathsf{Hess}_{(x,y)} V((u, v), (u, v)) &= g''(\langle x, y\rangle)\, (\langle x, v\rangle + \langle x, v\rangle)^2 \\ &+ g'(\langle x, y\rangle) \left(2\langle u, v\rangle - (\|u\|^2 + \|v\|^2)\langle x, y\rangle\right).\end{aligned}$$

Set $t = \langle x, y\rangle$ and assume (without loss of generality) that $\|u\|^2 + \|v\|^2 = 1$. Then $|2\langle u, v\rangle| \leqslant 1$ and the last term in the right hand side of the preceding equality is bounded below by $-tg'(t) - |g'(t)|$. Therefore the condition of Theorem 3 reads

$$tg'(t) + |g'(t)| < 2(n - 1) \tag{1}$$

while Theorem 2 would lead to

$$g^{(k)}(t) \geqslant 0\, \forall k \in \mathbb{N},\ |t| \leqslant 1. \tag{2}$$

Remark that condition (1) makes J a displacement-convex function while (2) makes J convex in the usual sense. Of course, none of these condition is optimal. For instance, suppose that $g(t) = at$. Then (1) reads $|a| < n - 1$, and (2) reads $a \geqslant 0$. On the other hand this example can be fully analyzed and it was shown in [2] that $\mu_t \to \lambda$ for $a > -(n + 1)$ while (μ_t) converges to a "Gaussian" measure with random center, for $a < -(n + 1)$.

This is joint work with Olivier Raimond.

References

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Hakima Bessaih (**University of Wyoming**)

Some results on the stochastic $2D$ Euler equation

We are interested in the $2D$ Euler equation perturbed by an additive noise and a multiplicative noise. In both cases, we study the existence and uniqueness of strong and weak solutions. For the additive noise, pathwise arguments are used, while for the multiplicative noise, martingale solutions are introduced. Some results on invariant measures will be given, in the case where we have some dissipation in the equation.

Philippe Blanchard (**Universität Bielefeld**)

Complex networks and random graphs: From structure to functions

After a short survey of recent developments in the theory of complex networks, we discuss a class of random graph models where the exceptional and extreme events play a crucial role in the formation of the network's architecture. The models are built on a principle we call the Cameo principle. According to this approach "The more rare you are the more attractive you become". The Cameo principle extends the concept of random graph introduced 1959 by Erdös and Renyi. We further discuss the interaction between graph structure and collective dynamics and present results about thresholds of epidemic processes. A new model of social contagion (opinion dynamics, innovation, corruption, cultural fads...) will also be presented.

Stefano Bonaccorsi (**Università di Trento**)

Volterra equations perturbed by a Gaussian noise

In a Hilbert space U, we consider a class of abstract linear Volterra equations of convolution with respect to the fractional integration kernel, perturbed by a cylindrical Gaussian process in U. To make use of the ability to treat a general kernel, we investigate what can be said when the behavior of the kernel is almost regular (*e.g.* $K(t,s) = o((t-s)^{\theta-1})$ for $\theta \in (\frac{1}{2}, 1)$ no matter how close to 1) or less regular than any of the fractional integration kernels (*e.g.* $K(t,s) \gg (t-s)^{\theta-1}$ for any θ no matter how close to $\frac{1}{2}$). We hope that the examples discussed here may be enlightening of the behavior of the system in these cases. Finally, in the last part, we modify the arguments to cover the case of a cylindrical fractional Brownian motion $B_H(t)$.

Brahim Boufoussi (**University of Marrakesh**)

An approximation result for a nonlinear Neumann boundary value problem via BSDE's

We prove a weak convergence result for a sequence of backward stochastic differential equations related to a semilinear parabolic partial differential equation under the assumption that the diffusion corresponding to the PDE's is obtained by penalization method converging to a normal reflected diffusion on a smooth and bounded Domain D. As a consequence we give an approximation result to the solution of semilinear parabolic partial differential equations with nonlinear Neumann boundary conditions. A similar result in the linear case was obtained by P. L. Lions, J. L. Menaldi and A. S. Sznitman in 1981.

Fabienne Castell (**Université de Provence, Marseille**)

Large deviations for Brownian motion in a random scenery

We investigate large deviations properties in large time for Brownian motion a in random scenery, *i.e.* for $\int_0^t v(B_s)\,ds$, where B is a d-dimensional Brownian motion, and v is a random stationary field from $\mathbb{R}^d$ with value in $\mathbb{R}$, independent of the Brownian motion. The problem is considered either in the quenched setting where a typical realization of v is fixed, or in the annealed one. Brownian motion a in random scenery is the central object in the study of diffusion processes with random drift $X_t = W_t + \int_0^t V(X_s)\,ds$, where V is a shear flow random field independent of the Brownian W.

Sandra Cerrai (**Università di Firenze**)

Stabilization by noise for a class of SPDEs with multiplicative noise

We prove uniqueness, ergodicity and strongly mixing property of the invariant measure for a class of stochastic reaction-diffusion equations with multiplicative noise, in which the diffusion term in front of the noise may vanish and the deterministic part of the equation is not necessary asymptotically stable. To this purpose, we show that the L^1-norm of the difference of two solutions starting from any two different initial data converges $\mathbb{P}$-a.s. to zero, as time goes to infinity.

We also consider the case of systems and we see what may be proved in this more complicate situation.

Serge Cohen (**Université Paul Sabatier, Toulouse**)

Approximation of small jumps of multivariate Lévy processes with applications to operator stable laws

Suppose we want to simulate trajectories of a process $\mathbf{X} = \{X(t) : t \in \mathbb{T}\}$ in $\mathbb{R}^d$. When an exact method for simulation of $\mathbf{X}$ is not available, we may consider an approximate one. Suppose we can write

$$\mathbf{X} = \mathbf{X}^{\epsilon} + \mathbf{X}_{\epsilon}$$

where the process $\mathbf{X}^{\epsilon}$ can be simulated exactly and $\mathbf{X}_{\epsilon}$ is negligible when ϵ is small. In a first approach one may simulate $\mathbf{X}^{\epsilon}$ instead of $\mathbf{X}$ with small ϵ, neglecting $\mathbf{X}_{\epsilon}$. However, if the error of approximation $\mathbf{X}_{\epsilon}$ is asymptotically normal, then it may be advantageous to not discard $\mathbf{X}_{\epsilon}$ but replace it by a Gaussian process, say $\mathbf{W}_{\epsilon}$. We will have

$$\mathbf{X} \approx \mathbf{X}^{\epsilon} + \mathbf{W}_{\epsilon}. \tag{3}$$

In this talk, we will concentrate on the case when $\mathbf{X} = \{X(t) : t \in [0, T]\}$ is a d-dimensional Lévy process. The case of one-dimensional Lévy processes was studied rigorously in Asmussen and Rosinski (2001), with $\mathbf{X}_{\epsilon}$ being a centered non Gaussian part of $\mathbf{X}$, with magnitudes of jumps not exceeding ϵ. However, a natural residual process $\mathbf{X}_{\epsilon}$ does not always come from the truncation of small jumps. We illustrate this point on examples of operator stable processes. Furthermore, the notion of a small jump in the multidimensional case may depend on the geometry of Lévy measure. Consequently, one must allow a more general form of truncation as well.

Laure Coutin (Université Paul Sabatier, Toulouse)

Good rough path sequences and applications to anticipating and fractionnal stochastic calculus

We consider anticipative Stratonovich stochastic differential equations driven by some stochastic process (not necessarily a semi-martingale). No adaptedness of initial point of vector fields is assumed. Under a simple condition on the stochastic process, we show that the unique solution of the above SDE understood in the rough path sense is actually a Stratonovich solution. This condition is satisfied by the Brownian motion and the fractional Bronwnian motion with Hurst parameter greater than 1/4. As application, we obtain rather flexible results such as support theorems, large deviation principle and Wong-Zakaï approximations for SDEs driven by fractionnal Brownian motion along anticipating vector fields. In particular, this unifies many results on anticipative SDEs.

This is joint work with Peter Friz and Nicolas Victoir.

Ana Bela Cruzeiro (IST Lisbon)

Geometrical numerical schemes for stochastic differential equations

We present some numerical schemes for diffusions associated to elliptic second order operators which are derived via geometric arguments. Of the same order as the Milstein schemes, they present the advantage of not involving the simulation of Itô iterated stochastic integrals.

The talk covers some results obtained in collaboration with P. Malliavin and A. Thalmaier and others with C. Alves.

Giuseppe Da Prato (Scuola Normale Superiore, Pisa)

Some results on the Kolmogorov equation related to the Burgers equation

We are concerned with the Burgers equation in $H = L^2(0,1)$ perturbed by a cylindrical white noise,

$$\begin{cases} dX(t) & = (AX(t) + b(X(t)))dt + dW(t), \quad t > 0,\ x \in H, \\ X(0,\cdot) & = x, \quad x \in H, \end{cases} \tag{4}$$

where

$$Ax = D_\xi^2,\ \forall\, x \in D(A) = H^2(0,1) \cap H_0^1(0,1), \quad b(x) = D_\xi(x^2).$$

Existence, uniqueness of a mild solution $X(t,x)$ as well as of the invariant measure ν are known.

In this talk we consider the corresponding Kolmogorov equation,

$$\begin{cases} D_t u(t,x) & = K_0 u(t,x), \quad t > 0,\ x \in H, \\ u(0,x) & = \varphi(x), \quad x \in H, \end{cases} \tag{5}$$

where K_0 is the differential operator

$$K_0\varphi(x) = \frac{1}{2}\,\mathrm{Tr}[D^2\varphi(x)] + \langle Ax + b(x), D\varphi(x)\rangle, \quad \varphi \in \mathcal{E}_A(H), \tag{6}$$

and $\mathcal{E}_A(H)$ is the span of all exponential functions of the type

$$\varphi_h(x) = \exp(i\langle x, h\rangle), \quad h \in D(A).$$

We prove that K_0 is dissipative in $L^2(H,\nu)$ and that its closure is m–dissipative.

As a consequence we construct the Sobolev space $W^{1,2}(H,\nu)$, we show that the domain $D(K)$ of the closure of K_0 is included in $W^{1,2}(H,\nu)$ and prove the following identity

$$\int_H K\varphi\varphi\, d\nu = -\frac{1}{2}\int_H |\varphi|^2\, d\nu, \quad \varphi \in D(K).$$

This is joint work with Arnaud Debussche.

Kenneth Elworthy (University of Warwick)

Diffusions and Connections

A. Levels of structure. There are several levels of geometric structure loosely corresponding to levels of structure in stochastic analysis:

- Semi-martingale theory belongs on a C^2 manifold as pointed out by L. Schwartz, and P. A. Meyer.
- Brownian motions and diffusions give rise to Riemannian, or sub-Riemannian geometry.
- Malliavin calculus on path spaces explicitly involves connections [Dri92, ELL99].
- Stochastic flows *are* connections (in a sense I shall explain).

B. Non-linear semi-connections. Consider a smooth surjective map $p : N \to M$ between manifolds N and M. Let E_x be a linear subspace of the tangent space T_xM to M at x, depending smoothly on the point x in the sense that $E := \bigcup_{x\in E} E_x$ forms a subbundle of TM. By a (non-linear) *semi-connection on* $p : N \to M$ *over* E we will mean a smooth horizontal lift map $\mathcal{H}$ giving for each $u \in N$ a linear mapping $\mathcal{H}_u : E_{p(u)} \to T_uN$ which is a right inverse to the derivative $T_up : T_uN \to T_{p(u)}M$ of p at u. For such a semi-connection let H_u denote the image of $\mathcal{H}_u$; this is the *horizontal subspace* at u. Let F_u be the sum of H_u with the *vertical subspace* $\operatorname{Ker} T_up$ and $\Pi_u : F_u \to H_u$ the projection. When $E = TM$ we have a *(non-linear) connection* [Mic91].

A semi-connection $\mathcal{H}$ over E determines a covariant differention $\nabla^{\mathcal{H}}$ in the E-directions acting on smooth sections $f : M \to N$ of p. For this note that the derivative T_xf at a point $x = p(u)$ of such a section maps E_x to $F_{f(x)}$. Then, by definition, $\nabla^{\mathcal{H}}_v := T_xf(v) - \Pi_{f(x)}T_xf(v) \in KerT_{f(x)}p$, for all $v \in E_x$. Also any curve σ in M with $\dot\sigma(t) \in E_{\sigma(t)}$ for all t has a unique maximal horizontal lift $\widetilde{\sigma}$ with $\sigma(0)$ any given point above $\sigma(0)$ and $\dot{\tilde\sigma}(t) \in H_{\tilde\sigma(t)}$ for all t for which it is defined. Using Stratonovich differentials there is the corresponding result for continuous semi-martingales in M.

C. Examples

1. [ELJL04, ELJL] Consider smooth diffusion generators $\mathcal{A}$ and $\mathcal{B}$ on the manifolds M and N respectively, intertwined by p. They have symbols $\sigma^{\mathcal{A}}$ and $\sigma^{\mathcal{B}}$ related by the commutative diagram

$$\begin{tikzcd} T_u^*N \arrow[r, "\sigma_u^{\mathcal{B}}"] & T_uN \arrow[d, "T_up"] \\ T^*_{p(u)}M \arrow[u, "(T_up)^*"] \arrow[r, "\sigma^{\mathcal{A}}_{p(u)}"'] & T_{p(u)}M. \end{tikzcd}$$

If we assume that $\sigma^{\mathcal{A}}$ has constant rank and so has image a subbundle E, we obtain a semi-connection over E characterised by the requirement

$$\mathcal{H}_u \circ \sigma^{\mathcal{A}}_{p(u)} = \sigma^{\mathcal{B}}(T_up)^*. \tag{7}$$

When $\mathcal{A}$ is along E this can be used to obtain a *canonical* decomposition of $\mathcal{B}$ into 'horizontal' and vertical parts and to describe the conditional law of the diffusion processes on N corresponding to $\mathcal{B}$ given their projections onto M. In special cases, see below, there is a corresponding, unique, skew product decomposition.

2. [ELL99] Consider a stochastic differential equation on M

$$dx_t = X(x_t) \circ dB_t + A(x_t)\, dt, \qquad 0 \leqslant t \leqslant T. \tag{8}$$

Here $X : \mathbb{R}^m \times M \to TM$ is smooth in x and $X(x) \in \mathbb{L}(\mathbb{R}^m; T_xM)$ for each x. Set $E_x = \text{Image}[X(x)]$ and assume that it has constant dimension. There is an inner product $\langle \cdot, \cdot \rangle_x$ induced on each E_x by $Y_x = (X(x)| \operatorname{Ker} X(x))^{-1} : E_x \to \mathbb{R}^m$. (This corresponds to the second level mentioned above, and is equally well given by the symbol of the generator $\mathcal{A}$, say, of the diffusion determined by our SDE.) We now take $E = N$ with p the projection, to relate to our earlier notation, and observe that X induces a (metric linear) connection on the vector bundle E with the covariant derivative of a section U of E in an arbitrary direction $v \in T_xM$ being given by

$$\nabla_v U = X(x)(d\{y \mapsto Y_y(U(y))\}(v)).$$

Such a connection determines a parallel transport in E along smooth curves, or semi-martingale sample paths, in M. The connection is linear in the sense that this is linear, and metric in that it preserves the inner products $\langle \cdot, \cdot \rangle_x$. It follows from [NR61] that every such connection on E arises this way, see also [Qui88]. For compact M there is essentially a bijection between SDE and stochastic flows [Bax84] and a special case of this "LJW- connection" was discovered for certain flows by LeJan and Watanabe [LW84], see also [AMV96].

A linear connection with covariant derivative ∇ on E determines a linear semi-connection on TM over E by

$$\acute{\nabla}_U V = \nabla_V U + [V, U]$$

for U a section of E and V a vector field.

One of the first results showing the relevance of this to stochastic differential equations was that if $T_{x_0}\xi_t : T_{x_0}M \to T_{x_t}M$ is the derivative, at time t, of the solution flow $\{\xi_t : t \geqslant 0\}$ of our SDE, then the conditional expectation$\{T\xi_t \colon \xi_s(x_0) : 0 \leqslant s \leqslant t\}$ of the derivative given the one point motion is just parallel translation using the adjoint semi-connection 'damped' by the Ricci curvature and the Hessian of the drift. For gradient systems, when the connection is the Levi-Civita connection, this was done in [EY93] and in retrospect is behind many of the estimates in [Li94] and [ER96]. This result is also given for the action of the flow on differential forms, but here more complicated curvature terms, the Weitzenbock curvatures, arise as the 'dampening' agents.

3. [ELL99] As with analysis on abstract Wiener spaces, following Gross, to do Malliavin calculus on path spaces on M with diffusion measure determined by $\mathcal{A}$ requires differentiating in certain "H-directions". Generalising from Driver [Dri92], these are obtained using parallel translation determined by the adjoint of an arbitrary metric connection on E.

D. Relationships between these examples: (a) the diffeomorphism bundle. For M compact, and $x_0 \in M$ let $p : DiffM \to M$ be the evaluation map at x_0 from the group of smooth diffeomorphisms of M. This is a principal bundle: in particular it can be considered as the quotient map, quotienting out by the right action, by composition, of the subgroup of diffeomorphisms fixing x_0. The flow of our SDE can be considered as a $DiffM$ -valued process and determines a right invariant diffusion generator $\mathcal{B}$ say on $DiffM$. This is intertwined with $\mathcal{A}$ by p. There is therefore an induced semi-connection over E which enables us to give a skew product decomposition to the flow, and so to describe the conditional law of the flow given its one point motion [ELJL04]. Moreover our bundle is a universal natural bundle over M, so this semi-connection determines a semi-connection over E on each natural bundle over M; these include jet bundles and the tangent bundle itself [KMS93]. That induced on the tangent bundle turns out to be the adjoint of the LJW connection of the SDE.

The mapping of stochastic flows into semi-connections on the diffeomorphism bundle is injective (in this sense a stochastic flow *is* a connection) and so all the properties of the flow such as its Lyapunov exponents are determined by the semi-connection. To what extent they are determined by that induced on finite dimensional natural bundles is an open question.

Since an SDE is essentially a map into the classifying space for $O(p)$ with p the dimension of E (see the Appendix to [ELL99]), it follows from the above and the discussion in [AB83] that the space of stochastic flows associated to a given generator $\mathcal{A}$ is close to being a classifying space for the gauge group of E.

E. Relationships: (b) Ito maps as charts. Let $\mathcal{I} : C_0(\mathbb{R}^m) \to C_{x_0}(M)$ be the Ito map of our SDE, so $\mathcal{I}(\omega)_t = x_t(\omega)$. It has an H-derivative $T_\omega \mathcal{I} : H \to TC_{\xi_\cdot(x_0)}M$, from the Cameron-Martin space to the tangent space to our path space. By Bismut's formula this can be expressed in terms of the derivative of the flow. This enables us to describe its conditional expectation given $\mathcal{I}$ itself, in terms of the LJW connection, which has proved a useful tool in analysis on the path space, starting with [AE95]. In particular in [EL05] these maps are used as charts, and it is shown that the composition of $\mathcal{I}$ with a function $f : C_{x_0}(M) \to \mathbb{R}$ which is in the Sobolev space $\mathbb{D}^{2,1}$ is in $\mathbb{D}^{2,1}$ of the flat Wiener space *provided* the LJW connection of the SDE agrees with that determining the H-directions on our path space (and with some technical conditions on the connection). This property does not appear to hold in general for compositions with the stochastic development map, see [Li03]. Moreover, under the same conditions, the composition of a function f with $\mathcal{I}$ is in $\mathbb{D}^{2,1}$ if and only if f is in the analogous weak Sobolev space in the sense of Eberle [Ebe99]. An important point is that it follows from Eberle [Ebe99] that Markov uniqueness holds for the Dirichlet operator on $C_{x_0}(M)$ if and only if the two Sobolev spaces coincide.

Finally we note that the use of SDE's as charts plays a fundamental role in approaches to an L^2 Kodaira-Hodge theory of differential forms on path spaces in [FF97], [EL00] and [EL03].

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Franco Flandoli (Università di Pisa)

A stochastic turbulence model

We start from stochastic Navier-Stokes equations having approximatively (at least at a formal level) a scaling property of K41 type, up to large deviations. Then a phenomenological rigorous model is provided: a random field composed of random eddies and filaments, having a number of properties similar to those of the previous stochastic Navier-Stokes equations. Finally, a dynamic is imposed on the model, inspired by observed mechanisms of energy transfer; the result is a new ensemble with multifractal scaling corrections.

Mihai Gradinaru (Université Henri Poincaré, Nancy 1)

A question concerning the linear stochastic heat equation

The mild solution of the stochastic heat equation $dX_t = \Delta X_t dt + dW_t$ is a stochastic convolution $X_t = \int_0^t e^{(t-s)\Delta} dW_s$. Here $\Delta = \partial^2/\partial x^2$ is the Laplace operator on $[0,1]$ with Dirichlet boundary conditions and W is the cylindrical Brownian motion (connected to the space-time white noise). The Markov process $\{X_t : t \geq 0\}$ with values in $H = \mathrm{L}^2([0,1])$ is not a semi-martingale, but is Hölder continuous of order $(1/4)^-$. Is it possible to get an Itô's type formula for $F(X_t)$, with F belonging in some (large) class of functionals on H ? What about a Tanaka's type formula ?

Martin Grothaus (Universität Kaiserslautern)

Elliptic diffusions with reflecting boundary condition and an application to continuous N-particle systems

We give a Dirichlet form approach for the construction and analysis of elliptic diffusions in $\overline{\Omega} \subset \mathbb{R}^n$ with reflecting boundary condition. The problem is formulated in an L^2-setting with respect to a reference measure μ on $\overline{\Omega}$ having an integrable, dx-a.e. positive, density ϱ with respect to the Lebesgue measure. The symmetric Dirichlet forms $(\mathcal{E}^{\varrho,a}, D(\mathcal{E}^{\varrho,a}))$ we consider are the closure of the symmetric bilinear forms

$$\mathcal{E}^{\varrho,a}(f,g) = \sum_{i,j=1}^{n} \int_{\Omega} \partial_i f \, a_{ij} \, \partial_j g \, d\mu, \quad f,g \in \mathcal{D},$$
$$\mathcal{D} = \{f \in C(\overline{\Omega}) \,|\, f \in W^{1,1}_{\mathrm{loc}}(\Omega), \mathcal{E}^{\varrho,a}(f,f) < \infty\},$$

in $L^2(\overline{\Omega}, \mu)$, where a is a symmetric, elliptic, $n \times n$-matrix-valued measurable function on $\overline{\Omega}$. Assuming that Ω is an open, relatively compact set with boundary $\partial\Omega$ of Lebesgue measure zero and that ϱ satisfies the Hamza condition, we can show that $(\mathcal{E}^{\varrho,a}, D(\mathcal{E}^{\varrho,a}))$ is a local, quasi-regular Dirichlet form. Hence, it has an associated self-adjoint generator $(L^{\varrho,a}, D(L^{\varrho,a}))$ and diffusion process $\mathbf{M}^{\varrho,a}$ (i.e, an associated strong Markov process with continuous sample paths). Furthermore, since $1 \in D(\mathcal{E}^{\varrho,a})$ (due to the Neumann boundary condition) and $\mathcal{E}^{\varrho,a}(1,1) = 0$, we obtain a conservative process $\mathbf{M}^{\varrho,a}$ (i.e., $\mathbf{M}^{\varrho,a}$ has infinite life time). Additionally, assuming that $\sqrt{\varrho} \in W^{1,2}(\Omega)$, we can show that the set $\{\varrho = 0\}$ has $\mathcal{E}^{\varrho,a}$-capacity zero. Therefore, in this case we even can construct an associated conservative diffusion process in $\{\varrho > 0\}$. This is essential for our application to continuous N-particle systems with singular interactions. Note that for the construction of the self-adjoint generator $(L^{\varrho,a}, D(L^{\varrho,a}))$ and the Markov process $\mathbf{M}^{\varrho,a}$ we do not need to assume any differentiability condition on ϱ and a. We obtain the following explicit representation of the generator for $\sqrt{\varrho} \in W^{1,2}(\Omega)$ and $a \in W^{1,\infty}(\Omega)$:

$$L^{\varrho,a} = \sum_{i,j=1}^{n} \partial_i(a_{ij}\partial_j) + \partial_i(\log \varrho) a_{ij} \partial_j.$$

Note that the drift term can be very singular, because we allow ϱ to be zero on a set of Lebesgue measure zero. Our assumptions even allow a drift which is not integrable with respect to the Lebesgue measure.

Massimiliano Gubinelli (Università di Pisa)

Explorations on rough paths

Rough path theory can be understood as a particular case of a theory of integration on algebras of non-smooth functions. In this talk we will illustrate this point of view trying to emphasize its algebraic and analytic aspects and the connection with probability. We will show how this approach provides a guiding principle to generalize the rough-path ideas to the multidimensional setting and to the construction of pathwise solutions to SPDEs. We describe some recent results in these directions.

Zbigniew Haba (**University of Wroclaw**)

Random fields defined by Green functions of operators with singular coefficients

We discuss Green functions $G(x;x')$ of some second order differential operators on R^{d+1} with singular coefficients depending only on one coordinate x_0. We express the Green's functions by means of the Brownian motion. Applying probabilistic methods we prove that when $x=(0,\mathbf{x})$ and $x'=(0,\mathbf{x}')$ (here $x_0=0$) lie on the singular hyperplanes then $G(0,\mathbf{x};0,\mathbf{x}')$ is more regular than the Green's function of operators with regular coefficients. We construct Gaussian and non-Gaussian random fields defined in some domains of R^{d+1} which are singular inside the domain but much more regular on the boundary of the domain.

Yueyun Hu (**Université de Paris 13**)

Directed polymers in random environment

Considering a $(d+1)$-dimensional directed polymer in a random environment, we shall discuss the asymptotic behaviors of this model including partition functions, large deviation principles and the volume and fluctuation exponents.

Thierry Huillet (**Université de Cergy-Pontoise**)

Dirichlet-Kingman partitions revisited

We consider the Dirichlet model for the random division of an interval. This model is parametrized by the number $n>1$ of fragments, together with a set of positive parameters $(\theta_1,\ldots,\theta_n)$. Its main remarkable properties are recalled, developed and illustrated.

Explicit results on the statistical structure of its sized-biased permutations are provided. This distribution appears in the sorting of items problem under the move-to-front rule. Assuming the parameters satisfy $\sum_{m=1}^{n}\theta_m \to \gamma < \infty$ as n tends to ∞, it is shown that the Dirichlet distribution has a Dirichlet-Kingman non-degenerate weak limit whose properties are briefly outlined.

This is joint work with Servet Martinez.

Gérard Kerkyacharian (**Université de Paris X**)

Approximation theory and learning : Upper and lower bounds

In the learning theory framework, we are dealing with a sequence of data $(X_1,Y_1),\ldots,(X_n,Y_n)$ of i.i.d. random variables and we want to estimate the regression function $f(X)$ of Y given X or to give an estimation of some functional of Y, given X. We will show that tools from approximation theory, like metric entropy, Kolmogorov width, linked with the Fano inequality, can give precise bounds on this problem.

Davar Khoshnevisan (**University of Utah**)

Images of the Brownian Sheet

We describe two new properties of the Brownian sheet. One has to do with the behavior of bridged (or pinned) sheets, and the other with an analogue of the Berman-Pitt notion of local non-determinism that we call 'sectorial local non-determinism'. By appealing to the said properties in different ways, we proceed to resolve two conjectures from the 1980's; one is due to Jean-Pierre Kahane and the other to Thomas S. Mountford.

This is joint work with Yimin Xiao (to appear in "The Transactions of the AMS").

Tomasz Komorowski (University of Lublin)

Diffusion in a weakly random Hamiltonian flow

In our talk we consider a particle that moves in an isotropic weakly random Hamiltonian flow with the Hamiltonian of the form $H_\delta(\mathbf{x},\mathbf{k}) = H_0(k) + \sqrt{\delta}H_1(\mathbf{x},k)$, $k = |\mathbf{k}|$, and $\mathbf{x}$, $\mathbf{k} \in \mathbb{R}^d$ with $d \geqslant 3$:

$$\frac{dX^\delta}{dt} = \nabla_{\mathbf{k}} H_\delta(X^\delta(t), K^\delta(t)), \quad \frac{dK^\delta}{dt} = -\nabla_{\mathbf{x}} H_\delta(X^\delta(t), K^\delta(t)) \quad X^\delta(0) = \mathbf{x}_0, \quad K^\delta(0) = \mathbf{k}_0. \tag{9}$$

Here $H_0(k)$ is the background Hamiltonian. We assume that it is a deterministic function $H_0 : [0,+\infty) \to [0,+\infty)$ that is $\mathcal{C}^3$-class of regularity in $(0,+\infty)$ with $H_0'(k) > 0$ for all $k > 0$. On the other hand $H_1 : \mathbb{R}^d \times [0,+\infty) \times \Omega \to \mathbb{R}$ is assumed to be a random field given over a certain probability space $(\Omega, \Sigma, \mathbb{P})$ that is measurable and strictly stationary in the first variable. This means that for any shift $\mathbf{x} \in \mathbb{R}^d$, $k \in [0,+\infty)$, and a collection of points $\mathbf{x}_1, \ldots, \mathbf{x}_n \in \mathbb{R}^d$, the laws of $(H_1(\mathbf{x}_1 + \mathbf{x}, k), \ldots, H_1(\mathbf{x}_n + \mathbf{x}, k))$ and $(H_1(\mathbf{x}_1, k), \ldots, H_1(\mathbf{x}_n, k))$ are identical. In addition, we assume that $\mathbb{E}H_1(\mathbf{x},k) = 0$ for all $k \geqslant 0$, $\mathbf{x} \in \mathbb{R}^d$. Here, $\mathbb{E}$ denotes the expectation with respect to $\mathbb{P}$.

It as been shown in [2] that when $H_\delta(\mathbf{x},\mathbf{k}) = \frac{k^2}{2} + \sqrt{\delta}V(\mathbf{x})$ and under certain mixing assumptions on the random potential $V(\mathbf{x})$, the momentum process $K^\delta(t/\delta)$ converges to a diffusion $K(t)$ on the sphere $k = k_0$, whose corresponding Kolmogorov equation is given by (16) below, and the rescaled spatial component $\widetilde{X}^\delta(t) = \delta X^\delta(t/\delta^{1+2\alpha})$ converges to $X(t) = \int_0^t K(s)\,ds$.

The principal topic we wish to discuss in our talk is the description of the motion in time scales that are longer than the time scale of the momentum diffusion. We show that under certains assumptions concerning mixing properties of H_1 in the spatial variable, see condition (11) below, there exists $\alpha_0 > 0$ so that the process $\delta^{1+\alpha}X^\delta(t/\delta^{1+2\alpha})$ converges to the standard Brownian motion in $\mathbb{R}^d$ for all $\alpha \in (0,\alpha_0)$. The main difficulty of the proof is to obtain error estimates in the convergence of $K^\delta(t/\delta)$ to the momentum diffusion on time scales of the order $O(\delta^{-1})$. The error estimates allow us to push the analysis to time scales much longer than $O(\delta^{-1})$ where the momentum diffusion converges to the standard Brownian motion. A quantum analogue of our result has been recently obtained by Erdös et al. in [1].

Let us describe more precisely our main results. Besides the assumptions concerning centering of $H_1(\mathbf{x},k)$ and its stationarity with respect to the $\mathbf{x}$ variable, we suppose that the realizations of $H_1(\mathbf{x},k)$ are $\mathbb{P}$-a.s. $\mathcal{C}^2$-smooth in $(\mathbf{x},k) \in \mathbb{R}^d \times (0,+\infty)$ and they satisfy

$$D_{i,j}(M) := \max_{|\alpha|=i} \operatorname*{ess\,sup}_{(\mathbf{x},k\omega)\in\mathbb{R}^d\times[M^{-1},M]\times\Omega} |\partial_{\mathbf{x}}^\alpha \partial_k^j H_1(\mathbf{x},k;\omega|) < +\infty, \quad i,j = 0,1,2. \tag{10}$$

We suppose further that the random field is strongly mixing in the uniform sense. More precisely, for any $R > 0$ we let $\mathcal{C}_R^i$ and $\mathcal{C}_R^e$ be the σ-algebras generated by random variables $H_1(\mathbf{x},k)$ with $k \in [0,+\infty)$, $|\mathbf{x}| \leqslant R$, and $|\mathbf{x}| \geqslant R$ respectively. The uniform mixing coefficient between the σ-algebras is defined as

$$\phi(\rho) := \sup[\mathbb{P}(A) - \mathbb{P}(B|A)| \colon R > 0, A \in \mathcal{C}_R^i, B \in \mathcal{C}_{R+\rho}^e],$$

for all $\rho > 0$. We suppose that $\phi(\rho)$ decays faster than any power: for each $p > 0$,

$$h_p := \sup_{\rho\geqslant 0} \rho^p \phi(\rho). \tag{11}$$

The two-point spatial correlation function of the random field H_1 is defined as $R(\mathbf{y},l) := \mathbb{E}[H_1(\mathbf{y},l)H_1(\mathbf{0},l)]$. We also assume that the correlation function $\mathbf{y} \mapsto R(\mathbf{y},l)$ is of $\mathcal{C}^\infty$-class for a fixed $l > 0$ and that for any fixed $l > 0$ the function $\mathbf{k} \mapsto \widehat{R}(\mathbf{k},l)$ does not vanish identically on any hyperplane $H_{\mathbf{p}} = \{\mathbf{k} \colon \mathbf{k}\cdot\mathbf{p} = 0\}$. Here $\widehat{R}(\mathbf{k},l) = \int R(\mathbf{x},l)\exp(-i\mathbf{k}\cdot\mathbf{x}\,d\mathbf{x}$ is the power spectrum of H_1.

We assume further that $H_0 : [0,+\infty) \to \mathbb{R}$ is a stricly increasing function satisfying $H_0(0) \geqslant 0$ and such that it is of $\mathcal{C}^3$-class of regularity in $(0,+\infty)$ with $H_0'(k) > 0$ for all $k > 0$, and let

$$h^*(M) := \max_{k\in[M^{-1},M]}(H_0'(k) + |H_0''(k)| + |H_0'''(k)|), h_*(M) := \min_{k\in[M^{-1},M]} H_0'(k). \tag{12}$$

Let the function $\phi_\delta(t,\mathbf{x},\mathbf{k})$ satisfy the Liouville euqation

$$\begin{aligned}\frac{\partial \phi^\delta}{\partial t} + \nabla_{\mathbf{x}} H_\delta(\mathbf{x},\mathbf{k})\cdot\nabla_{\mathbf{k}}\phi^\delta - \nabla_{\mathbf{k}} h_\delta(\mathbf{x},\mathbf{k})\cdot\nabla_{\mathbf{x}}\phi^\delta = 0,\\ \phi^\delta(0,\mathbf{x},\mathbf{k}) = \phi_0(\delta\mathbf{x},\mathbf{k}).\end{aligned} \tag{13}$$

Here, as we recall $H_\delta(\mathbf{x},\mathbf{k}) = H_0(|\mathbf{k}|) + \sqrt{\delta}H_1(\mathbf{x},|\mathbf{k}|)$. We assume that the initial data $\phi_0(\mathbf{x},\mathbf{k})$ is a compactly supported function four times differentiable in $\mathbf{k}$, twice differentiable in $\mathbf{x}$ whose support is contained inside a spherical shell $\mathcal{A}(M) = \{(\mathbf{x},\mathbf{k}) : M^{-1} < |\mathbf{k}| < M\}$ for some positive $M > 0$.

Let us define the diffusion matrix D_{mn} by

$$D_{mn}(\widehat{\mathbf{k}},l) = -\frac{1}{2}\int_{-\infty}^{+\infty}\frac{\partial^2 R(H_0'(l)s\widehat{\mathbf{k}},l)}{\partial x_n \partial x_m}\,ds = -\frac{1}{2H_0'(l)}\int_{-\infty}^{+\infty}\frac{\partial^2 R(s\widehat{\mathbf{k}},l)}{\partial x_n\partial x_m}\,ds, \quad m,n=1,\ldots,d. \tag{14}$$

Let also

$$\|G\|_{p,q} := \sum_{|\beta|=p,\|\gamma|=q}\sup_{(\mathbf{x},\mathbf{k})\in\mathbb{R}^{2d}}|\partial_{\mathbf{x}}^\beta\partial_{\mathbf{k}}^\gamma G(\mathbf{x},\mathbf{k}). \tag{15}$$

Then we have the following results.

Theorem 4 *Let ϕ^δ be the solution of (13) and let $\overline{\phi}$ satisfy*

$$\begin{aligned}\frac{\partial\overline{\phi}}{\partial t} = \sum_{m,n=1}^{d}\frac{\partial}{\partial k_m}\left(D_{mn}(\widehat{\boldsymbol{k}},k)\frac{\partial\overline{\phi}}{\partial k_m}\right) + H_0'(k)\widehat{\boldsymbol{k}}\cdot\nabla_{\boldsymbol{x}}\overline{\phi}\\ \overline{\phi}(0,\boldsymbol{x},\boldsymbol{k}) = \phi_0(\boldsymbol{x},\boldsymbol{k}).\end{aligned} \tag{16}$$

Then there exist two constants C, $\alpha_0 > 0$ such that for all $T \geqslant 1$, $M \geqslant 1$,

$$\sup_{(t,\boldsymbol{x},\boldsymbol{k})\in[0,T]\times K}\left|\mathbb{E}\phi^\delta\left(\frac{t}{\delta},\frac{\boldsymbol{x}}{\delta},\boldsymbol{k}\right) - \overline{\phi}(t,\boldsymbol{x},\boldsymbol{k})\right| \leqslant CT(1+\|\phi_0\|_{1,4})\delta^{\alpha_0} \tag{17}$$

for all compact sets $K \subset \mathcal{A}(M)$.

Let $w(t,\mathbf{x},k)$ be the solution of the spatial diffusion equation

$$\begin{aligned}\frac{\partial w}{\partial t} &= \sum_{m,n=1}^{d} a_{mn}(k)\frac{\partial^2 w}{\partial x_n\partial x_m},\\ w(0,\mathbf{x},k) &= \overline{\phi}_0(\mathbf{x},k)\end{aligned} \tag{18}$$

with the average initial data

$$\overline{\phi}_0(\mathbf{x},k) = \frac{1}{\Gamma_{d-1}}\int_{\mathbb{S}^{d-1}}\phi_0(\mathbf{x},\mathbf{k})\,d\Omega(\widehat{\mathbf{k}}).$$

Here $d\Omega(\widehat{\mathbf{k}})$ is the surface measure on the unit sphere $\mathbb{S}^{d-1}$ and Γ_n is the area of an n-dimensional sphere. The diffusion matrix $A := [a_{mn}]$ in (18) is given explicitely as

$$a_{mn}(k) = \frac{1}{\Gamma_{d-1}}\int_{\mathbb{S}^{d-1}} H_0'(k)\widehat{k}_n\chi_m(\mathbf{k})\,d\Omega(\widehat{\mathbf{k}}), \tag{19}$$

where the functions χ_j appearing above are the mean-zero solutions of

$$\sum_{m,n=1}^{d}\frac{\partial}{\partial k_m}\left(D_{mn}(\widehat{\mathbf{k}},k)\frac{\partial\chi_j}{\partial k_n}\right) = -H_0'(k)\widehat{k}_j. \tag{20}$$

The following theorem holds.

Theorem 5 *For every $0 < T_* < T < +\infty$ the re-scaled solution $\overline{\phi}_\gamma(t, \boldsymbol{x}, \boldsymbol{k}) = \overline{\phi}(t/\gamma^2, \boldsymbol{x}/\gamma, \boldsymbol{k})$ of (16) converge as $\gamma \to 0$ in $\mathcal{C}([T_*, T]; L^\infty(\mathbb{R}^{2d}))$ to $w(t, \boldsymbol{x}, \boldsymbol{k})$. Moreover, there exists a constant $C > 0$ so that we have*

$$\|w(t, \cdot) - \overline{\phi}_\gamma(t, \cdot)\|_{0,0} \leqslant C(\gamma T + \sqrt{\gamma})\|\phi_0\|_{1,1} \tag{21}$$

for all $T_* \leqslant t \leqslant T$.

As an immediate corollary of Theorems 1 and 2, we obtain the following result.

Theorem 6 *Let ϕ_δ be solution of (13) with the initial data $\phi_\delta(\delta^{1+\alpha}(0, \boldsymbol{x}, \boldsymbol{k}) = \phi_0(\delta^{1+\alpha}\boldsymbol{x}, \boldsymbol{k})$ and let $\overline{w}(t, \boldsymbol{x})$ be the solution of the diffusion equation (18) with the initial data $w(0, \boldsymbol{x}, k) = \overline{\phi}_0(\boldsymbol{x}, k)$. Then there exists $\alpha_o > 0$ and a constant $C > 0$ so that for all $0 \leqslant \alpha \leqslant \alpha_0$ and all $0 < T_* \leqslant T$ we have for all compact sets $K \subset \mathcal{A}(M)$*

$$\sup_{(t,\boldsymbol{x},\boldsymbol{k})\in[T_*,T]\times K} |w(t, \boldsymbol{x}, \boldsymbol{k}) - \mathbb{E}\overline{\phi}_\delta(t, \boldsymbol{x}, \boldsymbol{k})| \leqslant CT\delta^{\alpha_0\alpha}, \tag{22}$$

where $\overline{\phi}_\delta(t, \boldsymbol{x}, \boldsymbol{k}) := \phi_\delta(t/\delta^{1+2\alpha}, \boldsymbol{x}\delta^{1+\alpha}, \boldsymbol{k})$.

Theorem 3 shows that the movement of a particle in a weakly random quenched Hamiltonian is, indeed, approximated by a Brownian motion in the long-time space limit, at least for times $T \leqq \delta^{-\alpha_0}$. In fact we can allow T_* to vanish as $\delta \to 0$ by choosing $T_* = \delta^{3\alpha/2}$.

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Rémi Léandre (**Université de Bourgogne, Dijon**)

Recent developmemts in Malliavin calculi of Bismut's type

The talk is divided in two parts

- We give a geometrical hypoelliptic diffusion by using Langerock's connection and establish a geometrical Hörmander's theorem, by avoiding the use of Malliavin's matrix.
- We translate in semi-group theory Bismut's approach to Malliavin calculus, and we eliminate the probability language in Malliavin calculus. We apply this in an elliptic situation for sake of simplicity.

Paul Lescot (**Université de Picardie**)

Isovectors and Euclidean quantum mechanics: the general case

The isovectors for the heat equation were first computed by S. Lie, whose result was later rediscovered by Estabrook and Harrison. In a previous joint work with J.-C. Zambrini (Proceedings of Ascona 2002), we studied the structure of the Lie algebra of these isovectors, and applied that knowledge to the study of Bernstein diffusions arising in Euclidean Quantum Mechanics. In particular, we gave a new interpretation of a bilinear form first constructed by Zambrini using stochastic calculus, and we found algebraic analogues of Itô's formula and of a well-known result in classical Analytical Mechanics. I shall present an extension of these results to the case of the heat equation with linear drift and quadratic potential. The main new tool in the proofs is Rosencrans' concept of "perturbation algebra of an elliptic operator."

Hannelore Lisei (Babeş-Bolyai University)

Approximation of stochastic differential equations driven by fractional Brownian motion

In [1] it is proved that a fractional Brownian motion $B = (B(t))_{t\in[0,1]}$ with Hurst index $H \in (0,1)$ can be written as

$$B(t) = \sum_{n=1}^{\infty} \frac{\sin(x_n t)}{x_n} X_n + \sum_{n=1}^{\infty} \frac{1-\cos(y_n t)}{y_n} Y_n,$$

where $x_1 < x_2 < \cdots$ are the positive, real zeros of the Bessel function of first type J_{-H}, while $y_1 < y_2 < \ldots$ are the positive, real zeros of the Bessel function of first type J_{1-H}, $(X_n)_{n\in\mathbb{N}}$ and $(Y_n)_{n\in\mathbb{N}}$ are two independent sequences of centered Gaussian random variables such that

$$\operatorname{Var} X_n = \frac{2c_H^2}{x_n^{2H} J_{1-H}^2(x_n)}, \quad \operatorname{Var} Y_n = \frac{2c_H^2}{y_n^{2H} J_{-H}^2(y_n)},$$

where

$$c_H^2 = \frac{\sin(\pi H)}{\pi} \Gamma(1+2H).$$

Using this expansion of the fractional Brownian motion, we approximate the solutions of stochastic differential equations of the form

$$\begin{aligned} dX(t) &= F(X(t),t)dt + G(X(t),t)dB(t), \quad t \in [0,T], \\ X(0) &= X_0, \end{aligned}$$

where the random functions F and G satisfy with probability 1 the following conditions:

1. $F \in C(\mathbb{R}\times[0,T]), G \in C^1(\mathbb{R}\times[0,T])$;
2. for each $t \in [0,T]$ the functions $F(\cdot,t), \dfrac{\partial G(\cdot,t)}{\partial x}, \dfrac{\partial G(\cdot,t)}{\partial t}$ are locally Lipschitz.

The Hölder continuity of the sample paths of B ensures the existence of the integrals

$$\int_0^T G(X(t),t)dB(t),$$

defined in terms of fractional integration as investigated in [2] and [3]. The approximations given in this paper have practical relevance in financial mathematics, for example in the model for the price of risky assets.

This is joint work with Anna Soós.

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Ravi R. Mazumdar (University of Waterloo)

Boundary properties of reflected diffusions with jumps in the positive orthant

Reflected diffusions with jumps on the positive orthant arise in many applications such as finance and heavy-traffic limits in queueing networks. Characterizing the boundary behavior is key to being able to compute distributions.

In this talk I will present some new results on a local time characterization of the boundary properties for reflected-diffusions with jumps in wedges. We allow for random reflection matrices. We provide conditions on the reflection matrix for stationary distributions to exist and we provide sufficient conditions for a "product-form" to exist. We provide will discuss some typical examples of Lévy networks as well as the relation of these results to earlier results on Semi-martingale Reflected Brownian Motion (SRBM) due to Williams et al..

This is joint work with Francisco Piera and Fabrice Fuillemin.

Sylvie Méléard (Université de Paris X)

Individual-based probabilistic models and various time-scaling approximations in adaptive evolution

A distinctive signature of living systems is Darwinian evolution, that is, a propensity to generate as well as select individual diversity. To capture this intrinsic feature of life, new classes of mathematical models are emerging. These models are rooted in the microscopic, stochastic description of a population of discrete individuals characterized by one or several adaptive traits. Bolker and Pacala [1] and Dieckmann and Law [2] have offered appealing heuristics to scale the microscopic description of an evolving population as an individual-based stochastic process.

We start with a rigorous microscopic description of a population of discrete individuals characterized by one or several adaptive traits. The population is modelled as a stochastic point process whose generator captures the probabilistic dynamics in continuous time of birth, mutation and death, as influenced by each individual's trait values. The adaptive nature of a trait implies that an offspring usually inherits the trait values of her progenitor, except when a mutation occurs. In this case, the offspring makes an instantaneous mutation step at birth to new trait values. The interaction between individuals implies a trait competition, leading to selection and modelled by a death rate depending on the total population at each time.

We propose a rigorous algorithmic construction of the population point process as an individual-based model of adaptive evolution. This construction gives moreover an effective simulation algorithm. We prove some martingale properties satisfied by this measure-valued process, which are the key point of our approach. We are next interested in finding some more tractable approximations. One can follow different mathematical paths. The first approach, classical for evolutionary biologists, aims at deriving deterministic equations to describe the moments of trajectories of the point process, i.e. the statistics of a large number of independent realizations of the process. We explain the difficult hierarchy between these equations coming from the competition kernels and preventing, even in the simple mean-field case, decorrelations and tractable moment closure. The alternative approach involves renormalizations of the point process based on a large population limit. According to different time or mutation step scalings, we obtain different limiting partial differential equations, either deterministic or stochastic. These results are based on the semi-martingale decomposition of the measure-valued process describing the renormalized population. The interest of this approach is the unification of different models, pointing out how different time scalings may involve very different approximations. More precisely we assume that there exists a fixed amount of resources and we consider the following asymptotics:

- By itself, the large-population limit leads to a deterministic, nonlinear integro-differential equation.

- When combined with the acceleration of birth (hence mutation) and death and an asymptotic of small mutation steps, the large-population limit yields either a deterministic nonlinear reaction-diffusion model, or a stochastic measure-valued process (depending on the acceleration rate of the birth-and-death process). Hence we give a justification to the appearance of some demographic stochasticity, observed by biologists in case of fast birth-and-death processes.
- When this acceleration of birth and death is combined with a limit of rare mutations, the large-population limit yields a nonlinear integro-differential equation, either deterministic or stochastic, depending here again on the speed of the scaling of the birth-and-death process.
- We finally model in an initially monomorphic population a time scale separation between ecological events (fast births and deaths) and evolution (rare mutations). The competition between individuals takes place on the short time scale. This leads in a large population limit and on the mutation time scale to a jump process over the trait space, where the population stays monomorphic at any time.

We show how this approach may be generalized to spatially structured populations, where the individuals are moreover migrating, following a reflected diffusion in a bounded domain. The individuals are then characterized both by their position and trait value. We prove that if the population size tends to infinity, the renormalized individual-based process converges to the weak measure-valued solution of a nonlinear partial differential equation involving position and trait. The nonlinearity is nonlocal, depending on the spatial interaction range. Under some non-degenerescence and smoothness assumptions on the migration coefficient and initial density hypothesis, we prove using the associated evolution equation, that the measure-valued solution has at each time a density with respect to the Lebesgue measure. This density depends on the spatial interaction range and converges, as the latter tends to zero, to the solution of a spatially local nonlinear partial differential equation. Simulations show the intricate influences between migration, mutation and selection.

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Annie Millet (Université Paris 1)

Stochastic analysis and rough paths of the fractional Brownian motion

Using anticipating calculus, L. Coutin and Z. Qian [1] have constructed geometric rough paths above the trajectories of the fractional Brownian motion W^H with Hurst parameter $H > \frac{1}{4}$. In a joint work with M. Sanz-Solé, we study several aspects of these rough paths in the topology of p-variation, such as large deviations and Wong-Zakai approximations by means of elements of the reproducing kernel Hilbert space which are not linear interpolations of W^H. This extends results proved by M. Ledoux, Z. Qian end T. Zhang [2] for the rough paths of the Brownian motion. The *universal limit theorem* in [3] allows to transfer these properties to some dynamical systems driven by a fractional Brownian motion. As a by-product of our study, geometric rough paths above the elements of the reproducing kernel Hilbert space of the fractional Brownian motion are obtained and an explicit integral representation is given.

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Ivan Nourdin (**Université Henri Poincaré, Nancy 1**)

Absolute continuity in SDE's driven by a Lévy process or a fractional Brownian motion

We study the problem of absolute continuity in the SDE

$$X_t = x_0 + \int_0^t \sigma(X_s)\, dB_s^H + \int_0^t b(X_s)\, ds, \quad t \in [0,1],$$

where σ and b are real functions, $x_0 \in \mathbb{R}$ and B^H is a fractional Brownian motion with *any* Hurst index in $(0,1)$. More precisely, if

$$t_x = \sup\left\{ t \in [0,1] \colon \int_0^t |\sigma(x_s)|\, ds = 0 \right\},$$

where x is defined by $x_t = x_0 + \int_0^t b(x_s)\, ds$, then the law of X_t has a density with respect to the Lebesgue measure if and only if $t > t_x$.

We also study a companion problem, that is the problem of absolute continuity in the SDE

$$X_t = x_0 + \int_0^t b(X_s)\, ds + Z_t,$$

where Z is a real Lévy process without Brownian part and b a $\mathcal{C}^1$-function with bounded derivative. If we denote ν the Lévy measure of Z, we will explain why, when b is monotonous at x_0, we have $X_1 \ll \lambda \iff \nu$ is infinite. In full generality on b, we will prove $Z_1 \ll \lambda \implies X_1 \ll \lambda$.

This is joint work with Thomas Simon.

Étienne Pardoux (**Université de Provence, Marseille**)

Homogenization of PDEs with periodic degenerate coefficients

We study by a probabilistic argument the homogenization of linear PDEs with periodic coefficients. The novelty is that we allow the matrix of the coefficients of the second order PDE operator to degenerate and even possibly to vanish on a set with nonvoid interior.

Victor de la Peña (**Columbia University**)

An upper law of the iterated logarithm without moment or dependence conditions

In this talk I will present an LIL which as a special case gives a new upper LIL for self-normalized martingales. This result extends naturally the LIL's by Kolmogorov and Stout. The key to the development is a new class of exponential martingales. The optimality properties of our results will also be discussed.

This is joint work with M. J. Klass and T. L. Lai.

Edwin Perkins (University of British Columbia)

Uniqueness for degenerate SPDE's and SDE's

The work is motivated by the following parabolic SPDE:

$$\frac{\partial u}{\partial t}(t,x) = \frac{\Delta u}{2}(t,x) + (\gamma(u(t,\cdot),x))^{1/2}\dot{W}_{t,x}, \tag{23}$$

where $u_0 \in C_b(\mathbb{R}) \cap L^1(\mathbb{R})$, $u_0 \geqslant 0$, $\dot{W}_{t,x}$ is white noise on $\mathbb{R}_+ \times \mathbb{R}$, $u \geqslant 0$, $\gamma : C_b(\mathbb{R}_+) \times \mathbb{R} \to [\epsilon, \epsilon^{-1}]$ continuous. $u(t,x)dx$ arises as a scaling limit of empirical measures of a system of critical branching random walks whose branching rates at (t,x) is $\gamma(u_N(t,\cdot),x)$, where u_N is an approximate density for the branching rw's. If $\gamma(\cdot) \equiv \gamma^0$, the solution is unique in law by a well-known duality proof, and is the density of 1-dimensional super-Brownian motion with branching rate γ^0. Nonetheless most questions about uniqueness remain unresolved including the following:

(i) Are solutions pathwise unique?

(ii) Are solutions unique in law?

(iii) For $d > 1$, are solutions to the measure-valued martingale problem corresponding to (1) unique?

(i) remains unresolved even for the case where γ is constant. The problem of course is that $u \to \sqrt{\gamma(u)u}$ is non-Lipschitz and degenerate.

We will not solve any of these questions but will illustrate some potential tools which have resolved some of these questions in finite and countable dimensional settings (work with Rich Bass and Don Dawson) and also look at (23) in the context of coloured noise, where some progress can be made on (i) (work with Leonid Mytnik and Anja Sturm).

If in (23) $\mathbb{R}$ is replaced with $\{1,\dots,d\}$ then one gets a process $X_t \in \mathbb{R}_+^d$, satisfying the SDE:

$$dX_t^i = \sqrt{\gamma_i(X_t)X_t^i}\, dB_t^i + b_i(X_t)\, dt,\ i = 1,\dots,d. \tag{24}$$

Assume:
(Γ) $\gamma_i : \mathbb{R}_+^d \to (0,\infty)$ continuous.
(B) $b_i : \mathbb{R}_+^d \to \mathbb{R}$ continuous; $|b_i(x)| \leqslant c(1+|x|)$ and $b_i(x) \geqslant 0$ on $\{x_i = 0\}$.

Special Case. $\gamma_i(\cdot) \equiv \gamma_i^0$, $b_i(x) = \sum_j x_j q_{ji}$, (q_{ji}) is the Q-matrix of a MC on $\{1,\dots,d\}$. Then (2) has pathwise unique solutions (Yamada-Watanabe) and is the super-Q MC.

Theorem 1 (Bass-P, 03) If b_i and γ_i are locally Hölder continuous, then for each initial law on $\mathbb{R}_+^d$, there is a unique in law solution to (24).

The result is false if Hölder continuity is replaced by continuity.

There are a number of catalytic branching type interactions which do not satisfy the non-degeneracy condition in (Γ). For example consider the cyclically catalytic branching equations

$$dX_t^i = (c_i(X_t)X_t^{i+1}X_t^i)^{1/2}\, dB_t^i + b_i(X_t)\, dt, X^i \geqslant 0,\ i = 1,\dots,d,\ d+1 = 1. \tag{25}$$

Hence the branching rate of type i is proportional to amount of type $i+1$. If $c_i \equiv c_i^0$, $b_i(x) = \theta_i - x_i$, $\theta_i > 0$ the model is called the cyclically catalytic branching diffusion. If $d = 2$ this is a special case of the mutually catalytic branching (no spatial structure) which has been studied by a number of authors (Dawson-Perkins 98, Mytnik 98). Here uniqueness in law holds by a self duality relation which breaks down for $d > 2$.

Fleischmann-Xiong 01 studied the higher order models with spatial diffusion on the line but their results were limited without knowing uniqueness.

The general equation (25) with $d = 2$ arose in the Cox-Dawson-Greven program [CDG, Mem. AMS 04] of identifying the universality class for mutually catalytic branching upon multiple scale block renormalization of 2-type spatial systems.

We will describe recent joint work with Don Dawson which provides a general uniqueness result for SDE's for a more general system of branching catalytic networks which includes Theorem 1 and uniqueness in equation (25) above. The proofs use explicit calculations on additive Bessel-squared processes and also analytic ideas from Cannarsa and DaPrato 96. The hypotheses are Hölder continuity on the coefficients.

Returning to infinite dimensions, consider the following adaptation of (23):

$$\frac{\partial u}{\partial t}(t,x) = \frac{\Delta u}{2}(t,x) + \sigma(u(t,x))\dot{W}_{t,x}, \tag{26}$$
$$u_0 \in C_{\text{tem}} = \{f : \mathbb{R} \to \mathbb{R} : \forall \lambda > 0, |f|_\lambda = \sup_x e^{-\lambda|x|}|f(x)| < \infty\}.$$

Here $\sigma : \mathbb{R} \to \mathbb{R}$ Hölder continuous and we seek solutions s.t. $t \to u_t$ is in $C(\mathbb{R}_+, C_{\text{tem}})$.

Assume $\dot{W}_{t,x}$ is coloured noise, *i.e.*, $\{W_t(\phi) : \phi \in C_K^\infty(\mathbb{R}_+ \times \mathbb{R}), t \geqslant 0\}$ is a Gaussian martingale measure such that

$$E(W_\infty(\phi)W_\infty(\psi)) = \int_0^\infty \int\int \phi(s,x)\psi(s,y)k(x,y)dxdyds,$$

$|k(x,y)| \leqslant C(1 + |x-y|^{-\beta})$, $0 \leqslant \beta < 1$.

Note: If $k_\beta(x,y) = \frac{1-\beta}{2}|x-y|^{-\beta}$, then $\dot{W}^\beta \to \dot{W}$ (white noise) as $\beta \to 1-$.

Peszat-Zabczyk[97], Dalang[99], Sanz-Solé and Sarrà [02]: If σ is Lipschitz, $\beta < 1$, solutions to (SPDE) are pathwise unique and locally Hölder continuous of index $\frac{1-\beta/2}{2} - \epsilon$ in t and $1 - (\beta/2) - \epsilon$ in x. The following result is joint with Leonid Mytnik and Anja Sturm.

Theorem 2. Assume σ is α-Hölder continuous with $1 \geqslant \alpha > \frac{1+\beta}{2}$. Then there is a pathwise unique solution to (26).

Note: As $\beta \to 1-$, we have $\alpha \to 1-$ and above "converges" to classical uniqueness of Lipschitz σ for white noise.

The proof is an infinite dimensional version of the well-known pathwise uniqueness result for one-dimensional sde's of Yamada and Watanabe. It is an extension of a result of Viot for k bounded ($\beta = 0$) for Fleming-Viot processes. For $\beta > 0$ $u(t,x)$ is too rough to be a semimartingale in t and so the stochastic calculus argument is more delicate. The Hölder continuity results of Dalang [99] and Sanz-Sole and Sarra [02] and factorization method they use (DaPrato-Kwapien-Zabczyk [87])are used in the proof.

Dominique Picard (**Université de Paris 7**)

Estimation fonctionnelle dans le cadre de problèmes inverses

Nous considérerons le problème d'estimer une fonction qui est observée après passage par un opérateur régularisant puis bruitée. L'exemple le plus typique est celui d'une fonction d'abord régularisée par la convolution avec une fonction régulière puis bruitée par un bruit blanc de faible amplitude —ou encore estimer la densité de la variable X en observant $Z_1, \ldots, Z_n$, i.i.d., $Z_i = X_i + U_i$, les U_i étant indépendantes des X_i et de loi connue.

La difficulté essentielle de ce problème consiste en l'apparition de deux bases 'naturelles' mais éventuellement antagonistes : une base qui diagonalise l'opérateur (SVD), permet de faire des calculs explicites et respecte la structure décorrélée du bruit (dans le cas de la convolution, c'est la base de Fourier) et une base (typiquement une base d'ondelette WAVE) dans laquelle la régularité de la fonction s'exprime bien et permet des calculs en norme L_p.

Nous proposons une méthode (WAVE-VD) qui allie les intérêts des deux bases et permet d'estimer directement les coefficients du développement en ondelette puis de les seuiller en respectant la structure du bruit. Nous montrons que cette méthode nous permet d'obtenir des vitesses minimax pour une grande classe de contraintes fonctionnelles (espaces de Sobolev ou de Besov) en fonction de la régularité de l'opérateur.

Nous étudions les conditions sur l'opérateur qui permettent de décrire les vitesses d'estimation obtenues par la méthode. Nous montrons que dans le cas particulier d'une déconvolution par une fonction 'boîte' (de la forme $I\{x \in [a, a+1]\}$) ces propriétés s'expriment en fonction des propriétés diophantiennes du réel a.

Nous montrons aussi que cette méthode permet de considérer des opérateurs éventuellement aléatoires ou même seulement partiellement observés.

Nicolas Privault (**Université de La Rochelle**)

Convex concentration inequalities via forward-backward stochastic calculus

Given (M_t) and (M_t^*) respectively a forward and a backward martingale with jumps and continuous parts, we prove that $E[\phi(M_t + M_t^*)]$ is a non-decreasing function of t for all convex functions ϕ, provided the local characteristics of $(M_t)_{t\in\mathbb{R}_+}$ and (M_t^*) satisfy some comparison inequalities. We deduce convex concentration inequalities and deviation bounds for random variables admitting a predictable representation in terms of a Brownian stochastic integral and a non-necessarily independent point process component.

This is joint work with Thierry Klein and Yutao Ma.

Bernard Prum (**Génopôle Évry**)

Markov and Hidden Markov Models in genome analysis

Biological sequences essentially consist in DNA chains, the chromosomes of which transmit the information from a generation to the next, and proteic chains, the proteins being the essential component of all phenomena in living cells. The first ones are writen in a 4 letters alphabet a, c, g, t while the second ones contain 20 letters, the amino-acid. Daily, more than 20 million new deciphered letters arrive in the data banks and a challenge for the statisticians is to help biologists find the relevant information in this vast amount of data.

A first topic we are interested in consists in searching for words whose frequency is too high to let us believe that they result from pure randomness. As an example, a signal (called CHI) exists in bacterial genomes and participates in their netural defenses and must therefore be sufficiently frequent to be effective. Hence CHI's role is irrelevant for the usual genetic code but has another importance for the organism.

To search for these exceptionnal words, we look for a modelisation which could be both satisfactory for the biologist and tractable for the mathematician. One has to take into account the frequencies of the letters, of the 2-letters words, 3-letters words, etc., hence to work conditionnally on the sufficient statistics of a Markov chain model. In these models, for each word W, using a conditionnal approach, we compute the expectation and the variance of the number of occurrences and give results about its (asymptotic) law.

New models are also to be considered, where the length of the memory may depend on the context : on one side VLMC (variable length Markov chains) and, more generally, PMM (parsimonious Markov models) ; on an other side MTD (mixed transition models) : as they use less parameters, their criterion BIC is often better on real sequences.

A very relevant criticism against this modelisation is that it assumes the homogeneity of the sequence, and this hypothesis is less and less acceptable to biologists when they deal with larger and larger sequences. One way for answering these criticisms consists in allowing the simultaneous existence of more than one Markovian model and this led us to work with Hidden Markov Models (HMM) or, better SHMM (Semi HMM, where the law of each homogeneous segment may be chosen a priori). These models quickly turn out to be statistical tools permiting much more than the separate analysis of regions chosen to be homogeneous. The fact that, at the begining of the algorithm, we need not fix the Markovian transitions in each state or the positions of the various states implies that adjusting a HMM on a sequence produces its segmentation by allocating a common characteristic to all the segments related to a same state. An important drawback of the 'clasical' modelisation by HMM is that it implies that the areas corresponding to a same state must have length distributed according to an exponential law, and this is not at all verified in the reality of genomes. Semi-markovian models solve this difficulty : they allow every law for the length of the various area.

Joined with the use of charateristics of the biological context, these methods must significatively improve the performances of the predictions of homogeneous regions. We will present a few applications as search of "horizontal transfers" and "annotation". For some 10 years, it has been assumed that besides the vertical transmission (from parents to offspring), a phenomenon of horizontal transmission of genetic information plays an important role in the evolution of life. For example some viruses may copy a part of the genome of some individual and transport and incorporate it in the genome of another individual - maybe of an other species. The potential profit of this phenomenon is obvious : through such tranposons, a new beneficial gene can spread in a great number of species. As it is well known that each species leads to a different adjustment of a Markov model (frequencies of words change from one species to another), modelisation using HMM or HSMM - is perfectly adapted for searching for tranposons. The matter of "annotation" is to contribute to an automatic research in DNA sequences of coding parts, and within these of exons and introns (in "eucaryotes" - essentially every species except bacteriae - genes contain two kinds of regions : exon message is *in fine* translated into the proteins, while introns desappear during the maturation process). HMM is also a successful approach for this problem.

Mickael Röckner (**Universität Bielefeld**)

The stochastic porous media equation: a survey of recent results

The talk will be about a certain class of fully non-linear stochastic partial differential equations of type

$$dX_t = [\Delta\Psi(X_t) + \Phi(X_t)]dt + \sqrt{Q}dW_t \tag{27}$$

with values in the dual of the first order Sobolev space $H_0^1(D)$ with Dirichlet boundary conditions. Here D is a bounded open set in $\mathbb{R}^d$ and Ψ, $\Phi : \mathbb{R} \to \mathbb{R}$ are functions satisfying certain monotonicity and growth condition. In fact some of the recent results have been proved in a more general framework with a suitable Hilbert space H replacing $H_0^1(D)$ and the role of Laplacian being taken by a self-adjoint operator L with discrete spectrum. In particular, L can be the "Laplacian" on a fractal. In case $\Phi \equiv 0$ and $Q \equiv 0$, and e.g. Ψ is monomial, equation (27) is the classical porous medium equation. In the talk we shall present results both on weak and strong solutions of (27) as well as on their large time asymptotics and their invariant measures. We shall also discuss the corresponding Kolmogorov equations, Lyapunov functions of the generator and the infinite dimensional potential theory of the latter.

Bernard Roynette (**Université Henri Poincaré, Nancy 1**)

Penalization of a d-dimensional Bessel process ($0 < d < 2$) with a function of its local time at 0

Let (R_t) be a d-dimensional Bessel process started at 0 and let us denote by (L_t) its local time at 0. We first prove a penalization principle analogous to (30):

$$\lim_{t\to\infty} \frac{E\big[1_{\Gamma_u} h(L_t)\big]}{E[h(L_t)]} := Q^h(\Gamma_u) := E\big[1_{\Gamma_u} M_u^h\big], \quad \forall \Gamma_u \in \mathcal{F}_u \text{ and } u \geqslant 0, \tag{28}$$

where $h : \mathbb{R}_+ \to]0, +\infty[$ satisfies $\int_0^\infty h(x)\,dx = 1$ and (M_u^h) is the P-martingale

$$M_u^h = h(L_u)R_u^{2-d} + 1 - \int_0^{L_u} h(z)dz.$$

We also prove that $Q^h(L_\infty < \infty) = 1$ and we describe the distribution of (R_t) under Q^h.

Barbara Rüdiger (**Universität Koblenz-Landau**)

Stochastic differential equations with non-Gaussian additive noise on Banach spaces

Itô integrals of random Banach space-valued functions with respect to compensated Poisson random measures (cPrm) are discussed in [5]. The Lévy-Itô decomposition theorem states that any additive process can be uniquely decomposed into a continuous semimartingale driven by a Brownian motion and a pure jump semimartingale driven by a jump martingale obtained by an Itô integral with respect to a cPrm. In [1], we prove that the Lévy-Itô decomposition theorem holds also on separable Banach spaces of type 2. (It holds also on general separable Banach spaces (see Dettweiler [2]) but in this case the integral with respect to a cPrm is not defined in terms of an Itô integral.) These results permit us [4] to define non Gaussian additive noise and to study stochastic differential equations (SDEs)with non-Gaussian additive noise on separable Banach spaces. Existence and uniqueness is proven under local Lipshitz conditions for the drift and noise coefficients. The Itô formula for Banach space-valued functions applied to the solutions of such SDEs is proven in [6]. This permits us to analyze solutions of others SDEs [3] .

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Marta Sanz-Sole (**Universitat de Barcelona**)

An approximation scheme for the stochastic wave equation

We study strong approximations for the non-linear stochastic wave equation

$$\frac{\partial^2 u}{\partial t^2}(t,x) = \frac{\partial^2 u}{\partial x^2}(t,x) + f(t,x,u(t,x)) + \sigma(t,x,u(t,x))\frac{\partial^2 W}{\partial t \partial x}(t,x), \tag{29}$$

$t > 0$, $x \in (0,1)$, with initial conditions $u(0,x) = u_0$, $\dfrac{\partial u}{\partial t}(0,x) = v_0$, and Dirichlet boundary conditions, by means of a sequence obtained as follows: for any $n \geqslant 1$, we fix the spatial grid $x_k = \frac{k}{n}$, $k = 1, \dots, n-1$, and consider the system of SDE's with the corresponding discretized Laplacian and freezing the evolution equation at the points of the grid. With linear interpolation, this provides an implicit evolution scheme.

Assuming that $u_0 \in H^{\alpha,2}([0,1])$, $v_0 \in H^{\beta,2}([0,1])$, with $\alpha > \frac{3}{2}$, $\beta \geqslant \frac{1}{2}$, we prove convergence in any $L^p(\Omega)$, uniformly in t, x, to the solution of (29) with a rate of order $n^{-\rho}$, $0 < \rho < \frac{1}{3} \wedge (\alpha - \frac{3}{2}) \wedge (\beta - \frac{1}{2})$. As a preliminar, we study the Hölder continuity of the sample paths of the solution to (29). In comparison with parabolic examples, the rate of convergence differs substantially from the Hölder continuity order. We have checked with a numerical analysis that one cannot expect better results.

This is joint work with Lluís Quer-Sardanyons.

Michael Scheutzow (**Technische Universität Berlin**)

Attractors for ergodic and monotone random dynamical systems

In this talk, we relate ergodicity, monotonicity and attractors of a random dynamical system (rds). The main result is that under suitable conditions, ergodicity and monotonicity together imply the existence of a weak random attractor which consists of a single random point only. We also show that ergodicity alone does not suffice to guarantee the existence of a weak attractor – not even when the one-point motion is a diffusion on $\mathbb{R}^d$ for $d \geqslant 2$.

Our motivation for studying this problem is to provide a rather general sufficient condition for a random attractor. Indeed several authors have proved the existence of random attractors for particular systems using ad hoc methods and in many cases the systems were in fact monotone and ergodic. When an rds is monotone, then this property is usually very easy to prove. Ergodicity is not always easy to prove – but in most cases still considerably easier than to prove the existence of an attractor. The result below provides a stronger conclusion than most other results on attractors in that we show that the attractor is trivial (i.e. consists of a single point). On the other hand, due to the generality of the set-up, we can only show that the attractor attracts all deterministic compact sets in probability while some authors prove almost sure attraction for a larger class of sets.

We now state the assumptions and the main result. Let (X, d) be a complete, separable metric space and let $\mathbb{T}$ be either $\mathbb{R}$ or $\mathbb{Z}$. Further $\mathbb{T}_+$ denotes the set of nonnegative elements from $\mathbb{T}$. We denote by $\mathcal{B}(X)$ the Borel σ-algebra of subsets of X. By definition, a *random dynamical system* with time $\mathbb{T}_+$ and state space X is a pair (ϑ, φ) consisting of the following two objects:

- A *metric dynamical system* (mds) $\vartheta \equiv (\Omega, \mathcal{F}, \mathbb{P}, \{\vartheta(t), t \in \mathbb{T}\})$, *i.e.* a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a family of measure preserving transformations $\vartheta \equiv \{\vartheta(t) : \Omega \to \Omega, t \in \mathbb{T}\}$ such that
  - **(a)** $\vartheta(0) = \mathrm{id}$, $\vartheta(t) \circ \vartheta(s) = \vartheta(t+s)$ for all t, $s \in \mathbb{T}$;
  - **(b)** the map $(t, \omega) \mapsto \vartheta(t)\omega$ is measurable and
  - **(c)** $\vartheta(t)\mathbb{P} = \mathbb{P}$ for all $t \in \mathbb{T}$.
- A (perfect) *cocycle* φ over ϑ of continuous mappings of X with one-sided time $\mathbb{T}_+$, *i.e.* a measurable mapping
$$\varphi : \mathbb{T}_+ \times \Omega \times X \to X, \quad (t, \omega, x) \mapsto \varphi(t, \omega)x$$
  such that the mapping $x \mapsto \varphi(t, \omega)x$ is continuous for every $t \geqslant 0$ and $\omega \in \Omega$ and it satisfies the cocycle property:
$$\varphi(0, \omega) = \mathrm{id}, \quad \varphi(t+s, \omega) = \varphi(t, \vartheta(s)\omega) \circ \varphi(s, \omega)$$
  for all t, $s \geqslant 0$ and $\omega \in \Omega$.

We call an rds *ergodic* if there exists a probability measure π on X such that for each $x \in X$, the law of $\varphi(t, \omega)x$ converges weakly to π. Note that we do not assume that the one-point motion is Markovian.

Next, we introduce the concept of a *random attractor.*

Definition 1 Let $\mathbb{C}$ be the family of compact subsets of X. The mapping $\mathbb{A} : \Omega \to \mathbb{C}$ is called an *invariant random compact set* if

(i) $\omega \mapsto d(x, \mathbb{A}(\omega))$ is measurable for each $x \in X$, where $d(x, \mathbb{A}) = \inf_{y \in \mathbb{A}} d(x, y)$.

(ii) There exists a set $\widetilde{\Omega} \in \mathcal{F}$ of full measure which is invariant under $\vartheta(t)$ for each $t \in \mathbb{T}$ such that $\varphi(t, \omega)(\mathbb{A}(\omega)) = \mathbb{A}(\vartheta(t)\omega)$ for all $\omega \in \widetilde{\Omega}$ and $t \in \mathbb{T}_+$.

An invariant random compact set is called a *pullback attractor* if for each compact set $B \subseteq X$

$$\lim_{t\to\infty} \sup_{x\in B} d(\varphi(t, \vartheta(-t)\omega, x), \mathbb{A}(\omega)) = 0 \quad \text{a. s.},$$

and a *weak attractor* if for each compact set $B \subseteq X$

$$\lim_{t\to\infty} \sup_{x\in B} d(\varphi(t, \vartheta(-t)\omega, x), \mathbb{A}(\omega)) = 0 \quad \text{in probability.}$$

Since almost sure convergence implies convergence in probability, every pullback attractor is also a weak attractor. The converse is not true (see, e.g., [5] for examples). The concept of a pullback attractor was proposed independently in [1] and [6]. Weak attractors were introduced in [4]. If an attractor (weak or pullback) exists, then it is unique up to sets of measure zero.

In the talk, we will provide examples, namely isotropic Brownian flows with drift on $\mathbb{R}^d$, $d \geqslant 2$, showing that an ergodic rds does not necessarily admit a weak attractor.

To introduce a *monotone* rds, we need a partially ordered state space X. We will assume that (X, d) is a suitable subset of an ordered Banach space V.

Let V be a real separable Banach space with a cone $V_+ \subset V$. By definition, V_+ is a closed convex set in V such that $\lambda v \in V_+$ for all $\lambda \geqslant 0$, $v \in V_+$ and $V_+ \cap (-V_+) = \{0\}$. The cone V_+ defines a partial order relation on V via $x \leqslant y$ iff $y - x \in V_+$ which is compatible with the vector space structure of V. If V_+ has nonempty interior $\mathrm{int}V_+$, we say that the cone V_+ is *solid*. For elements a and b in V such that $a \leqslant b$ we define the (conic) closed *interval* $[a, b]$ as the set of the form

$$[a, b] = \{x \in V : a \leqslant x \leqslant b\}.$$

If the cone V_+ is solid, then any bounded set $B \subset V$ is contained in some interval. A cone V_+ is said to be *normal* if every interval $[a, b]$ is bounded.

An rds (ϑ, φ) taking values in a subset X of V is called *monotone* or *order preserving*, if (possibly up to a universal set of measure zero) $x \leqslant y$ implies $\varphi(t, \omega)x \leqslant \varphi(t, \omega)y$ for all $t \in \mathbb{T}_+$ and $\omega \in \Omega$.

Theorem 7 *Let (ϑ, φ) be an ergodic and monotone rds taking values in a separable Banach space V with a solid and normal cone V_+. Then the rds has a weak attractor which consists of a single (random) point.*

The theorem remains true if V is replaced by a suitable subset $X \subseteq V$ (see [2]), but it does not hold for arbitrary subsets X of V (we will provide an example in the talk). The assumptions of the theorem do not guarantee that the rds has a pullback attractor. We point out that another paper dealing with attractors for an ergodic rds (with independent increments) is [3].

Our result can be applied to one-dimensional rds, to multi-dimensional rds with *cooperative* drift, to certain stochastic delay differential equations, certain parabolic spde's and some (monotone) interacting particle systems like the (dynamical) Ising model.

We will conclude the talk with some remarks about the relationship between the existence of a singleton attractor and the convergence of the *Propp-Wilson algorithm* – an algorithm for the perfect simulation of the invariant measure of a finite discrete time Markov chain via *coupling from the past*.

This is joint work with Igor Chueshov, see [2].

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Isabel Simao (**Universidade de Lisboa**)

Regularity of the transition semigroup associated with a diffusion process in a Hilbert space

Let P_t, $t \geqslant 0$ be the transition semigroup determined by the stochastic evolution equation

$$\begin{cases} dX(t) &= [AX(t) + F(X(t)]dt + dW(t), \\ X(0) &= x, \end{cases}$$

on a separable Hilbert space H, where W is a cylindrical Wiener process on H, A is a selfadjoint operator with a trace class inverse, and $F : H \to H$ is measurable, with at most linear growth. Under the assumption that $F = DU$, where $U : H \to \mathbb{R}$ is a Gateaux differentiable function in the domain of the Ornstein-Uhlenbeck generator, we give an explicit formula for the kernel of P_t, with respect to the centered Gaussian measure on H with covariance $-\frac{1}{2}A^{-1}$. This formula is then applied to prove smoothing properties of the transition semigroup.

Wilhelm Stannat (**Universität Bielefeld**)

On stability of the filter equation for nonergodic signals

Stability of the optimal filter with respect to its initial condition is studied for a nonlinear signal process observed with independent additive noise. We show that exponential stability holds if there exists a uniformly strictly log-concave ground state associated with the generator of the signal process and the square of the observation. An a priori lower bound on the exponential rate is given depending mainly on the mass gap of the corresponding ground state transform. Ergodicity of the signal process is not needed.

Karl-Theodor Sturm (**Universität Bonn**)

Mass transportation, equilibration for nonlinear diffusion, and Ricci curvature

We will give a survey on recent developments in optimal mass transportation. Among others:

- We explain how nonlinear diffusions (e.g. porous medium equation, fast diffusion, McKean-Vlasov equation) on $\mathbb{R}^n$ or on a Riemannian manifold M are related to gradient flows of certain functionals S (e.g. Rényi entropy functionals) on $\mathcal{P}_2(M)$, the L_2-Wasserstein space of probability measures on M.

- We show how convexity properties of S on $\mathcal{P}_2(M)$ imply functional inequalities (e.g. Talagrand inequalities, logarithmic Sobolev inequality) and equilibration of the underlying diffusion on M. In particular, we deduce the analogue to the Bakry-Émery condition for nonlinear diffusions.

- We introduce and analyze generalized Ricci curvature bounds for metric measure spaces (M, d, m), based on convexity properties of the relative entropy $Ent(.|m)$. For Riemannian manifolds, $Curv(M, d, m) \geq K$ if and only if $Ric_M \geq K$ on M. For the Wiener space, $Curv(M, d, m) = 1$.

  One of the main results is that these lower curvature bounds are stable under convergence. This solves one of the basic problems in this field, open for many years.

  The notion of convergence comes from a complete separable metric D on the space of normalized metric measure spaces, again defined in terms of mass transportation.

- We construct a canonical stochastic process on the Wasserstein space $\mathcal{P}_2(\mathbb{R})$ associated with any given probability measure m on $\mathbb{R}$. This process has an invariant measure which may be characterized as the 'uniform distribution' on $\mathcal{P}_2(\mathbb{R})$ with weight function $\frac{1}{Z}\exp(-\beta \cdot Ent(.|m))$.

Sami Tindel (**Université Henri Poincaré, Nancy 1**)

Young integrals and stochatic PDEs

In this talk, we will review some recents results obtained in collaboration with Massimiliano Gubinelli and Antoine Lejay, aiming at a pathwise definition of stochastic PDEs. We will first mention some (local) existence and uniqueness results obtained with the semi-group approach. We will then give the strategy to follow in the multiparametric setting, which should yield some improvements in the definition of a PDE driven by an infinite dimensional rough path.

Aubrey Truman (**University of Wales Swansea**)

A one dimensional analysis of real and complex turbulence and the Maxwell set for the stochastic Burgers equation

We consider the stochastic viscous Burgers equation with body forces white noise in time

$$\begin{aligned} \frac{\partial v^\mu}{\partial t} + (v^\mu \cdot \nabla)\, v^\mu &= \frac{\mu^2}{2}\Delta v^\mu - \nabla V(x) - \epsilon \nabla k_t(x) \dot{W}_t, \\ v^\mu(x,0) &= \nabla S_0(x) + \mathrm{O}(\mu^2), \end{aligned}$$

where $v^\mu(x,t) \in \mathbb{R}^d$ denotes the velocity field, $\dot{W}_t$ is white noise, μ^2 is the coefficient of viscosity and $x \in \mathbb{R}^d$, $t > 0$.

We are interested in the advent of discontinuities in the velocity as we take the inviscid limit ($\mu \to 0$). As we shall show, our analysis of these discontinuities produces a rigorous explanation of the turbulent behaviour of the Burgers fluid. To find the discontinuities, we use the Hopf-Cole transformation

$$v^\mu(x,t) = -\mu^2 \nabla \ln u^\mu(x,t),$$

to transform the Burgers equation into the corresponding Stratonovich heat equation:

$$\begin{aligned} \frac{\partial u^\mu}{\partial t} &= \frac{\mu^2}{2}\Delta u^\mu + \mu^{-2} V(x) u^\mu + \epsilon \mu^{-2} k_t(x) u^\mu \circ \dot{W}_t, \\ u^\mu(x,0) &= \exp\left(-\frac{S_0(x)}{\mu^2}\right) T_0(x), \end{aligned}$$

where the convergence factor T_0 is related to the initial Burgers fluid density.

Following Donsker, Freidlin et al., as $\mu \to 0$

$$-\mu^2 \ln u^\mu(x,t) \to \inf_{X(0)} \left[S_0(X(0)) + A(X(0), x, t)\right] = \mathcal{S}(x,t),$$

where

$$\begin{aligned} A(X(0),x,t) &= \inf_{\substack{X(s)\\ X(t)=x}} A[X], \\ A[X] &= \frac{1}{2}\int_0^t \dot{X}^2(s)\,ds - \int_0^t V(X(s))\,ds - \epsilon \int_0^t k_s(X(s))\,dW_s. \end{aligned}$$

This gives the minimal entropy solution of Burgers equation. To find the path X which extremises $\mathcal{A}[X] = A[X] + S_0(X(0))$, we require

$$d\dot{X}(s) + \nabla V(X(s))\,ds + \epsilon \nabla k_s(X(s))\,dW_s = 0, \qquad \dot{X}(0) = \nabla S_0(X(0)).$$

When we then minimise $\mathcal{A}[X]$ over $X(0)$ we find $\mathcal{S}(x,t)$, the minimal solution of the Hamilton-Jacobi equation

$$dS_t + \left(\frac{1}{2}|\nabla S_t|^2 + V(x)\right)\,dt + \epsilon k_t(x)\,dW_t = 0, \qquad S_{t=0}(x) = S_0(x).$$

For our analysis of the discontinuities we introduce the classical mechanical flow map by defining $\Phi_s : \mathbb{R}^d \to \mathbb{R}^d$,

$$d\dot{\Phi}_s + \nabla V(\Phi_s)\,ds + \epsilon \nabla k_s(\Phi_s)\,dW_s = 0, \qquad \Phi_0 = \mathrm{id}, \qquad \dot{\Phi}_0 = \nabla S_0.$$

Since $X(t) = x$ by definition, $X(s) = \Phi_s \Phi_t^{-1} x$, where we accept that $x_0(x,t) = \Phi_t^{-1} x$ is not necessarily unique. It is this non-uniqueness which will give rise to discontinuities. Given some regularity and boundedness, the global inverse function theorem gives a caustic time $T(\omega) > 0$ such that for $0 < s < T(\omega)$, Φ_s is a random diffeomorphism. For $t < T(\omega)$,

$$v^0(x,t) = \dot{\Phi}_t\left(\Phi_t^{-1} x\right),$$

is a classical solution of Burgers equation with probability one.

Discontinuities appear in the solution when this preimage is not unique. When $\Phi_t^{-1}\{x\} = \{x_0(1)(x,t), x_0(2)(x,t), \ldots, x_0(n)(x,t)\}$, where each $x_0(i)(x,t) \in \mathbb{R}^d$, the Feynman-Kac formula and Laplace's method in infinite dimensions give for a non-degenerate critical point:

$$u^\mu(x,t) = \sum_{i=1}^n \theta_i \exp\left(-\frac{S_0^i(x,t)}{\mu^2}\right),$$

where

$$S_0^i(x,t) = S_0\left(x_0(i)(x,t)\right) + A\left(x_0(i)(x,t), x, t\right),$$

θ_i being an asymptotic series in μ^2. Using a semi classical expansion, Truman and Zhao demonstrated that,

$$v^\mu(x,t) \sim \nabla S_0(x,t) + \mathrm{O}(\mu^2),$$

so that the dominant term in the asymptotic expansion for $v^0(x,t)$ comes from the $x_0(i)(x,t)$ which minimises the action. Therefore, jump discontinuities can occur if the minimiser suddenly changes. There are two distinct ways in which this can occur:

1. When we cross the caustic surface where an infinitesimal volume dx_0 is focussed into a zero volume $dX(t)$ by the flow map Φ_t, we have two real $x_0(i)(x,t)$'s coalescing and disappearing (becoming complex). When one of these is the minimiser we describe that part of the caustic as cool and the velocity field has a jump discontinuity.

2. When we cross the Maxwell set we have two real distinct $x_0(i)(x,t)$'s returning the same value of the action. If one of these corresponds to the minimiser then we describe the Maxwell set as cool and we again have a jump discontinuity in the velocity field.

In this paper we recapitulate the geometrical results established by Davies, Truman and Zhao relating the classical mechanical caustic to level surfaces of the Hamilton-Jacobi function and their algebraic preimages under the classical mechanical flow map. This demonstrates that cusps on the level surfaces correspond to intersections of the algebraic preimages of the caustic and level surface. Using their result that

$$\Phi_t x_0 = x \qquad \Leftrightarrow \qquad \nabla_{x_0}\mathcal{A}(x_0, x, t) = 0,$$

we develop a one dimensional analysis of the problem using the reduced (one-dimensional) action function. A complete analysis of the d-dimensional Burgers velocity field is then presented in terms of this reduced action function. We characterise those parts of the caustic which are singular (cool) and also analyse the geometry of the caustic. By considering the double points of the level surfaces in the two dimensional polynomial case, we find an explicit formula for the Maxwell set. This is later extended to provide a simple expression for the Maxwell set in any dimension as the double discriminant of the reduced action function, thereby solving a long standing problem for Hamiltonian dynamical systems. The chaotic nature of the solution in the presence of body forces which are white noise in time is manifested in the appearance of two new forms of turbulence, real and complex.

In real turbulence the prelevel surface touches the precaustic. This causes the number of cusps on the level surface to change infinitely rapidly, producing turbulent behaviour. It is shown that times when this occurs are zeros of a“zeta process” which is simply the reduced action function evaluated at points on the cool caustic where the Burgers velocity field has zero scalar product with the tangent to the caustic. The intermittence of this turbulence is then demonstrated by showing that this ”zeta process” is recurrent. An explicit formula for the cusp density on both the caustic and level surface is also given.

In complex turbulence, we use a circle of ideas due to Arnol'd, Cayley and Klein, to demonstrate that when the real part of the precaustic touches its complex counterpart, swallowtails form and disappear infinitely rapidly on the caustic. This not only alters the shape of the cool part of the caustic, but also typically creates small Maxwell sets within each swallowtail across which the velocity is also discontinuous. It is demonstrated that this occurs at zeros of a“resultant eta process” given by the resultant of the third and fourth derivatives of the reduced action.

It is also shown that this is a particularly turbulent form of the real turbulence outlined previously.

Gerald Trutnau (Universität Bielefeld)

Time inhomogeneous diffusions on monotonely moving domains

We construct and analyze in a very general way time inhomogeneous (possibly also degenerate or reflected) diffusions in monotonely moving domains $E \subset \mathbb{R} \times \mathbb{R}^d$, *i.e.* if $E_t := \{x \in \mathbb{R}^d \colon (t,x) \in E\}$, $t \in \mathbb{R}$, then either $E_s \subset E_t$, for all $s < t$, or $E_s \supset E_t$, for all $s < t$, s, $t \in \mathbb{R}$. Our major tool is a further developed $L^2(E, m)$-analysis with well chosen reference measure m. Among a few examples of completely different kinds, such as e.g. singular diffusions with reflection on moving Lipschitz domains in $\mathbb{R}^d$, non-conservative and exponential time scale diffusions, degenerate time inhomogeneous diffusions, we present an application to what we name skew Bessel process on γ. Here γ is either a monotonic function or a continuous Sobolev function. These diffusions form a natural generalization of the classical Bessel processes and skew Brownian motions, where the local time refers to the constant function $\gamma \equiv 0$.

This is joint work with F. Russo.

Ciprian Tudor (Université de Paris 1)

Statistical aspects of the fractional stochastic integration

We apply the techniques of stochastic integration with respect to the fractional Brownian motion and the Gaussian theory of regularity and supremum estimation to study the maximum likelihood estimator (MLE) for the drift parameter of stochastic processes satisfying stochastic equations driven by fractional Brownian motion with any level of Hölder-regularity (any *Hurst* parameter). We prove existence and strong consistency of the MLE for linear and nonlinear equations.

Pierre Vallois (**Université Henri Poincaré, Nancy 1**)

Limiting laws associated with Brownian motion perturbed by its one sided-maximum. An extension of Pitman's theorem

Let P_0 denote the Wiener measure defined on the canonical space $\big(\Omega = \mathcal{C}(\mathbb{R}_+, \mathbb{R}),\ (X_t)_{t\geq 0},\ (\mathcal{F}_t)_{t\geq 0}\big)$, and (S_t) be the one sided-maximum process. Let us consider Borel functions $\varphi : \mathbb{R}_+ \to]0, +\infty[$ such that $\int_0^\infty \varphi(x)dx = 1$.

The first main result is the following:

$$\lim_{t\to\infty} \frac{E_0\big[1_{\Gamma_u}\varphi(S_t)\big]}{E_0[\varphi(S_t)]} := Q_0^\varphi(\Gamma_u) := E_0\big[1_{\Gamma_u} M_u^\varphi\big], \quad \forall \Gamma_u \in \mathcal{F}_u \text{ and } \ u \geq 0, \tag{30}$$

where (M_u^φ) is the P_0-martingale : $M_u^\varphi = \varphi(S_u)(S_u - X_u) + 1 - \int_0^{S_u} \varphi(z)dz$.

We determine the law of (X_t) under the p.m. Q_0^φ defined on $\big(\Omega, \mathcal{F}_\infty\big)$ by (30). We prove in particular that $Q_0^\varphi(S_\infty < \infty) = 1$

The second main result deals with Pitman's Theorem. Although $(X_t)_{t\geqslant 0}$ is not a Markov process under Q_0^φ, we prove that $(R_t = 2S_t - X_t)_{t\geq 0}$ is, under Q_0^φ, in its own filtration, a three dimensional Bessel process started at 0.

Frederi Viens (**Purdue University**)

Some applicaltions of the Malliavin Calculus and Gaussian-analysis

The *Malliavin calculus*, also known as the stochastic calculus of variations, is a most prolific tool in contemporary stochastic analysis. Popular areas of application include anticipating stochastic calculus, fractional Brownian motion, financial mathematics, infinite-dimensional stochastic analysis, and many others. The following is a short selection of references: [2], [10], [16], [18], [19], [20], [22], [24], [36].

With few exceptions, these applications rely heavily on the Gaussian framework; however, this calculus is flexible enough to be useful in situations which are strikingly non-Gaussian (see [36]). Simultaneously, the theory of *Gaussian fields* can be invoked to provide sharp conditions for pathwise regularity and boundedness (see [1], [12], [27]); yet the fundamental underlying constructs can be adapted to non-Gaussian situations (see [17]). Our purpose here is to summarize several results – recently published or under review – illustrating the interplay between the Malliavin calculus and Gaussian analysis along the lines of the above ideas, including some results in non-Gaussian situations.

At the onset of our presentation, we give some background material on the Malliavin derivative, and on the notions of Gaussian and sub-Gaussian processes. We then present applications of seldom-quoted estimates found for example in Üstünel's textbook [36], yielding in particular a clear criterion for the sub-Gaussian property using Malliavin derivatives; a brief discussion of non-sub-Gaussian issues is also included. With these preliminaries in hand, we present a sample of applications.

The *stochastic Anderson model* in continuous space-time is a stochastic heat equation with linear multiplicative noise: see e.g. [6], [14], [25]. Its large-time exponential behavior (almost-sure Lyapunov "exponent") depends heavily on the potential's spatial regularity. While the first evidence of this property appears in our work [32] with S. Tindel done in 2001, we show ([13], with I. Florescu) stronger evidence by studying the solution's Feynman-Kac formula, and investigating the sub-Gaussian property and the average behavior of the solution's logarithm. Upper and lower bounds, which are sharp in some scales, are obtained for the exponential behavior in the region of small diffusivity. Open problems and current research directions are mentioned, including statements made continuously in time, questions regarding non-Brownian increments, non-sub-Gaussian situations, and large diffusivity.

One of the major successes of the Malliavin calculus is the possibility to define *Skorohod integration*, i.e. stochastic integration with respect to arbitrary Gaussian processes, such as *fractional Brownian motion* (fBm), especially for irregular processes (*Hurst* parameter $H < 1/2$): see Nualart's book [22], and the article of Alòs, Mazet, and Nualart [2] for fBm. However, the standard Skorohod integral definition was notorious for not allowing basic stochastic calculus (Itô's formula) to be established when $H \leq 1/4$. An idea of P. Cheridito and D. Nualart [7] can be used to extend the Skorohod stochastic calculus beyond this critical level. We show ([21], with O. Mocioalca) how to go even beyond the fBm scale, including non-uniformly continuous processes, one basic success of our method being that it is universal and free of fractional calculus. We discuss applications to existence and representation of local times, and a conjecture regarding their time-regularity.

For *multiparameter Gaussian fields*, the existence of a Skorohod stochastic calculus can again run into problems for small H (see C.A. Tudor et. al [11]). We show ([33], [34], with C.A. Tudor) that the extended Skorohod theory can be applied in such a situation, by specializing to the fractional Brownian sheet. The power of our technique is evidenced for example by proving a Tanaka formula for the sheet's local time, which holds for all $H > 0$ in this two-parameter situation, surpassing the level $H > 1/3$ which was previously achieved for the one-parameter fBm using non-extended integration.

A scalar stochastic differential equation driven by fractional Brownian motion can be solved using a Girsanov-type theorem if the noise term is additive, even if the drift is non-linear (see Nualart and Ouknine [23]). It is natural to answer the question of estimating the scalar intensity (amplitude) of the drift by defining a *Maximum Likelihood Estimator* (MLE) (see Kleptsyna and le Breton [15]). We use Gaussian and sub-Gaussian properties – established by calculating Malliavin derivatives, to show ([35], with C.A. Tudor) that the MLE exists even for non-linear drifts, and is a strongly consistent estimator of the drift intensity. Professor Ciprian Tudor is to provide more details on this result in this conference. The main open problem is to give a truly implementable estimator; we propose a partial answer.

Any *stochastic heat equation* (SHE) with linear additive noise has a unique evolution solution which can be expressed explicitly (see the classical works [9], [37]), using for example a spatial Fourier expansion of the noise (our work with S. Tindel and C.A. Tudor [28]). If the noise is fractional Brownian in time, we establish (with S. Tindel and C.A. Tudor [29], and with Y. Sarol [26]) conditions on the spatial regularity of the noise which characterize corresponding regularity conditions for the solution, both in space and time. In the case of compact one-dimensional parameter, our characterizations are sharp, which we prove by establishing a sharpening of the classical Dudley-Fernique-Talagrand Gaussian regularity theory (see [12], [27]). There should be no obstacle to considering regularity questions for SHEs with non-sub-Gaussian noises. Fairly sharp characterizations could be obtained thanks to Malliavin derivatives estimations.

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Alessandro Villa (**Université Joseph Fourier, Grenoble**)

Detection of dynamical systems from noisy multivariate time series

The pattern detection algorithm (PDA) [Tetko and Villa, 1997, 1999, 2001] is applied to study sets of time series produced by mathematical mappings (e.g., Hénon, Kaplan-Yorke, Ikeda). We show that this algorithm is particularly well suited to detect deterministic dynamics in the presence of noise. With an increase of noise (points are deleted at random, points are added at random, points are shifted in time by some jitter) PDA was able to detect spatio-temporal patterns of events that repeated more frequently than expected by chance. These patterns were related to the generating attractors even if classical dynamical system algorithms were unable to detect the underlying deterministic behavior. On the basis of this result we propose a filtering procedure aimed at decreasing the amount of noisy events in time series. This algorithm may improve the quality of the data for subsequent study, e.g. by classical dynamical system analytical methods, which is of considerable interest for specialists working with practical applications of time series analysis.

This approach is applied to the study of spike trains – the multivariate time series obtained by recording the epochs of the neuronal discharges in real brains networks – usually characterized by large embedded noise. Chaotic determinism in the dynamics of spiking neural networks has been observed in experimental data. This behavior was theoretically predicted and is considered as an important mechanism for representation of learned stimuli in large scale distributed networks. The synfire chain theory [Abeles, 1982, 1991], based on topological assumptions of diverging/converging feed-forward layers of neurons, suggests that whenever the same process repeats in a cell assembly in the brain, the same spatio-temporal firing patterns should appear. Synfire chains may exhibit structures in which a group of neurons excite themselves and maintain elevated firing rates for a long period. Let us note that the synfire chain theory emphasizes the importance of precise timing of spikes (precise temporal coding), while theories of attractor neural networks, generally speaking, do not require it (noisy rate coding). The present study may contribute to understanding whether these two theories represent two faces of the same coin.

This is joint work with Yoshiyuki Asai.

John B. Walsh (**University of British Columbia**)

Some remarks on the rate of convergence of numerical schemes for the stochastic wave equation

We examine a numerical scheme for the stochastic wave equation

$$\frac{\partial^2 U}{dt^2} = \frac{\partial^2 U}{dt^2} + f(x,t,u(x,t)) + g(x,t,u(x,t))\,\dot{W}$$

with Lipschitz coefficients and appropriate initial and boundary conditions.

We show that it converges with order $\sqrt{h}$ in the L^2 norm, uniformly in compact sets, and that for any $\epsilon > 0$ it converges with order $h^{1/2-\epsilon}$ a.s. pointwise, where h is the size of the space and time step. We also show that this rate is optimal in the sense that any other scheme which depends only on the same increments of the white noise will have a locally uniform L^2 rate of convergence which is at best $O(\sqrt{h})$.

Moshe Zakaï (**Technion-Haïfa**)

The Clark-Ocone formula for vector valued Wiener functionals

The classical representation of random variables as the Itô integral of nonanticipative integrands is extended to include Banach space valued random variables on an abstract Wiener space equipped with a filtration induced by a resolution of the identity on the Cameron-Martin space.

The Itô integral is replaced in this case by an extension of the divergence to random operators, and the operators involved in the representation are adapted with respect to this filtration in a suitably defined sense.

A complete characterization of measure preserving transformations in Wiener space is presented as an application of this generalized Clark-Ocone formula.

This is joint work with E. Meyer-Wolf.

Lorenzo Zambotti (**Politecnico di Milano**)

A renewal approach to periodic copolymers

We consider a directed copolymer in two solvents separated by an interface. The copolymer is given by a periodic chain of different monomers, each preferring one of the two solvents. This system displays a localization/delocalization transition. We give a detailed description of the infinite volume measure, computing Brownian scaling at all phases, including the critical case.

Jean-Claude Zambrini (**Universidade de Lisboa**)

Stochastic quadratures of diffusion processes

In [1], the notion of "stochastic quadrature" was introduced, for a class of diffusion processes relevant to free quantum dynamics. This notion provides, in particular, a systematic way to reinterpret most explicit relations between diffusion processes along a geometric-algebraic line inspired by quantum symmetries. We shall generalize the results of [1] beyond the free case, show how they unify dynamically a number of representations already known, and allow to discover new ones.

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Boguslaw Zegarlinski (Imperial College London)

Nonlinear Markov Semigroups for large interacting systems

We will present recent results on construction and ergodicity properties of nonlinear Markov semigroups for large interacting systems.

ABSTRACTS

of the

Minisymposium on Stochastic Methods in Financial Models

Jean-Pierre Aubin (Université de Paris Dauphine)

A tychastic approach to financial problems

We prensent the viability/capturability approach for studying the problem of dynamic valuation and management of a portfolio with transaction costs in the framework of tychastic control systems (or dynamical games against nature) instead of stochastic control systems. Indeed, the very definition of the guaranteed valuation set can be formulated directly in terms of guaranteed viable-capture basin of a dynamical game. Hence, we shall "compute" the guaranteed viable-capture basin and find a formula for the valuation function involving an underlying criterion, use the tangential properties of such basins for proving that the valuation function is a solution to Hamilton-Jacobi-Isaacs partial differential equations. We then derive a dynamical feedback providing an adjustment law regulating the evolution of the portfolios obeying viability constraints until it achieves the given objective in finite time. We shall show that the Pujal/Saint-Pierre viability/capturability algorithm applied to this specific case provides both the valuation function and the associated portfolios.

Olé E. Barndorf-Nielsen (University of Aarhus)

Recent results in the study of volatility

The advent of commonly available trade prices and quote data has led to new questions, models and tools regarding inference and forecasting of volatility. Some of the recent developments, using continuous time modelling frameworks, will be reviewed, with particular focus on the theory and applications of (realised) multipower variation and the roles of jumps and microstructure effects.

This is joint work with Neil Shephard.

Sara Biagini (Università di Perugia)

Utility maximization in a general framework and properties of the optimal solution

We develop some new aspects in the utility maximization from terminal wealth in (very) general, incomplete, market models. In a previous article of ours (Biagini and Frittelli, *Utility maximization in incomplete markets for unbounded processes* Fin. and Stoch., forthcoming) we extended the existing theory to cover the case where:

- $u : \mathbb{R} \to \mathbb{R}$ is a strictly concave, regular utility function;
- the underlying semimartingale X *can be possibly non locally bounded.*

The extension relies upon a new definition of admissible strategies, for agents who are willing to take more risk. Hence we built a perfectly sensible utility maximization problem and we showed that the optimal claim $\widehat{f}$ admits an integral representation as soon as the minimax measure is equivalent to P. Namely, $\widehat{f} = (\widehat{H} \cdot X)_T$.

Unfortunately, the strategy $\widehat{H}$ which leads to the terminal optimal wealth may not be admissible, even in our wider sense. This phenomenon is not surprising, as it appears also in the locally bounded case.

So, we investigate on the properties of the optimal process $\widehat{H} \cdot X$. We prove that $\widehat{H} \cdot X$ is in fact a *supermartingale* (*true martingale* if the utility is exponential) with respect to the relevant pricing measures in our general setting, *i.e.* the *σ-martingale* measures for X with finite entropy.
This result can be seen as the fourth step in the following path:

1. Six Authors' paper 2002. When X is locally bounded, the utility is exponential and a technical condition holds (the reverse Hölder inequality), they proved that the optimal wealth process is a *true martingale* with respect to every local martingale measure Q with finite entropy.
2. Kabanov and Stricker 2002 removed the superfluous reverse Hölder inequality.

3. Schachermayer 2003 proved that if $\widehat{Q} \sim P$, then $\widehat{H} \cdot X$ is a *supermartingale* under every local martingale measure with finite entropy (the *true martingale* property of the solution is lost when u is general).

Then the supermartingale (martingale in case of exponential u) property of the optimal portfolio continues to hold even in the general, possibly non locally bounded, case.

This is joint work with M. Frittelli.

Nicolas Bouleau (**ENPC Paris**)

Dirichlet forms methods in finance

The lecture presents some recent and unpublished advances on Dirichlet forms methods for studying error propagation and sensitivity analysis in stochastic calculus with a special focus on financial models. We will recall, in a first part, the main lines of the theory of error structures as constructed in the book [1]. Our starting point will be to display by simulation the propagation of errors though some dynamical systems and its axiomatization thanks to Dirichlet forms in the spirit of some ideas of Gauss at the beginning of the nineteenth Century. The superiority of Dirichlet forms techniques with respect to other methods for studying the propagation of errors stands in the closedness property of Dirichlet forms, which plays in this theory a similar role to σ-additivity in probability theory. It allows propagating errors not only through closed formulas but also through limit objects such as stochastic integrals etc. In this rigorous mathematical framework of error structures, we shall develop the tools of the Lipschitzian functional calculus, of finite and infinite products, of the gradient operator and of integration by parts formulas. Then we will give an outlook on error structures on the Monte Carlo space, on the general Poisson space but with a special emphasis on the construction of error structures on the Wiener space. We end this part by explaining the links between error structures and statistical data thanks to the Fisher information. A recent very significant result makes a connection between the discrete approach and the continuous time approach, often used in finance. It is the Donsker theorem on the convergence in law of a random walk to the Brownian motion extended to the case where the random walk is the sum of centered i.i.d. random variables which are also erroneous. At the limit is obtained the Ornstein-Uhlenbeck structure as error structure on the Wiener space. This quite natural result had not been yet proved. It is based on a tricky extension of the Donsker theorem to functions with quadratic growth. This new invariance principle allows to pass from discrete time to continuous time for some results on the Malliavin's gradient of the uniform norm obtained by Nualart and Vives [8]. For operational computations in finance the most useful recent result is certainly that the asymptotic error due to solving a stochastic differential equation by the Euler scheme (cf. [7], [6]) may be represented by an error structure on the Wiener space. It is an Ornstein-Uhlenbeck structure with a stochastic adapted weight. This structure may be shown to be closable under usual hypotheses. It allows therefore to propagate the error due to the Euler scheme through financial models like pricing and hedging models with local volatility. The results obtained up to now deal only with the variance of the error (the squared field operator of the structure). About the bias of the error, we can say that its propagation meets serious mathematical difficulties which are not yet overcome.

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René Carmona (**University of Princeton**)

Energy trading: new challenges in financial mathematics

Due to the physical nature of commodities, commodities trading is necessarily very different from that of equities. The first part of the lecture will emphasize these differences by describing the mechanics of the forward markets and the misleading similarities with the fixed income markets and by explaining the role of weather, physical storage and convenience yield.

The second part of the lecture will concentrate on energy trading and especially electricity trading in deregulated markets. We will discuss the fundamental role of the "stack" in price formation and we will give statistical evidence for the limitations of the Black-Scholes theory in this context. The lecture will conclude with a review of the mathematical challenges posed by some of the most popular instruments. These include spread and swing options, plant valuation, tolling agreements and gas storage.

Rama Cont (**École Polytechnique, France**)

Parameter uncertainty in diffusion models

Option pricing with diffusion models requires the specification of a functional diffusion coefficient -the "local volatility function". In the limit case where prices of call options are observed for a continuum of strikes and maturities, Dupire (1994) showed how to recover the local volatility function by an inversion formula. In practice, however, only a finite number of option prices are observed and the local volatility function cannot be completely identified. This leads to two issues:

1. how can one assess the uncertainty on the local volatility function, given a (finite) set of option prices?

2. How does this uncertainty affect the option prices and the corresponding hedge ratios?

We answer both of these questions using a probabilistic approach. To answer 1) using a Bayesian point of view, we start from a prior distribution on the local volatility function and a set of option prices and propose a particle method to update the prior and simulate a sample of local volatility functions which are calibrated to the observed option prices. Using a "propagation of chaos" argument allows to analyze the uncertainty on the local volatilities using the dispersion of the sample. The estimated local volatility function is then a random field. To answer the second question, we estimate the rate of convergence of option prices, which are computed as solutions of a parabolic PDE with random coefficients, to those of the underlying model. We show that the impact of parameter uncertainty on option prices is related to the estimation error on the inverse of the squared diffusion coefficient and give some examples where this result can be useful.

This is joint work with Sana Benhamida and Moeiz Rouis.

José Manuel Corcuera (**Universitat de Barcelona**)

Power variation of some integral long-memory processes

In this work we consider a process of the form $\int_0^t u_s\,dB_s^H$, where B^H is a fractional Brownian motion with Hurst parameter $H > \frac{1}{2}$, and u is a stochastic process with paths of finite q-variation, $q < \frac{1}{1-H}$. The integral is a pathwise Riemann-Stieltjes integral. We study the asymptotic behavior of the realized power variation:

$$\xi_t^{(n)} = n^{-1+pH} \sum_{i=1}^{[nt]} \left| \int_{(i-1)/n}^{i/n} u_s\,dB_s^H \right|^p .$$

We establish the convergence in probability of the random variable $\xi_T^{(n)}$ to the random variable

$$\xi_T = E(|B_1^H|^p) \int_0^T |u_s|^p\,ds.$$

Also we obtain the convergence in distribution of the fluctuations $\sqrt{n}(\xi_t^{(n)} - \xi_t)$ to a process of the form $v_1 \int_0^t |u_s|^p\,dW_s$, where W is a Brownian motion independent of the fractional Brownian motion B^H. This result holds if $H \in (1/2, 3/4)$, and it is a stable convergence in $\mathcal{D}([0,T])$. For $H = 3/4$ a similar result can be obtained but with an additional normalizing factor equal to $(\ln n)^{-1/2}$. To prove these results we make use of a CLT for functionals of a stationary Gaussian sequence whose correlation function decreases slowly to zero (see, for instance, [1]). This kind of central limit theorem is also obtained by a direct argument from a general convergence result for multiple stochastic integrals proved in [2], [3]. For $H > 3/4$, the problem is more involved because non-central limit theorems are required and in this case we consider u_t constant.

This is joint work with David Nualart and Jeannette H. C. Woerner.

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Ernst Eberlein (**Universität Freiburg**)

Symmetries and pricing of exotic options in Lévy models

We discuss symmetry relations for option pricing formulas in the context of models driven by Lévy processes. The following types of options are considered: floating and fixed strike Asian and Lookback options, power call and put options as well as Margrabe and Quanto call options.

This is joint work with Antonis Papapantoleon.

Hélyette Geman (**ESSEC and Université de Paris-Dauphine**)

From local volatility to local Lévy models I

Local volatility models have existed for more than a decade and exhibit quite interesting features and limits as well. We define here a class of local Lévy processes. These are Lévy processes time changed by an inhomogeneous local speed function. The local speed function is a deterministic function of time and the level of the process itself. We show how to reverse engineer the local speed function from traded option prices of all strikes and maturities. The local Lévy processes generalize the class of local volatility models. Closed forms for local speed functions in a variety of cases are also presented.

This is joint work with Peter Carr, Dilip Madan and Marc Yor.

Paolo Guasoni (Boston University)

No arbitrage under transaction costs and small ball probabilities

We establish a criterion for the absence of arbitrage opportunities in a market where the asset price follows a continuous-time stochastic process, and proportional transaction costs are present. This criterion essentially requires that the log-price process has strictly positive small ball probabilities over arbitrary stochastic intervals.

We then verify this criterion for different classes of log-price processes: Markov process with regular points, continuous self-similar processes with stationary increments, and processes supported on the entire Wiener space.

In particular, we show that proportional transaction costs of any size eliminate arbitrage opportunities from a market where log prices follow fractional Brownian motion.

Arturo Kohatsu-Higa (INRIA Rocquencourt)

Another approach to proving weak convergence

In this talk we present joint work with E. Clement and D. Lamberton where we consider a modification of the usual method to prove weak approximations results for the Euler scheme. This modification gives a simple proof of V. Bally and D. Talay's results on approximations for densities of diffusions. Furthermore the present method can be used for non-Markovian systems.

Furthermore we discuss some extensions to Lévy processes. This second part is joint work with N. Yoshida.

François LeGland (IRISA/INRIA Rennes)

Filtering a diffusion process observed in singular noise

We consider a nonlinear filtering problem in which a diffusion process is observed in additive noise, modelled as a Brownian motion with degenerate covariance matrix. For simplicity, we consider the special extreme case where observations are noise–free, and we also assume that the quadratic variation of the observation process does not bring any additional information that is not already present in the observation process itself. As long as the observations are regular values of (a function related to) the observation function, we derive an equation for the density (with respect to the canonical Lebesgue measure on the corresponding level set) of the conditional probability distribution of the state, given the past history of the observation process. This equation can be seen as a degenerate SPDE on the whole state space, or as a nondegenerate SPDE on a submanifold of reduced dimension. The proof is based on the idea of decomposition of solutions of SDE's, as introduced by Kunita. Two numerical approximation schemes based on interacting particle systems are presented. The first scheme follows the approach proposed by Del Moral, Jacod and Protter: in the mutation step, independent trajectories of the pair (hidden) diffusion process and observation process are simulated jointly, and each pair is validated in the selection step against the current observation process. It may very well happen that all the simulated pairs are rejected, in which case the sequential particle algorithm introduced by LeGland and Oudjane could be used, which automatically keeps the particle system alive. The second scheme corresponds to the second interpretation of the filter equation as a nondegenerate SPDE on a submanifold of reduced dimension: in the mutation step, independent trajectories of a conditional diffusion process on a submanifold are simulated, which are multiplied/discarded in the selection step according the value of the likelihood function.

Dilip Madan (**University of Maryland**)

From local volatility to local Lévy models II

We define the class of local Lévy processes. These are Lévy processes time changed by an inhomogeneous local speed function. The local speed function is a deterministic function of time and the level of the process itself. We show how to reverse engineer the local speed function from traded option prices of all strikes and maturities. The local Lévy processes generalize the class of local volatility models. Closed forms for local speed functions for a variety of cases are also presented. Numerical methods for recovery are also described.

This is joint work with Peter Carr, Hélyette Geman and Marc Yor.

Paul Malliavin (**Académie des Sciences, Paris**)

Non parametric statistics on market evolution

Processes associated to the risk-free measure have drift which vanishes or which is computable from the volatility matrix. The volatility matrix can be estimated pathwise by using non-parametric statistics. Therefore it is possible to estimate non-parametrically and pathwise the infinitesimal generator of the risk-free process. Furthermore, by Itô calculus, the derivatives of any function can be estimated pathwise from their observation along the path of the diffusion. As consequence, the Greek Delta can be estimated non-parametrically pathwise.

A new market liquidity indicator, the feedback volatility rate, is defined and its non-parametric estimation is realized. In an interest rate model, a pathwise non-parametric estimation of the separation between maturities can be constructed.

Franco Moriconi (**Università di Perugia**)

The no-arbitrage approach to embedded value and embedded options valuation in life insurance. An application to real life portfolio

It is well-known that traditional life insurance policies contain financial guarantees in the form of a minimum investment return guaranteed to the policyholder in each year of the contract. In most cases these guarantees are equivalent to long-term cliquet options sold from the insurer to the policyholder at the issuance of the policy and then embedded in the outstanding portfolios. Given the almost continuous interest rates decrease in the last decade, many of the minimum guarantee options embedded in the policy portfolios of European insurance companies are currently near-at-the-money (or have just gone in the money).

If an option pricing theory approach is used the cost of minimum guarantees that can result may be very high, substantially increasing the value of the outstanding liabilities of an insurance company and then sensibly reducing the value of the firm. This effect is usually expressed as a corresponding reduction of the so-called Embedded Value (EV) which the major companies usually compute as a measure of future profits generated by the in-force policies. Until recently however there was no general consensus on if and how the embedded options have to be valued.

In May 2004 the CFO Forum –a discussion group attended by the Chief Financial Officers of major European insurance companies– agreed to adopt "European Embedded Value Principles" in order to provide an international guidance on the implementation of EV reporting. As concerning the embedded options, the Principles prescribe that starting from 2006 the European insurance companies should provide EV measures reduced by the cost of financial options and guarantees (Principle 6) and that this cost should be derived by "stochastic techniques consistent with the methodology and assumptions used in the underlying embedded value" (Principle 7).

This paper illustrates how these requirements have been met by a leading group of Italian insurance companies in its EV disclosure on March 2005, thus anticipating by about one year the CFO Forum deadline. All the outstanding policy portfolios of the group and the related investment portfolios have been analysed. The financial component of the EV and the cost of the embedded options have been derived using an arbitrage pricing model under interest rates and stock price uncertainty. The model has been calibrated on market data at the valuation date and the "Value of Business In Force" (VBIF), net and gross of embedded options, has been derived by Monte Carlo methods simulating future profits under the risk-neutral probability measure. In generating the annual profits from the outstanding asset-liability portfolios both the investment strategy chosen at time zero by the insurer and the details of the accounting rules determining the contractual return on investments have been taken into account. As prescribed by the CFO Forum a time-value/intrinsic-value splitting of the price of embedded options has been derived.

From practitioners' point of view, an interesting open issue is how the results obtained with a risk-adjusted probability approach can be interpreted in terms of the risk-adjusted discounting approach which is traditional in capital budgeting. An example of a possible harmonization between the two methods has been proposed in the disclosure.

Marek Musiela (BNP Parisbas, London)

Dynamic risk preferences and optimal behaviour

We examine the optimal behaviour of investors whose risk preferences exhibit time-varying characteristics. Allowing for a dynamic risk preference structure leads to the development of a concise methodology for optimal investments and quantification of value. This approach is applicable to complete and incomplete markets and also provides a natural connection to the theory of indifference pricing

Bernt Øksendal (University of Oslo)

The value of information in stochastic control and finance

We present a general mathematical model for anticipative stochastic control/optimal stopping, based on Malliavin calculus and stochastic forward integrals. We use this to study stochastic control problems and optimal stopping problems for a financial trader with inside information about the market (e.g. information about the future values of the stocks), and we compare his/her value function with the corresponding value function for an honest trader (*i.e.* a "non-anticipating" trader, with no inside information about the future). The difference between the two value functions may be regarded as the value of the additional information available to the insider.

Maurizio Pratelli (Università di Pisa)

Generalizations of "Merton's mutual fund theorem" in infinite dimensional financial models

The original proof of the celebrated "Merton's mutual fund theorem" is based on stochastic control methods (solution of an Hamilton-Jacobi-Bellmann equation): in this talk, I will show how an easy proof of this result can be given with "stochastic calculus" methods (representation of martingales in a Brownian filtration). This method can be applied to infinite dimensional situations: the so called "large financial markets" (where a sequence of assets is taken into account) and "bond market models" (where there is a continuum of assets). The talk will insist on related infinite dimensional stochastic integration problems.

Wolfgang Johann Runggaldier (Universitá di Padova)

On portfolio optimization in discontinuous markets and under incomplete information

We consider the problem of portfolio optimization in a multi-asset market where prices evolve along purely discontinuous trajectories according to

$$dS_t^i = S_{t-}^i \left[\sum_{j=1}^{M} (e^{a_{ij}-1})\, dN_t^j \right] , \quad i = 1, \ldots, N\,; \;\; M \geq N.$$

The driving Poisson jump processes N_t^j are independent and their intensities may be stochastic processes themselves (doubly stochastic Poisson) and they may not be directly observable (incomplete information).

We analyze the log- and power-utility cases showing that, for the given class of models, a certainty equivalence property holds not only, as expected, for the log-utility but also for a power utility. We furthermore discuss the actual computation of an optimal investment strategy.

This a joint work with G. Callegaro and Giovanni Battista Di Massi.

Wolfgang M. Schmidt (HfB - Business School of Finance, Frankfurt)

Modeling default dependence and pricing credit baskets

The credit derivatives business has seen a dramatic growth over the last decade. Credit default swaps (CDS) are the dominating plain-vanilla credit derivative product which serves also as a building block for credit linked notes and other synthetic credit investments. A credit default swap is a derivative contract that offers protection against default of a certain underlying entity over a specified time horizon. A premium, the CDS spread, s is paid on a regular basis (e.g., on a semi-annual, act/360 basis) and on a certain notional amount N as an insurance fee against the losses from default of a risky position of notional N, e.g., a bond. The payment of the premium s stops at maturity or at the random time τ of default of the underlying credit, whichever comes first. At the time of default before maturity of the trade the protection buyer receives the payment $N(1-R)$, where R is the recovery rate of the underlying credit risky instrument.

More advanced credit derivative products are linked to several underlying credits $i = 1, \ldots, n$ and the payoff is a function $P(\tau_1, \ldots, \tau_n)$ of the default times τ_i of the involved credits. Examples are basket default swaps, synthetic CDOs or default swaps on certain tranches of losses from a portfolio. What is common to these basket derivative products is that their modelling and pricing requires a model on the dependencies between the underlying credits.

The most important inputs for any credit derivative pricing model are the market observed fair CDS spreads $s^i(0,T)$ for (in principle) all maturities T and all credits i. From these spread curves one can back out the market implied (risk neutral) distributions of the default time, $F_i(t) = \mathbb{P}(\tau_i < t)$, $t \geqslant 0$. Now, by the general no-arbitrage pricing principle, the valuation of a multi-credit derivative with payoff $P(\tau_1, \ldots, \tau_n)$ at time T calls for calculating the risk-neutral expectation

$$\mathbb{E}\left(\exp(-\int_0^T r_s\, ds) P(\tau_1, \ldots, \tau_n) \right), \tag{31}$$

with (r_t) as the riskless short rate. However this requires a model for the joint distribution

$$\mathbb{P}(\tau_1 < t_1, \ldots, \tau_n < t_n)$$

of the default times, where the marginal distributions F_i are given by the "market". A common technique in practice is to link the marginal distribution assuming a certain copula. The calculation of the expectation above is then done either by Monte-Carlo simulation, or, in case of certain low-factor dependencies, by quasi-analytical methods.

In this talk we start by investigating the relationship between the dynamics of the fair CDS spread $s(t,T)$ as well as prices $V(t,T)$ of credit default swaps on one hand and important quantities related to the conditional distribution of the default time τ on the other hand.

For the purpose of hedging a complex credit derivative by CDS we then introduce new securities based on strategies which discretely and continuously rebalance the CDS position to be fair. It turns out that these new securities are very convenient for approaching the problem of hedging a basket credit derivative.

There are two sources of risk to hedge against. The first one is the so-called spread risk, which is the risk that the market quoted fair CDS spreads $s^i(t,T)$ change as time t evolves. A changed spread impacts the distribution of the respective default time and thus the joint distribution and the mark-to-market valuation of the considered basket derivative. The spread risk is thus the risk of changing default probabilities without an actual default of this name having occurred. The second source of risk is the so-called default risk, which is the impact of an actual default on the basket derivative contract. Both sources of risk have to be hedged simultaneously.

Analyzing both sources of risk simultaneously requires a model that goes beyond the joint distribution of the default times which covers just a static snapshot at time $t = 0$. As time t evolves, the flow of information and the stochastic modelling of the actual defaults as well as the stochastic dynamics of the market observables are essential ingredients of the model that determine the hedging strategies. One important quantity in the dynamics of the CDS spreads $s^i(t,T)$ that measures dependencies is the impact of default of one of the credits on the spreads of the remaining ones. Given the copula describing the joint distribution one can derive the conditional distribution $\mathbb{P}(\tau_i > t|\tau_j = t)$ and the fair spread for credit i after the occurrence of the default of credit j.

In the special case of a pure jump filtration we present an elegant and very efficient approach to the pricing of credit baskets. We derive an easily solvable system of equations for the integrands in the hedge representation of a basket product. Given default induced spread jumps, *i.e.*, the functional dependence of the default intensities of each credit on the default times of the others, we show existence of such a model.

Agnès Sulem (INRIA - Rocquencourt)

Utility maximization in an insider influenced market

We study a controlled stochastic system whose state is described by a stochastic differential equation with anticipating coefficients. This setting is used to model markets where insiders have some influence on the dynamics of prices. We give a characterization theorem for the optimal logarithmic portfolio of an investor with a different information flow from that of the insider. We provide explicit results in the partial information case which we extend in order to incorporate the enlargement of filtration techniques for markets with insiders. Finally, we consider a market with an insider who influences the drift of the underlying price asset process. This example gives a situation where it makes a difference for a small agent to acknowledge the existence of an insider in the market.

This is joint work with A. Kohatsu-Higa.

Esko Valkeila (Helsinki University of Technology)

Asymmetric information – a Bayesian approach

We recall how the Girsanov theorem and dynamical Bayesian modelling are related to initial enlargement of filtration. The Bayesian approach allows us to deal with an unified approach initial enlargement, *i.e.* strong insider information, and the weak insider information introduced by Baudoin.

Tiziano Vargiolu (Università di Padova)

Robustness of the Hobson-Rogers model

In the Hobson-Rogers model, the evolution of the price of the risky asset depends not only on the current value but also on the values assumed in all the past history, via the so-called *offset function*. Though in principle it is possible to reduce the model to a Markov system with more state variables than the number of risky assets, in practice one can observe the history of the price process only in a finite time window $[-R, 0]$, thus the initial condition for the offset function is misspecified. In this paper we show how this misspecification affects the prices of the derivative assets, and we give bounds for R if one wants the error to be smaller than a certain threshold. We finally give some numerical examples in three particular specifications of the volatility.

This is joint work with Vera Hallulli and Alessandro Platania.

Jochen Wolf (BaFin, Bonn)

On the valuation of the interest rate guarantee in with-profit life insurance

The interest rate guarantee may be viewed as an embedded option in life insurance contracts. For with-profit contracts the life insurance company can influence the value of this guarantee by adjusting its strategy of profit sharing.

Adopting a combined view of biometrical and financial risk in life insurance we investigate the value of the interest rate guarantee depending on different strategies of profit sharing.

LIST OF PARTICIPANTS

Daniel Andersson
Department of Mathematics
Royal Institute of Technology
S-10044 Stockholm
Suède
daniean@math.kth.se

Jean-Pierre Aubin
Université de Paris Dauphine
14, rue Domat
F-75005 Paris cedex 16
France
J.P.Aubin@wanadoo.fr

Khaled Bahlali
CPT-Toulon, CNRS
Univ. du Sud-Toulon-Var
B.P. 20132
F-83957 La Garde Cedex
bahlali@univ-tln.fr

Olé E. Barndorff-Nielsen
Department of Mathematical Sciences
University of Aarhus
Ny Munkegade
DK-8000 Aarhus
Denmark
oebn@imf.au.dk

Claas Becker
Deutsche Bank AG
Taunusanlage 12
D-60325 Frankfurt
Germany
claas.becker@db.com

Mohamed Ben Alaya
Institut Galilée, Mathématiques
Université de Paris 13
99 avenue Jean Baptiste Clément
F-99430 Villetaneuse
France
mba@math.univ-paris13.fr

Michel Benaïm
Faculté des Sciences, Mathématiques
Université de Neuchâtel
Rue Emile-Argand 11
CH-2007 Neuchâtel
Switzerland
michel.benaim@unine.ch

Violetta Bernyk
Institut de mathématiques
EPF-Lausanne
Station 8
1015 Lausanne
Switzerland
violetta.bernyk@epfl.ch

Hakima Bessaih
Department of Mathematics
University of Wyoming
Ross Hall 210
Laramie WY 82071
U.S.A.
bessaih@uwyo.edu

Sara Biagini
Dipartimento Economia Sezione Finanza
Università di Perugia
Via A. Pascoli 20
I-06123 Perugia
Italy
s.biagini@sns.it

Philippe Blanchard
Fakultät für Physik
Universität Bielefeld
Universitätsstr. 25
D-33615 Bielefeld
Germany
blanchard@physik.uni-bielefeld.de

Stefano Bonaccorsi
Dipartimento di Matematica
Università di Trento
Sommarive 14
I-38050 Povo-Trento TN
Italy
bonaccor@science.unitn.it

Brahim Boufoussi
Semlalia Fac. of Sciences, Mathematics
Cadi Ayyad University
P.B.O 2390 Marrakesh
Morocco
boufoussi@ucam.ac.ma

Nicolas Bouleau
Annales des Ponts
ENPC Paris
28, rue des Saints Peres
F-75013 Paris
France
bouleau@mail.enpc.fr

René Carmona
Bendheim Center for Finance
Princeton University
ORFE
Princeton NJ 08544
U.S.A.
rcarmona@princeton.edu

Fabienne Castell
LATP-CMI
Université de Provence
39, rue F. Joliot-Curie
F-13453 Marseille cédex 13
France
castell@cmi.univ-mrs.fr

Sandra Cerrai
DIMADEFAS
Università di Firenze
Via C. Lombroso, 6/17
I-50134 Firenze
Italy
sandra.cerrai@dmd.unifi.it

Serge Cohen
Laboratoire de Statistique et Probabilités
Université Paul Sabatier
118, route de Narbonne
F-31062 Toulouse Cedex 4
France
scohen@cict.fr

Rama Cont
Centre de mathématiques appliquées
Ecole Polytechnique
91128 Palaiseau Cedex
France
rama.cont@polytechnique.fr

Daniel Conus
Institut de mathématiques
EPF-Lausanne
Station 8
1015 Lausanne
Switzerland
daniel.conus@epfl.ch

Jose-Manuel Corcuera
Facultat de Matematiques
Universitat de Barcelona
Gran Via 585
08014 Barcelona
Spain
jmcorcuera@ub.edu

Laure Coutin
Laboratoire de Statistique et Probabilités
Université Paul Sabatier
118, route de Narbonne
F-31062 Toulouse cedex 4
France
coutin@cict.fr

Rosanna Coviello
Institut Galilée, Mathématiques
Université de Paris 13
99, av. JB Clément
F-93430 Villetaneuse
France
coviello@sns.it

Jacky Cresson
Bureau B 403-UFR Sciences Techniques
Université de Franche-Comté
16, route de Gray
F-25000 Besançon
France
cresson@math.univ-fcomte.fr

Ana Bela Cruzeiro
GFM and Dep. Mathematics
IST Lisbon
Av. Rovisco Pais
P-1049-001 Lisboa codex
Portugal
abcruz@math.ist.utl.pt

Giuseppe Da Prato
Scuola Normale Sup. di Pisa
P. dei Cavalieri 7
I-56126 Pisa
Italy
daprato@sns.it

Robert C. Dalang
Institut de Mathématiques
EPF-Lausanne
Station 8
CH-1015 Lausanne
Switzerland
robert.dalang@epfl.ch

Sébastien Darses
Université de Franche-Comté
13, rue Renau
F-25000 Besançon
France
darses@math.univ-fcomte.fr

Victor de la Pena
Department of Statistics
Columbia University
618 Math Building - Box 10
New York NY 10027
U.S.A.
vp@stat.columbia.edu

Latifa Debbi
Institut Elie Cartan, Dept. de Math.
Université Henri Poincaré Nancy I
BP 239
F-54506 Vandoeuvre-les-Nancy Cedex
France
ldebbi@yahoo.fr

Marco Dozzi
Département de Mathématiques
Université de Nancy II
B.P. 239
F-54506 Vandoeuvre-les-Nancy
France
marco.dozzi@iecn.u-nancy.fr

Ernst Eberlein
Institut für Mathematische Stochastik
Universität Freiburg
Eckerstr. 1
D-79104 Freiburg
Germany
eberlein@stochastik.uni-freiburg.de

Kenneth David Elworthy
Maths. Institute
University of Warwick
Coventry CV47AL
United Kingdon
kde@maths.warwick.ac.uk

Franco Flandoli
Dipartimento di Matematica Applicata
Università di Pisa
Via Bonanno 25 bis
I-56126 Pisa
Italy
flandoli@dma.unipi.it

Hélyette Geman
Finance Department
ESSEC + Dauphine
Av. Bernard Hirsch
F-95021 Cergy Pontoise
France
geman@essec.fr

Mihaï Gradinaru
Institut Elie Cartan
Université de Nancy I
BP 239
F-54506 Vandoeuvre-les-Nancy Cedex
France
mihai.gradinaru@iecn.u-nancy.fr

Martin Grothaus
Mathematics Department
Universität Kaiserslautern
P.O. Box 3049
D-67653 Kaiserslautern
Germany
grothaus@mathematik.uni-kl.de

Paolo Guasoni
Department of Maths. and Stats.
Boston University
111 Cummington St.
Boston MA 02215
U.S.A.
guasoni@bu.edu

Massimiliano Gubinelli
Dip. di Matematica Applicata "U. Dini"
Università di Pisa
Via Bonanno 25/B
I-56125 Pisa
Italy
m.gubinelli@dma.unipi.it

Zbignew Haba
Institute of Theoretical Physics
University of Wroclaw
Pl. Maxa Borna 9
P-50-205 Wroclaw
Poland
zhab@ift.uni.wroc.pl

Astrid Hilbert
Matematiska och systemtek. inst. (MSI)
Växjö Universitet
Vejdesplats 7
S-351 95 Växjö
Sweden
astrid.hilbert@msi.vxu.se

Yueyun Hu
Institut Galilée, Mathématiques
Université de Paris 13
99, av. J.-B. Clément
F-93430 Villetaneuse
France
yueyun@math.univ-paris13.fr

Thierry Huillet
Lab. de phys. théorique et modélisation
Univ. de Cergy Pontoise
Site de St-Martin, 2 av. Adolphe-Chauvin
F-95302 Cergy-Pontoise
France
thierry.huillet@ptm.u-cergy.fr

Davar Khoshnevisan
Department of Mathematics
University of Utah
155 South 1400 East JWB 233
Salt Lake City UT 84112-009
U.S.A.
davar@math.utah.edu

Tomasz Komorowski
University of Lublin
Kiepury 11/113
20-838 Lublin
Poland
komorow@hektor.umcs.lublin.pl

Ida Kruk
Université de Paris 13
72, rue de Meyrin
F-01210 Ferney Voltaire
ida_kruk@yahoo.com

Rémi Léandre
Institut de Mathématiques de Bourgogne
Université de Bourgogne
B.P. 47870
F-21078 Dijon Cedex
France
remi.leandre@u-bourgogne.fr

François LeGland
IRISA Rennes
Campus de Beaulieu
F-35042 Rennes Cedex
France
legland@irisa.fr

Paul Lescot
INSSET
Université de Picardie
48, rue Raspail
F-02100 Saint-Quentin
France
paul.lescot@insset.u-picardie.fr

Hannelore Lisei
Faculty of Mathematics and Comp. Sci.
Babes-Bolyai University
Str. Kogalniceanu Nr. 1
RO-400084 Cluj-Napoca
Romania
hannelore.3@gmx.net

Eva Lütkebohmert
Universität Bonn
Neefestr. 11
D-53115 Bonn
Germany
eva.lubo@gmx.de

Dilip Madan
Smith School of Business
University of Maryland
College Park
MD 20742
U.S.A.
dmadan@rhsmith.umd.edu

Paul Malliavin
Académie des Sciences Paris
10, rue St. Louis en l'Ile
F-75004 Paris
France
sli@ccr.jussieu.fr

Wolfgang Marty
Crédit Suisse Asset Management
Uetlibergstrasse 231
8070 Zurich
wolfgang.j.marty@csam.com

Ravi R. Mazumdar
Dept. of Electrical and Computer Eng.
University of Waterloo
200 University Ave. W
Waterloo Ont. N2L 3G1
Canada
mazum@ece.uwaterloo.ca

Sabrina Mulinacci
Ist. di Econometria e Matematica
Universita Cattolica di Milano
Via Necchi 9
I-20123 Milano
Italy
sabrina.mulinacci@unicatt.it

Edwin Perkins
Department of Mathematics
Univ. of British Columbia
121-1984 Mathematics Road
Vancouver B.C. V6T 1Y2
Canada
perkins@math.ubc.ca

Sylvie Méléard
MODAL'X
Université de Paris X
200, av. de la République
F-92000 Nanterre
France
sylvie.meleard@u-paris10.fr

Marek Musiela
BNP Paribas, London
10 Harewood Avenue
London NW 1
UK
marek.musiela@bnpparibas.com

Francisco Piera
School of Electrical and Computer Eng.
Purdue University
West Lafayette IN 47907
U.S.A.
fpieraug@purdue.edu

Danillo Merlini
CERFIM
Casella Postale 2004
Via F. Rusca 1
6600 Locarno
Switzerland

Ivan Nourdin
Institut Elie Cartan
Université de Nancy 1
BP 239
F-54506 Vandoeuvre-lès-Nancy Cedex
nourdin@iecn.u-nancy.fr

Maurizio Pratelli
Dipartimento di Matematica
Università di Pisa
Via Buonarroti 2
I-56127 Pisa
Italy
pratelli@dm.unipi.it

Annie Millet
SAMOS Centre Pierre Mendès France
Université Paris 1
90, rue de Tolbiac
F-75634 Paris Cedex 13
France
amil@ccr.jussieu.fr

Eulalia Nualart
Institut Galilée, Mathématiques
Université de Paris 13
99, av. J.B. Clément
F-93430 Villetaneuse
France
nualart@math.univ-paris13.fr

Nicolas Privault
Département de Mathématiques
Université de La Rochelle
Avenue Michel Crépeau
F-17042 La Rochelle Cedex 1
France
nicolas.privault@univ-lr.fr

Oana Mocioalca
Department of Mathematical Sciences
University of Kent
P.O. Box 5190
Kent OH 44242
U.S.A.
oana@math.kent.edu

Bernt Oksendal
Department of Mathematics
University of Oslo
PO Box 1053 Blindern
N-0316 Oslo
Norway
oksendal@math.uio.no

Bernard Prum
Labo Statistiques et Génome
Génopôle Evry
523 Place des Terrasses
F-91000 Evry
France
prum@genopole.cnrs.fr

Franco Moriconi
Facoltà di Economia e Commercio
Università di Perugia
Via Pascoli 1
I-06100 Perugia
Italy
moriconi@unipg.it

Etienne Pardoux
LATP-CMI
Université de Provence
39 rue F. Joliot-Curie
F-13453 Marseille cedex 13
France
etienne.pardoux@cmi.univ-mrs.fr

Michael Röckner
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131
D-33501 Bielefeld
Germany
roeckner@mathematik.uni-bielefeld.de

Arturo Rodriguez
Department of Mathematics
Stockholm University
106-91 Stockholm
Sweden
arturo@math.su.se

Bruno Saussereau
Département de Mathématiques
Faculté des Sci. et Tech. de Besançon
F-25030 Besançon
France
bruno.saussereau@univ-fcomte.fr

Isabel Simao
CMAF-UL
Universidade de Lisboa
Ava. Prof. Gama Pinto, 2
P-1649-003 Lisboa Codex
Portugal
isimao@ptmat.fc.ul.pt

Bernard Roynette
Institut Elie Cartan
Université Henri Poincaré Nancy 1
BP 239
F-54506 Vandoeuvre-les-Nancy Cedex
France
roynette@iecn.u-nancy.fr

Giacomo Scandolo
Dip. di Matematica per le Decisioni
Università di Firenze
via Lombroso 6/17
50134 Firenze
Italy
giacomo.scandolo@unifi.it

Wilhelm Stannat
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131
D-33501 Bielefeld
Germany
stannat@mathematik.uni-bielefeld.de

Barbara Ruediger
Mathematisches Instiut
Universität Koblenz -Landau
Campus Koblenz; Universitaetsstr. 1
D-56070 Koblenz
Germany
ruediger@uni-koblenz.de

Michael Scheutzow
Fak. II, MA 7-5
Technische Univ. Berlin
D-10623 Berlin
Germany
ms@math.tu-berlin.de

Christophe Stricker
Faculté des Sciences
Université de Franche-Comté
16, route de Gray
F-25030 Besançon Cedex
France
stricker@math.univ-fcomte.fr

Wolfgang J. Runggaldier
Dipart. di Matematica Pura e Applicata
Università di Padova
Via Belzoni 7
I-35131 Padova
Italy
runggal@math.unipd.it

Wolfgang Schmidt
Business school of finance & manag.
HfB, Frankfurt
Sonnemannsstrasse 9-11
D-60314 Frankfurt am Main
Germany
schmidt@hfb.de

Karl-Theodor Sturm
Universität Bonn
Wegelerstrasse 6
53125 Bonn
Germany
sturm@uni-bonn.de

Francesco Russo
Institut Galilée, Mathématiques
Université de Paris 13
99, av. JB Clément
F-93430 Villetaneuse
France
russo@math.univ-paris13.fr

Jürgen Schmiegel
Department of Mathematical Sciences
Aarhus University
DK-8000 Aarhus
Denmark
schmiegl@imf.au.dk

Agnès Sulem
INRIA - Rocquencourt
Rocquencourt research unit B.P. 105
F-78153 Le Chesnay Cedex
France
agnes.sulem@inria.fr

Marta Sanz-Solé
Facultat de Matemàtiques
Universitat de Barcelona
Gran Via 585
E-08028 Barcelona
Spain
marta.sanz@ub.edu

Simone Scotti
ENPC-Cermics
6, av. Blaise Pascal
F-77455 Champs s/ Marne la Vallée
France
scotti@cermics.enpc.fr

Jens Svensson
Department of Mathematics
Royal Institute of Technology
S-100 44 Stockholm
Suède
jenssve@math.kth.se

Michèle Thieullen
Laboratoire de Probabilités, Boîte 188
Université de Paris 6
4, place Jussieu
F-75252 Paris Cedex 05
France
mth@ccr.jussieu.fr

Sami Tindel
Institut Elie Cartan
Université Henri Poincaré (Nancy)
BP 239
F-54506 Vandoeuvre-les-Nancy Cedex
France
tindel@iecn.u-nancy.fr

Aubrey Truman
Dept. of Mathematics
Univ. of Wales Swansea
Singleton Park
Swansea SA2 8PP
U.K.
a.truman@swansea.ac.uk

Gerald Trutnau
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131
D-33501 Bielefeld
Germany
trutnau@mathematik.uni-bielefeld.de

Ciprian Tudor
SAMOS/MATISSE
Université de Paris 1 Panthéon-Sorbonne
90, rue de Tolbiac
F-75634 Paris cedex 13
France
tudor@ccr.jussieu.fr

Esko Valkeila
Institute of Mathematics
Helsinki Univ. of Technology
P.O. Box 1100
FI-02015 TKK
Finland
esko.valkeila@helsinki.fi

Pierre Vallois
Institut Elie Cartan, Dept. of Math.
Université Henri Poincaré Nancy I
BP 239
F-54506 Vandoeuvre-les-Nancy Cedex
France
vallois@iecn.u-nancy.fr

Tiziano Vargiolu
Dip. di Matematica Pura e Applicata
Università di Padova
via Belzoni 7
I-35131 Padova
Italy
vargiolu@math.unipd.it

Frederi G. Viens
Department of Mathematics
Purdue University
150 N. University St.
West Lafayette IN 47907-206
U.S.A.
viens@purdue.edu

Alessandro Villa
Laboratory of Neurobiophysics
Univ. J. Fourier - Grenoble 1
INSERM U318-CHUG MichallonBP217
F-38043 Grenoble cedex 9
France
alessandro.villa@ujf-grenoble.fr

Andrew B. Vizcarra
Department of Mathematics
Purdue University
150 N. University St.
West Lafayette IN 47907-206
U.S.A.
avizcarr@math.purdue.edu

John B. Walsh
Dept. of Mathematics
University of British Columbia
121-1984 Mathematics Road
Vancouver B.C. V6T 1Y4
Canada
walsh@math.ubc.ca

Jochen Wolf
Bundesanstalt für Finanzdienst.
BaFin
Georg-von-Boeselager-Str. 25
53117 Bonn
Germany
jochen.wolf@bafin.de

Moshe Zakai
Department of Electrical Engineering
Technion - Haifa
Israel Institute of Technology
Haifa 32000
Israel
zakai@ee.technion.ac.il

Lorenzo Zambotti
Dip. di Matematica
Politecnico di Milano
Piazza Leonardo da Vinci 32
I-20133 Milano
Italy
zambotti@mate.polimi.it

Jean-Claude Zambrini
Grupo de Fisica-Matematica
Universidade de Lisboa
Complexo II, Av. Gama Pinto 2
P-1649-003 Lisboa codex
Portugal
zambrini@cii.fc.ul.pt

Boguslaw Zegarlinski
Dept. of Mathematics
Imperial College Huxley Building
180 Queens Gate
London SW7 2BZ
U.K.
b.zegarlinski@imperial.ac.uk

Fifth Seminar on Stochastic Analysis, Random Fields and Applications
May 30 - June 3, 2005
Program summary

	Monday		Tuesday		Wednesday	
	Room A	Room B	Room A	Room B	Room A	Room B
8:30 - 8:40	*Opening*					
8:40 - 9:25	Truman		Perkins		Prum	
9:30 - 9:55	Röckner		Pardoux		Blanchard	
9:55 - 10:20	Da Prato		Zegarlinski		Villa	
10:50 - 11:35	Elworthy		Viens	Bénaïm	Scheutzow	
11:40 - 12:05	Vallois	Huillet	Tindel	Hu	Cerrai	Cohen
12:10 - 12:35	Roynette	Stannat	Thieullen	Zambotti	Bessaïh	Léandre
12:45 - 14:10	*Lunch*		*Lunch*		*Lunch*	
14:10 - 14:55	Méléard		Komorowski		Walsh	
15:00 - 15:25	Cruzeiro		Flandoli		Khoshnevisan	
15:25 - 15:50	Sanz		Sturm		Zakaï	
16:20 - 16:45	Millet	Privault	Mazumdar	Gradinaru	Rüdiger	de la Pena
16:50 - 17:15	Gubinelli	Bahlali	Grothaus	Nourdin	Corcuera	Zambrini
17:20 - 17:45	Coutin	Boufoussi	Trutnau	Tudor	Hilbert	Lescot
17:50 - 18:15	Bonaccorsi	Castell	Simao	Haba		
	19:30 Dinner		*19:30 Dinner*		***18:40 Bus to Reception***	

	Thursday
	Room A
8:30 - 8:40	*Opening*
8:40 - 9:25	Malliavin
9:30 - 9:55	Eberlein
9:55 - 10:20	Legland
10:50 - 11:35	Bouleau
11:40 - 12:05	Pratelli
12:05 - 12:30	Biagini
12:45 - 14:10	*Lunch*
14:10 - 14:15	*Opening*
14:15 - 15:00	Musiela
15:05 - 15:30	Wolf
15:35 - 16:00	Moriconi
16:05 - 16:30	Geman
	Public lectures
17:00 - 17:20	*Welcome*
17:20 - 17:45	P. Rossi
17:45 - 18:30	R. Carmona
	Aperitive
	20:30 Dinner

	Friday
	Room A
8:40 - 9:25	Schmidt
9:30 - 9:55	Barndorff-Nielsen
9:55 - 10:20	Cont
10:50 - 11:35	Oksendal
11:40 - 12:05	Sulem
12:05 - 12:30	Valkeila
12:45 - 14:10	*Lunch*
14:10 - 14:55	Madan
15:00 - 15:25	Guasoni
15:25 - 15:50	Stricker
16:20 - 17:05	Runggaldier
17:10 - 17:35	Vargiolu
17:35 - 18:00	Aubin

Posters : in bar Roccia, every day

Coviello	Lisei	Lütkebohmert	Nualart E.
Rodriguez	Schmiegel	Scotti	